Spin and Orbital Rotation of Electrons and Photons via Spin-Orbit Interaction:

Why electrons and photons and not so very different after all

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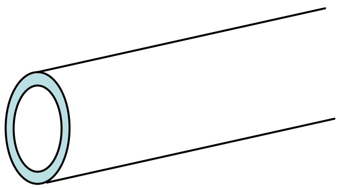
OUTLINE (elevator pitch)



1: **Spin-orbit interaction for electrons:**

-well-known to occur when an electron moves along any curved path
-must occur in an **inhomogeneous** electromagnetic potential (or no curved path)
-this is a relativistic effect
-Example: hydrogen atom--the proton field causes the curved electron orbital path

2: **Spin-orbit interaction for photons:**

-Less well known: occurs when a photon moves along any curved path
-must occur in an **inhomogeneous** dielectric medium (or no curved path)
-Has nothing to do with electromagnetic forces, which don't exist for photons!

3.) **Short definition of "spin-orbit interaction":**

-Particle's trajectory path ("orbit") depends upon its spin
-Particle's spin orientation depends upon its trajectory ("orbit")
-Usually these spin and orbital degrees of freedom are thought of as independent!

4.) In a cylindrical waveguide, the spin-orbit interaction dynamics are described by a **single** expression applying to **both** electrons **and** photons

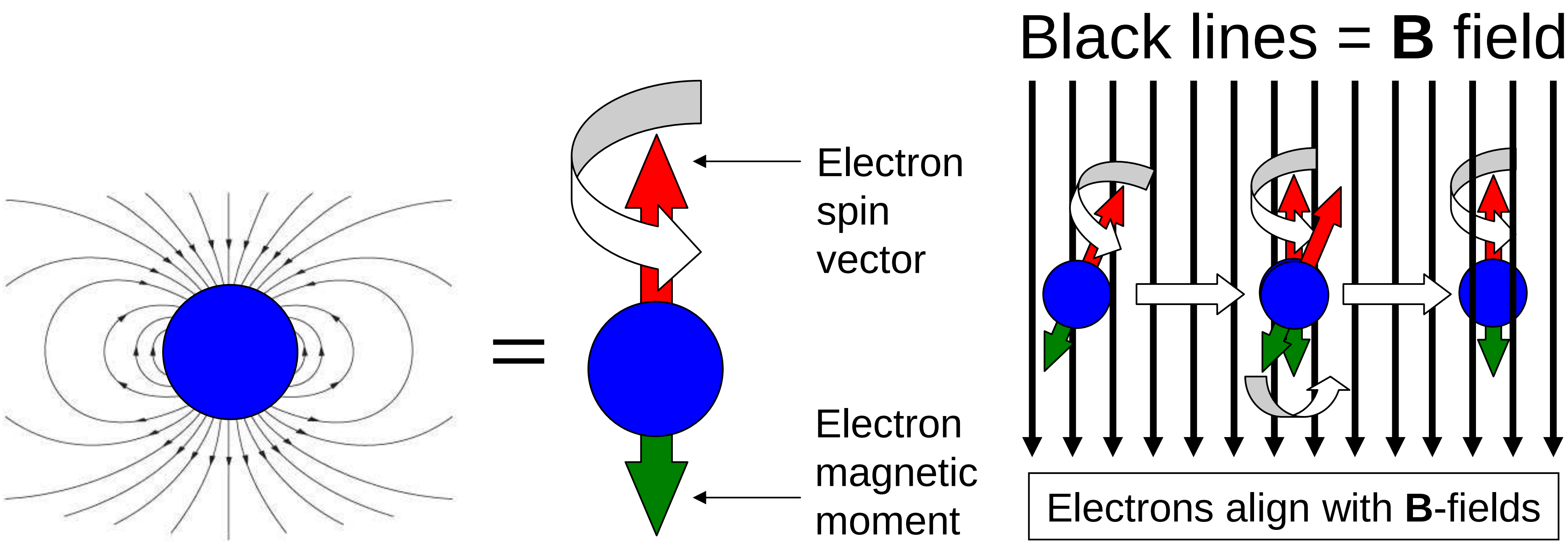
Question: Why such a close analogy?

The answer is geometric in nature…

quant-ph > arXiv:0905.3778

But what is spin, really?

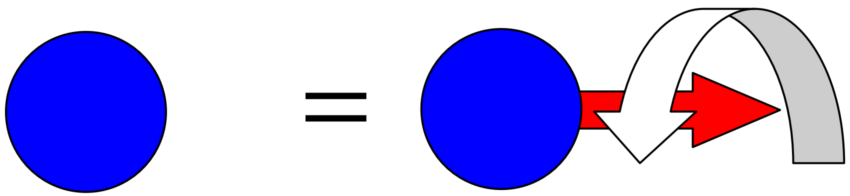
- Electron spin manifests itself through a **magnetic moment**
- You may therefore think of an electron as a bar magnet
- If there is a **B**-field, the electron will interact with it, and rotate to line up with it



And what about photons?

- Electron spin manifests itself through a **magnetic moment**
- You may therefore think of an electron as a bar magnet
- If there is a **B**-field, the electron will interact with it, and rotate to line up with it
- **But,** recall: *photons have spin as well, but no magnetic moment!*
  So where is the analogy? (We'll come back to this later)

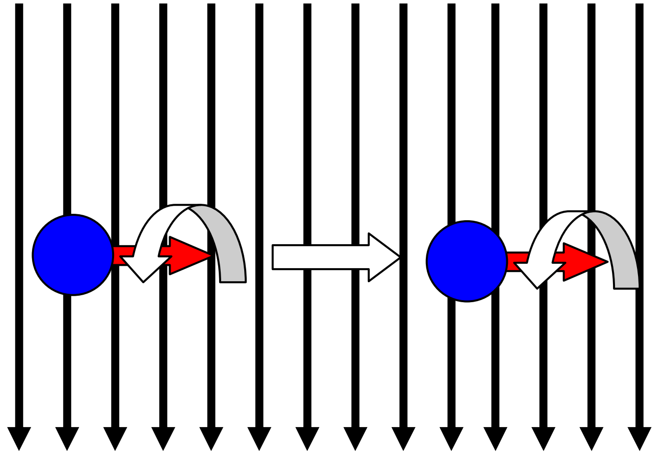
A photon still has spin
(points along motion)



No field lines
for a photon…

…and no magnetic moment

Black lines = **B** field

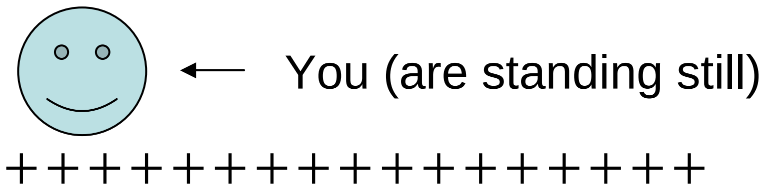


Electrons align with **B**-fields

Simple example: A hydrogen atom

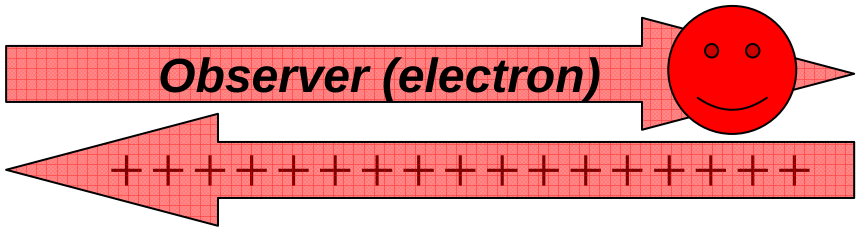
- A hydrogen atom is an electron moving in a proton’s electric field
- **But**, then there’s no magnetic field, right? **WRONG!**
- There **is** a magnetic field due to *relativity* (in the electron frame!)

Charges at rest in your lab...



You see an **E**- field **only**

Are a **current** to a moving observer



The observer sees a **B**-field too!

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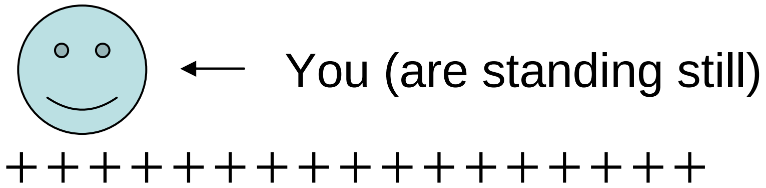

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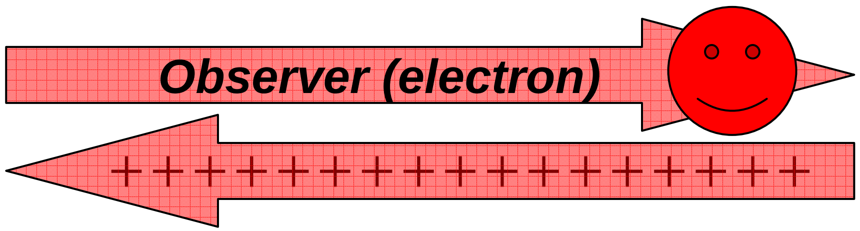

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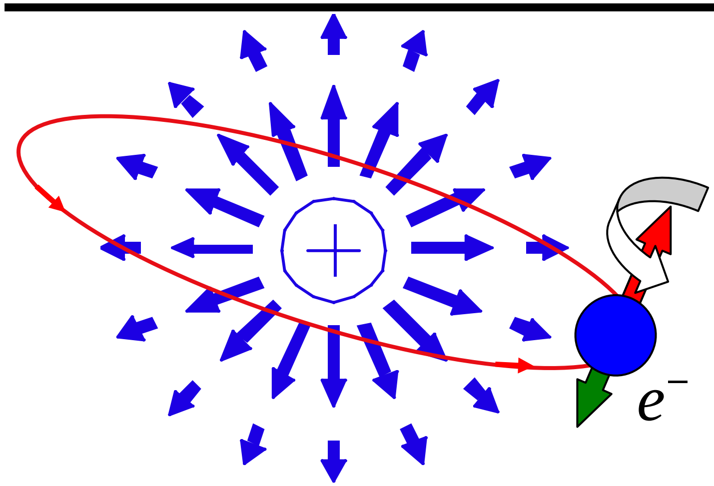
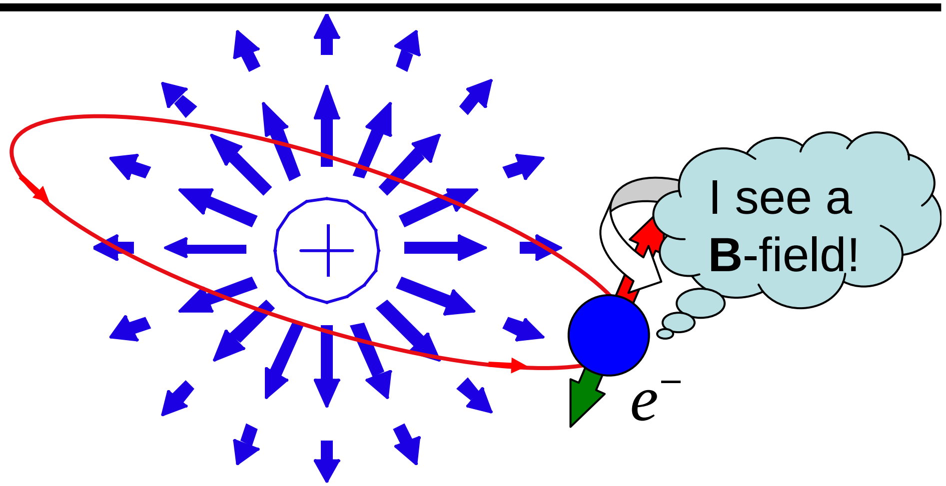




Spin-Orbit Interaction (SOI) in Spherical Potentials:

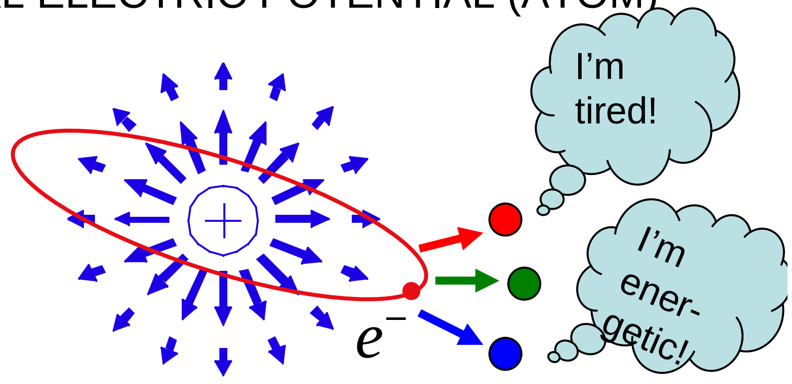
- ELECTRON IN AN INHOMOGENOUS SPHERICAL ELECTRIC POTENTIAL (ATOM)

$$H' = -\frac{e^2}{2m^2c^2}\frac{1}{r}\frac{\partial V}{\partial r}\mathbf{S}\cdot\mathbf{L}$$

$$\mathbf{S} = SAM$$

$$\mathbf{r}\times\mathbf{p} = \mathbf{L} = OAM$$

Electron "bound": Quantized energy levels; gives off radiation (photons)



(atomic fine structure)

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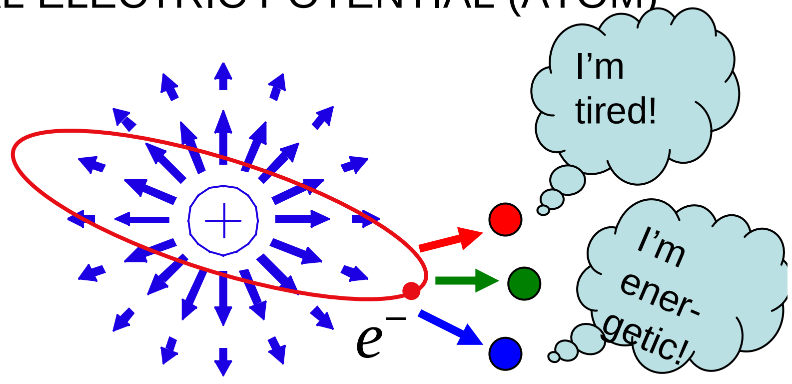

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The previous is well-known physics
The following is new

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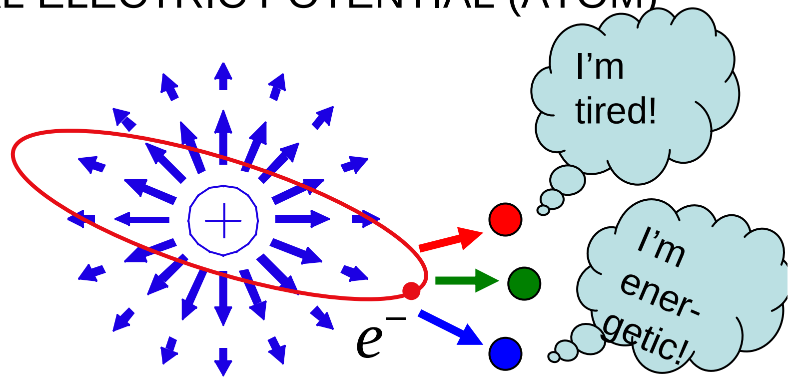

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Spin-Orbit Interaction (SOI) in Cylindrical Potentials?

- Why study *that*? Its not like there are cylindrical atoms!
- Hey, it's the only other simple, non-boring symmetry left, right?
- Also, **photons** in cylinders are all around us! (Optical fibers)
- Maybe there is an illuminating analogy…
- **Weird: after 80 years of spin-orbit, no one had solved this!**

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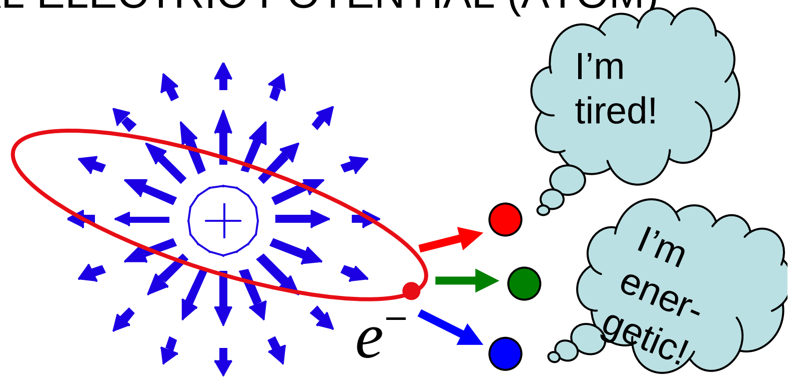

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C Leary, D Reeb, M Raymer, NJP, 10, 103022 (2008).

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$$S_z = \mathbf{S}\cdot\hat{\mathbf{z}}$$

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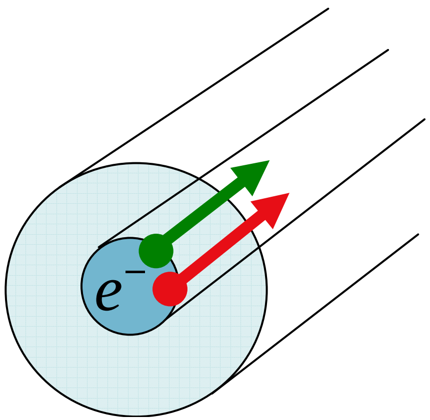
Hamiltonian is total energy

Electron "free!":Continuous energy levels; electron is a *traveling wave:*

$$\Psi \propto e^{i\left(\beta_0 z-\frac{E}{\hbar}t\right)}$$

The (phase) "velocity" β_0 **is "quantized!" (shifted) by the "effective" B-field**



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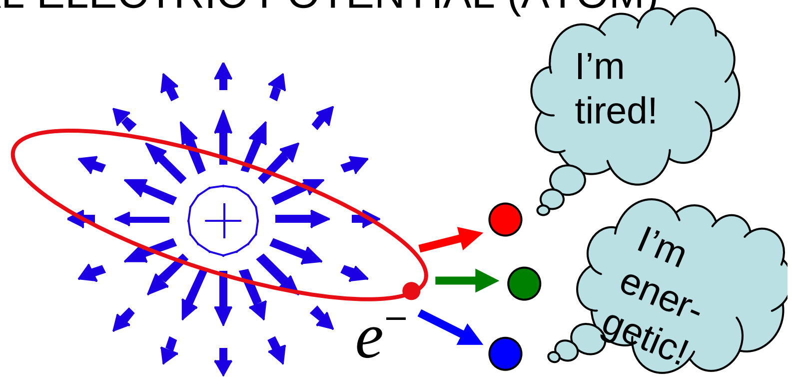

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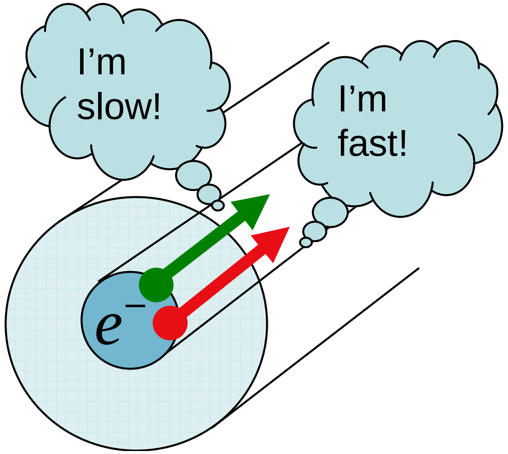
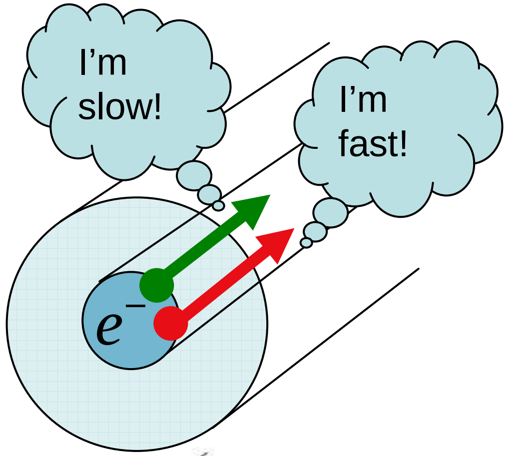


The cylinder is just like our hydrogen example:

- Fig. 1: the lab frame of a charged cylinder:
  - You see an **E**-field only
- Fig. 2: the electron frame:
  - The electron sees a **B**-field also!
  - Electron spin lines up parallel or anti-parallel with the **B**-field– this causes the velocity difference!



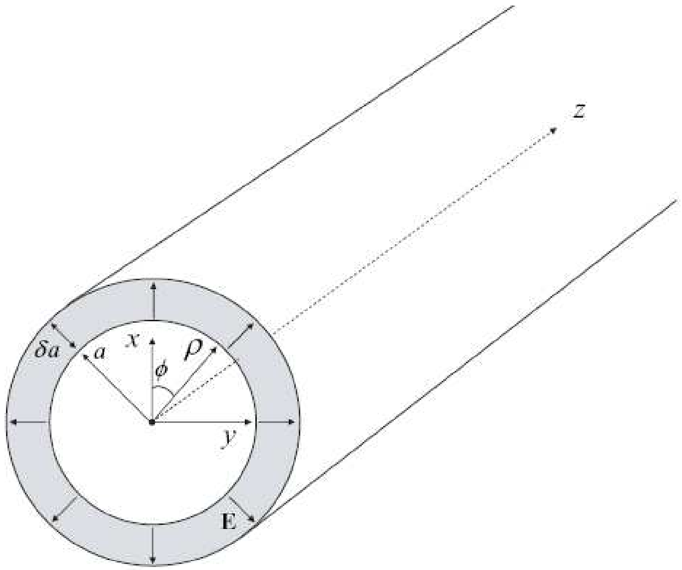


Figure 1. Two concentric cylindrical surfaces with nearly equal radii a and $a + \delta a$. The inner (outer) cylinder is positively (negatively) charged, thereby giving rise to an approximately constant electric field pointing radially outward between the cylinders, as expressed in equation (1). The electric field is zero elsewhere.

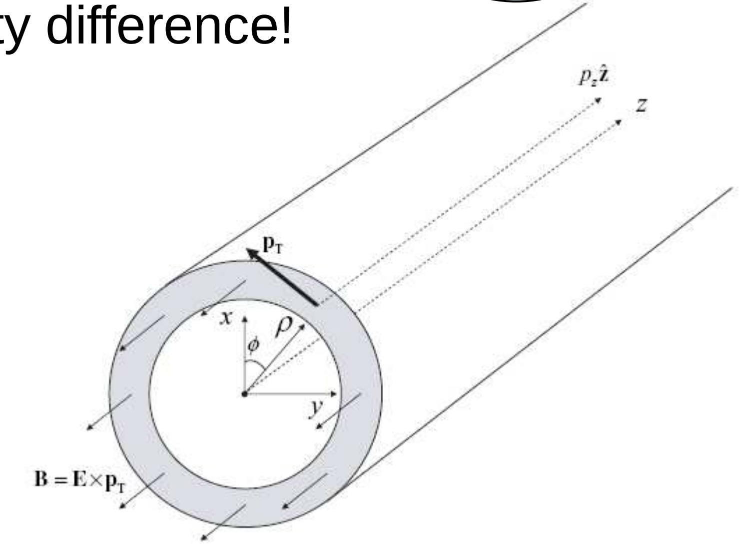
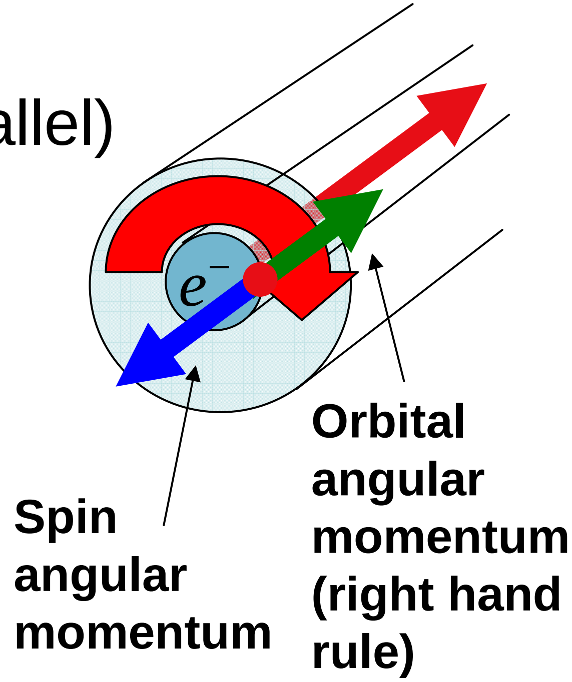
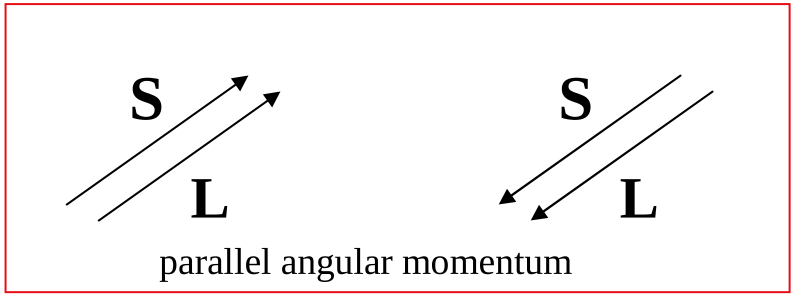


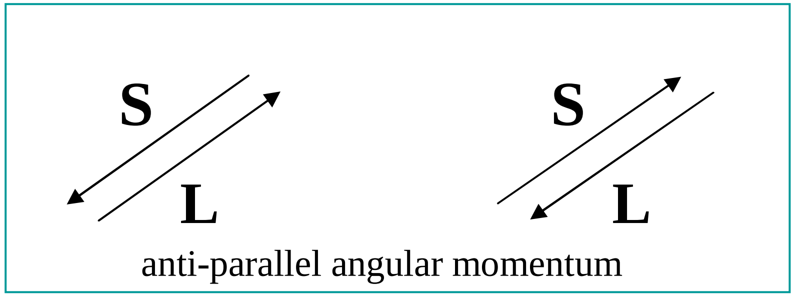
Figure 2. The magnetic field contribution due to an electron propagating paraxially between the cylinders of the waveguide with nonzero p_ϕ, as experienced in the electron's rest frame. As discussed in the main text of the paper, we ignore the contribution due to p_z (represented by the dotted arrow in the figure), so that the field shown in the figure is that due only to the transverse component of momentum $\mathbf{p}_T$ (represented by the bold arrow in the figure). This effective magnetic field points in the negative z-direction for anti-clockwise p_ϕ (as shown above), and in the positive z-direction for clockwise p_ϕ.

Parallel and anti-parallel spins and orbits

- The electron can be "spinning" clockwise or anti-clockwise (pointing parallel or anti-parallel)
- It can also **orbit** either way
- The electron propagates (classically) in a **spiral** trajectory
- Quantum mechanically, no trajectory!
- Electrons are waves
- Wave propagation speed (phase velocity) depends upon the spin and orbital orientation:







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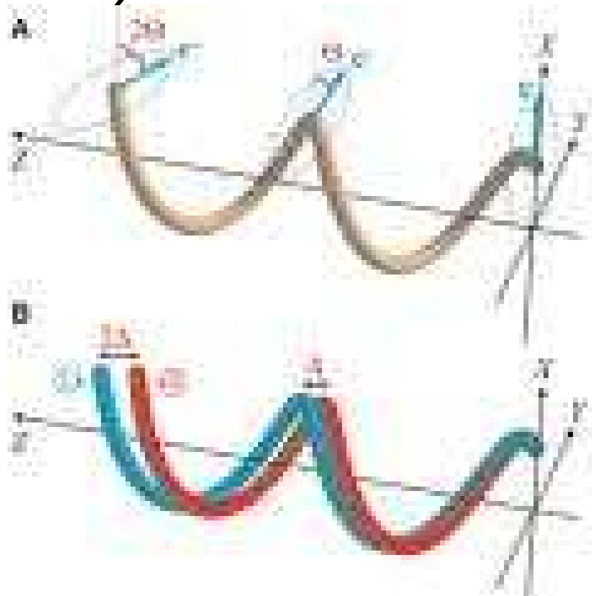
Different speeds



Bliohk, Nat. Ph. 2008

parallel angular momentum

anti-parallel angular momentum

Intermission: so much for electrons in a charged cylinder--
But what about photons in a glass cylinder?

- They must act quite different from electrons, right?
- I mean:
  - Electrons have mass (Catholic) photons don't (Protestant)
  - Electrons have a magnetic moment, photons don't
  - Electrons have charge, photons don't
  - Photons travel at speed of light, electrons can't
- Surprise! The photon motion is *completely* analogous
- Where is the connection?
  - The electrostatic potential V (which gives **E** via $\mathbf{E} = -\nabla V$) for electrons plays the same role as the permittivity ε for photons! Note that $\varepsilon = \varepsilon_0 n^2$ where n is the refractive index
  - So, **Snell's law** effectively replaces the **Lorentz Force law**

To what extent is there a photon-electron analogy? An Example:

1. Spin Hall Effect for Electrons: opposite spin accumulation on opposing lateral surfaces of a current-carrying sample. Its origin is spin-orbit interaction.

Dyakonov and Perel (1971) Sov. Phys. JETP Lett. 13, 467

Hirsch (1999) PRL 83, 1834

2. Spin Hall Effect for Light: spin-dependent displacement perpendicular to the refractive index gradient for photons passing through an air-glass interface.

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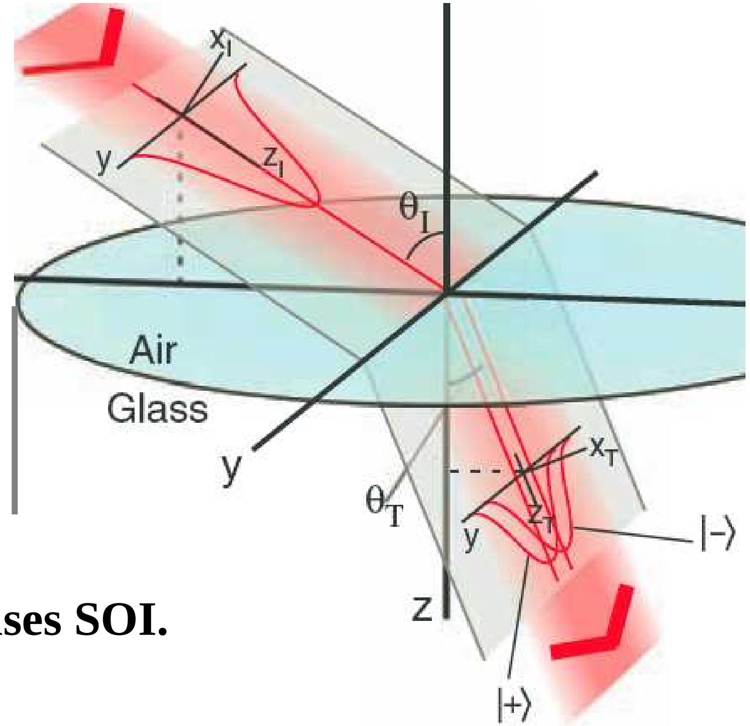

Inhomogeneity in refractive index causes SOI.

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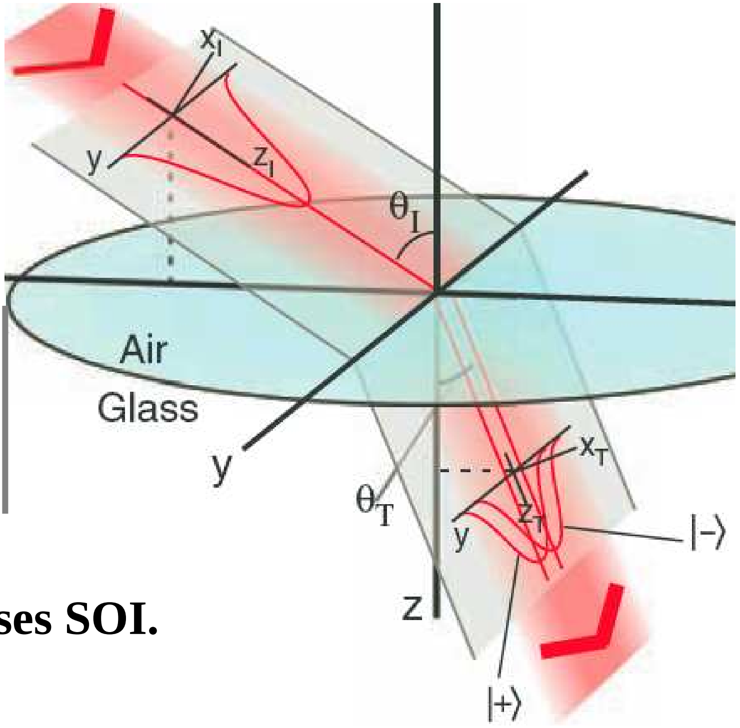
The spin hall effect is easily observed For electrons.

But it is very small for photons... How to amplify it?

Use optical fibers! Many (total internal) Reflections multiplies the effect.



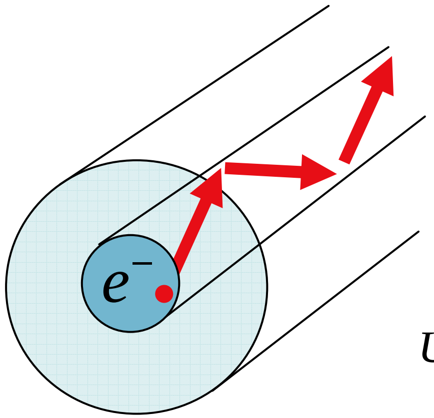
Inhomogeneity in refractive index causes SOI.

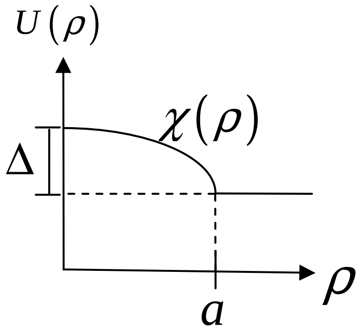
Perhaps the cylindrical electron case is not so surprising…

$$H\Psi = \beta^2 \Psi \qquad H = H_0 + H'$$

$$\Psi_0 \propto e^{i\left(\beta_0 z - \frac{E}{\hbar} t\right)}$$

Solve Schrodinger (Dirac) equation:





$$U(\rho) = \left(U(0) - \Delta\chi(\rho)\right)$$

$$H'_{e^-} = -\frac{\Delta}{4\beta_0} \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} S_z L_z$$

After all, the spin-orbit effect is just an electron moving in an **E**-field

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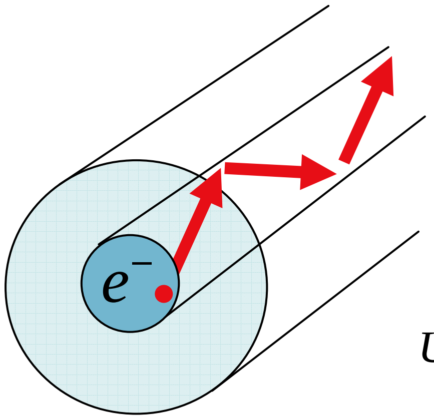

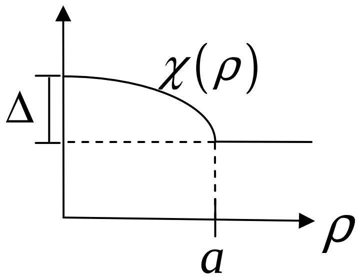

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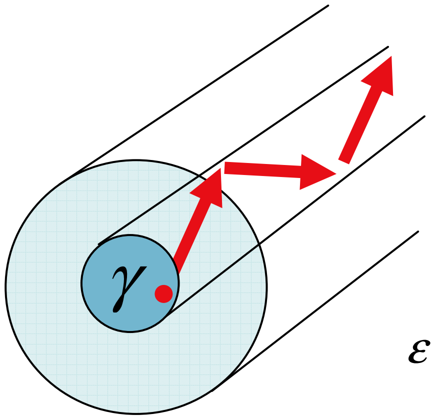

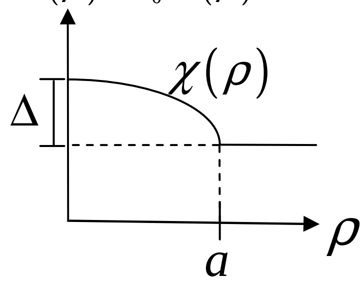

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But it is remarkable that the photon SOI is completely analogous!

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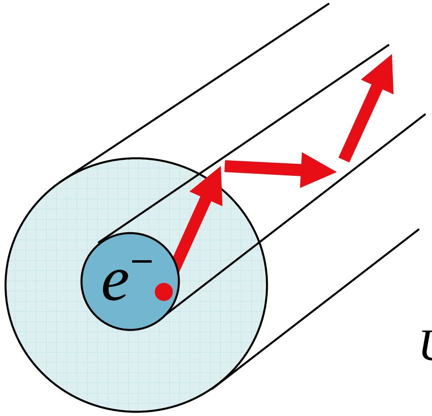

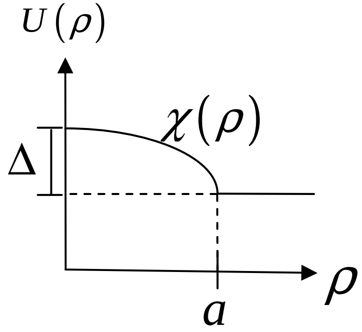

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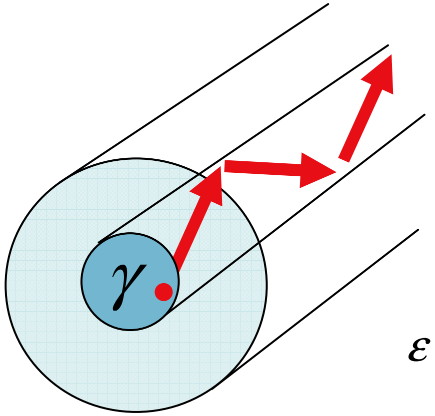

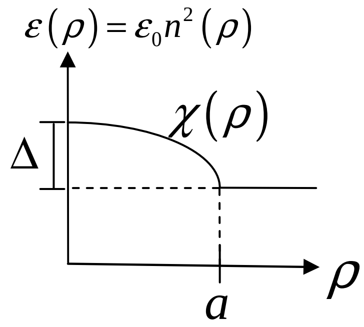

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Main Point: the $S_z L_z$ part of the Hamiltonian means that β will undergo a small positive/negative shift depending upon whether S_z and L_z are oriented parallel/antiparallel to each other

This slide common to both particle types

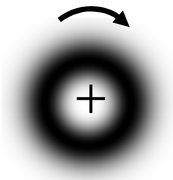
Wavefunction: $\Psi_{\sigma m_\ell} = \psi(\rho) e^{-i m_\ell \phi} e^{i(\beta_0 z - \omega t)} \hat{\mathbf{e}}_\sigma$

($e^{-i m_\ell \phi}$ ↑ OAM; $\hat{\mathbf{e}}_\sigma$ ↑ SAM)

$m_\ell = 0, \pm 1, \pm 2, \ldots =$ OAM quantum number $\sigma = \pm 1 =$ SAM quantum number

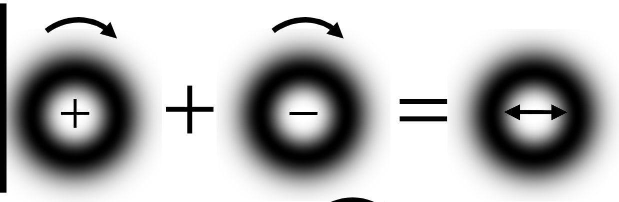
Hamiltonian: $H'_{SOI} = -\frac{\Delta}{4\beta_0} \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} S_z L_z \propto S_z L_z$

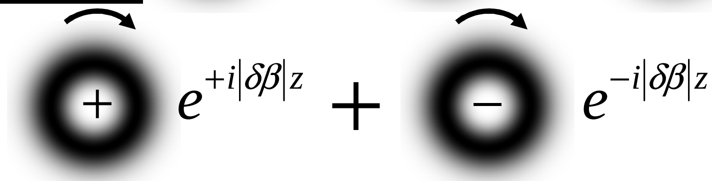
Perturbation theory: $\delta\beta = \langle \Psi_{\sigma m_\ell} | H'_{SOI} | \Psi_{\sigma m_\ell} \rangle \propto \sigma m_\ell \Rightarrow \Psi_{\sigma m_\ell} \to \Psi_{\sigma m_\ell} e^{i\delta\beta z}$

"Notation": $\Psi_{+1,+2} = \psi(\rho) e^{+2i\phi} e^{i(\beta_0 z - \omega t)} \hat{\mathbf{e}}_{+1} \Leftrightarrow$  etc.

"Donuts" have Orbital AM

"Beating" effect:

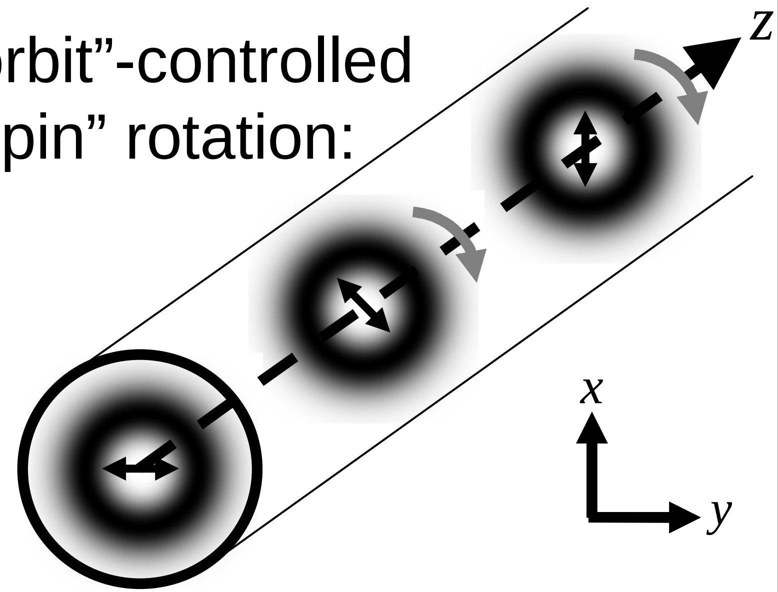


$\longrightarrow$ 

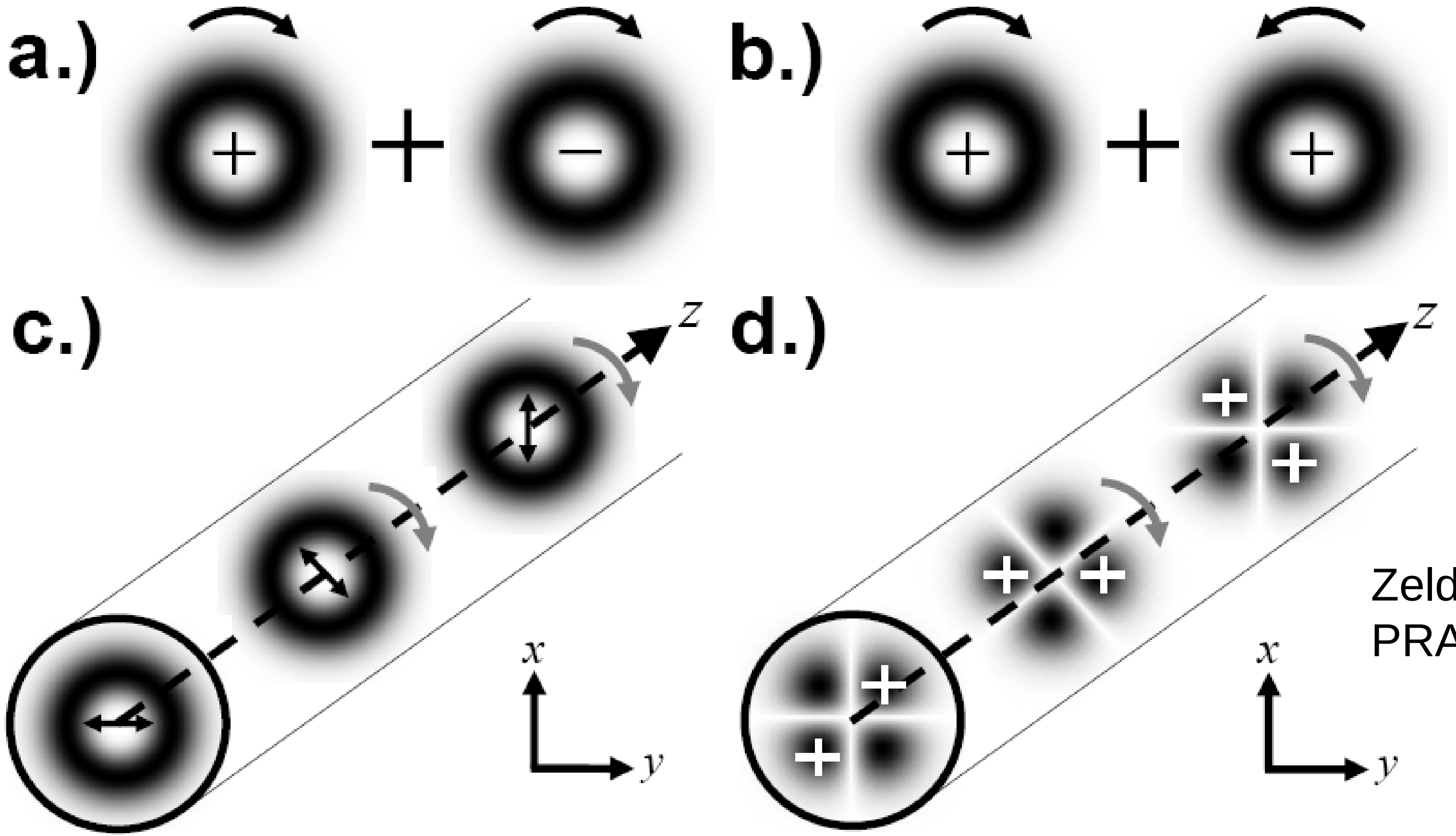
$\propto \cos(|\delta\beta| z)\hat{\mathbf{x}} + \mu \sin(|\delta\beta| z)\hat{\mathbf{y}}$

Spin/polarization rotation direction controlled by sign of $\mu = \pm 1 \equiv \frac{m_\ell}{|m_\ell|}$

"orbit"-controlled "spin" rotation:



Complementary spin-orbit effects



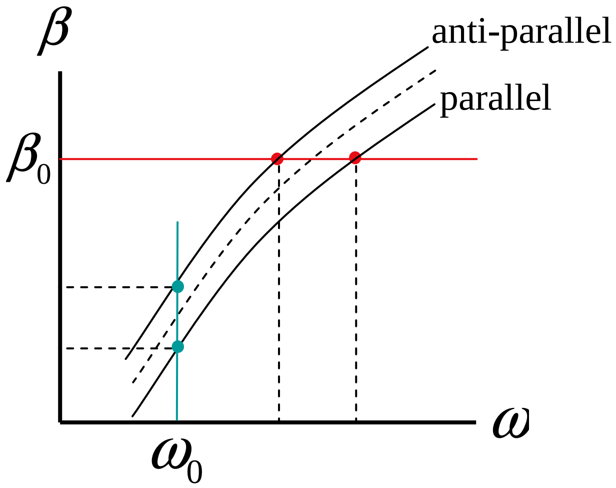
Zeldovich, PRA '91

"orbit"-controlled "spin" rotation (OAM eigenstate)

"spin"-controlled "orbit" rotation (SAM eigenstate)

Both of these effects may occur either in space or time

Spatial vs. Temporal Rotation



• The SOI may be thought of a a splitting of the dispersion curve for “parallel” vs. “anti-parallel” states.

• For a given frequency, there are two β values, so that the SOI splits the propagation constant (spatial rotation)

• If the both the frequencies and propagation constants of the parallel and anti-parallel states are originally slightly different, than the SOI acts to *restore* the propagation constants to degeneracy (temporal rotation)

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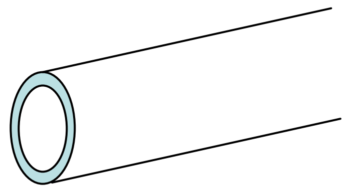

1: **Spin-orbit interaction for electrons:**

-well-known to occur when an electron moves along any curved path
-must occur in an **inhomogeneous** electromagnetic potential (or no curved path)
-this is a relativistic effect
-Example: hydrogen atom--the proton field causes the curved electron orbital path

2: **Spin-orbit interaction for photons:**

-Less well known: occurs when a photon moves along any curved path
-must occur in an **inhomogeneous** dielectric medium (or no curved path)
-Has nothing to do with electromagnetic forces, which don't exist for photons!

3.) **Short definition of "spin-orbit interaction":**

-Particle's trajectory path ("orbit") depends upon its spin
-Particle's spin orientation depends upon its trajectory ("orbit")
-Usually these spin and orbital degrees of freedom are thought of as independent!

4.) In a cylindrical waveguide, the spin-orbit interaction dynamics are described by a **single** expression applying to **both** electrons **and** photons

Question: Why such a close analogy?

quant-ph > arXiv:0905.3778

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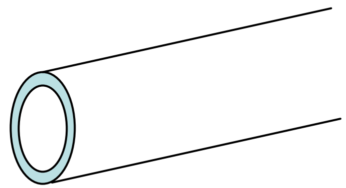

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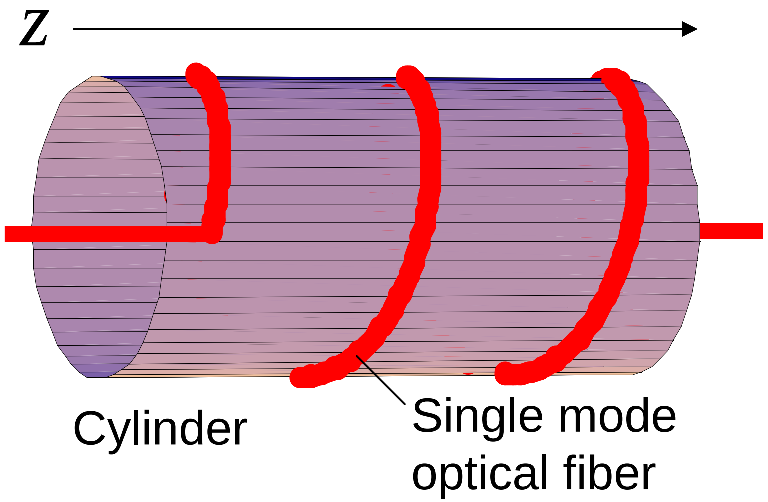
Question: Why such a close analogy?

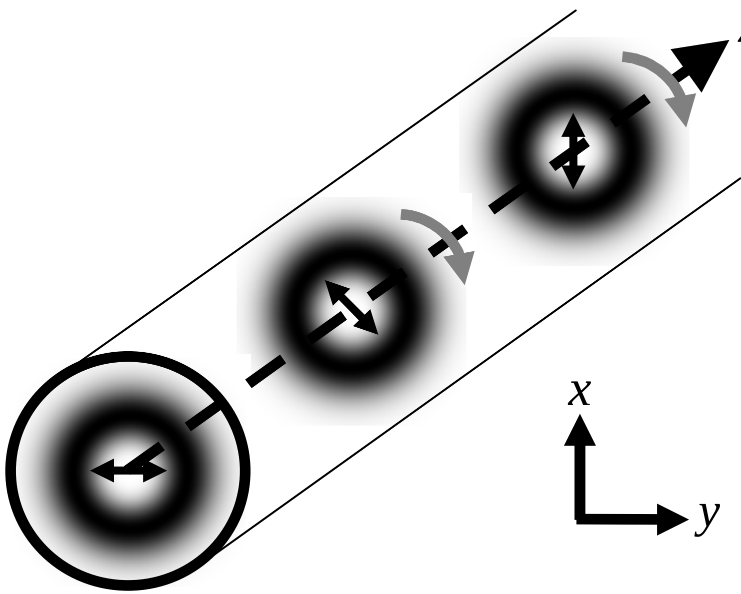
Answer: The spin-orbit interaction is a purely geometric effect (Berry phase)

A famous experiment

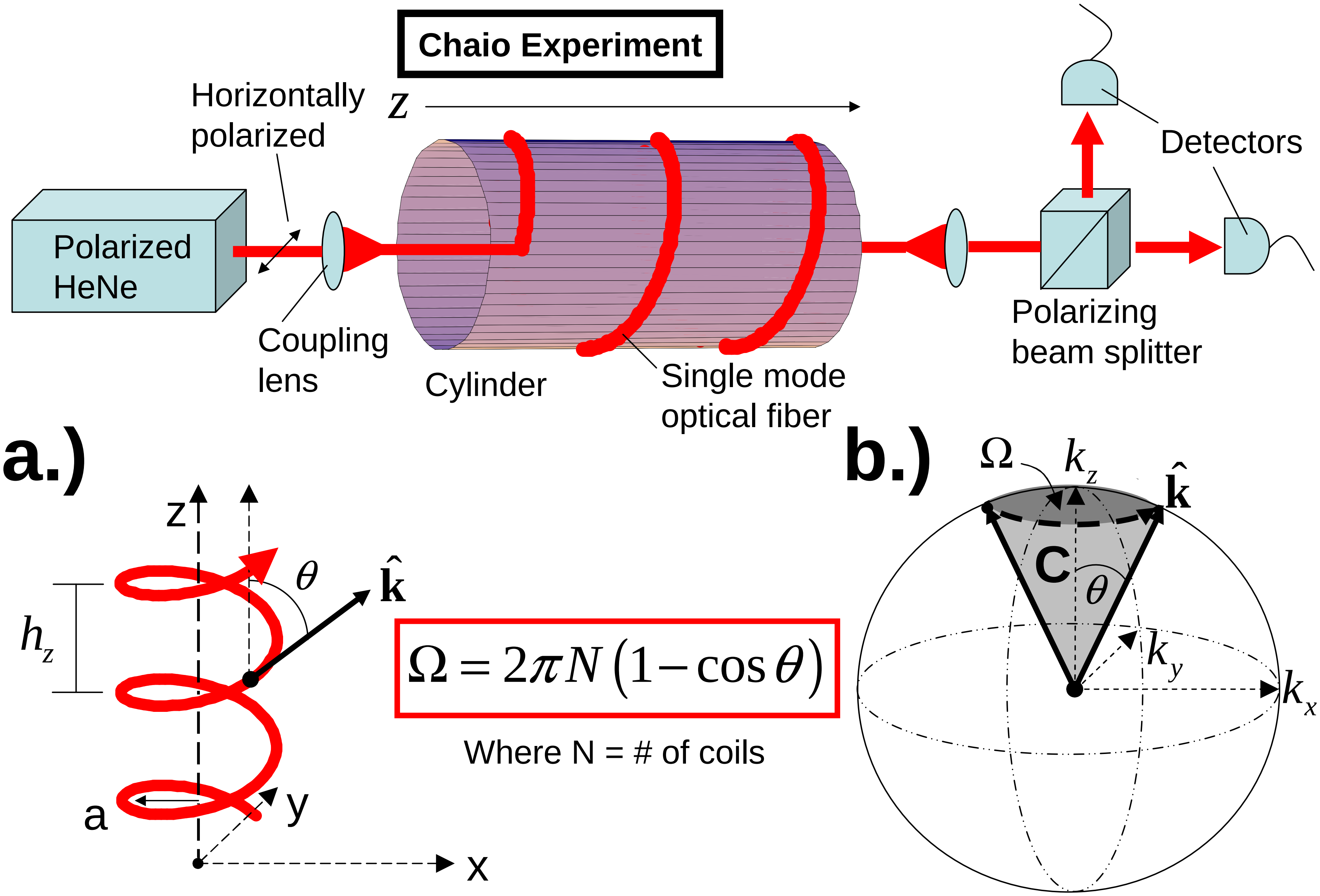
The polarization rotation effect is reminiscent of an experiment of Chaio, in which the geometric (Berry) phase was first observed as polarization rotation in a coiled fiber.

Chaio Experiment





However, the Chaio effect involves only the fundamental Gaussian mode, which has zero OAM, and predicts an accumulated geometric phase of zero for a straight fiber! Nevertheless, considering this effect provides a way in which to understand the straight fiber rotation phenomena for modes with OAM.



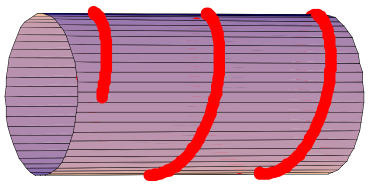
The geometric phase equals the surface area enclosed by the curve C, which is traced out by the momentum vector **k**. This phase causes a “beating” effect: polarization rotation

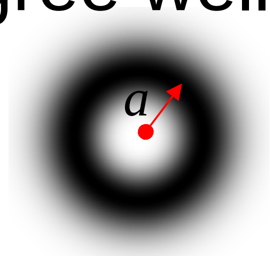
Geometric phase approach

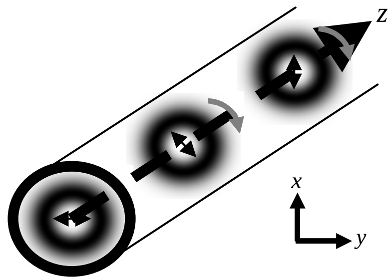
- The “classical” geometric phase is: $\Omega = 2\pi N\left(1 - \cos\theta\right)$
  (For a “classical” particle with spin; Bialynicki-Birula)
- For quantum states, we quantize: $\Omega \rightarrow \hat{\Omega} \equiv -\sigma\mu \frac{\Delta}{2\beta_0 a^2} \nabla_T$
- The geometric phase $\gamma = \delta\beta z$ is then given by the expectation of Ω in the unperturbed states:

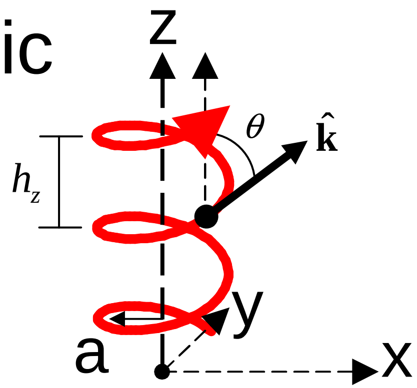
$$\gamma = \delta\beta z = \left\langle \Psi_{\sigma m_\ell} \middle| \hat{\Omega} \middle| \Psi_{\sigma m_\ell} \right\rangle \propto \sigma m_\ell \Rightarrow \Psi_{\sigma m_\ell} \rightarrow \Psi_{\sigma m_\ell} e^{i\gamma} = \Psi_{\sigma m_\ell} e^{i\delta\beta z}$$

- If *a* is set equal to the radius of the wavefunction “peak”, then the perturbative and geometric approaches agree well.







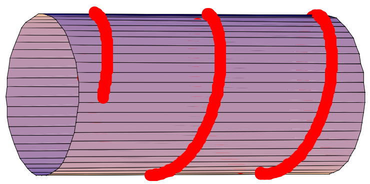


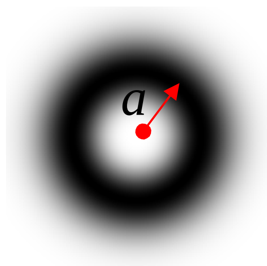
Geometric phase approach

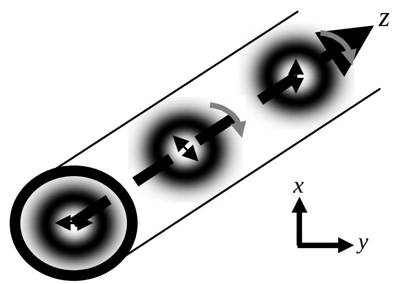
- Specifically, one gets a particularly simple form for the accumulated phase $e^{i\delta\beta z}$ in the special case of a step-index/potential of large radius *a*, for OAM states which are “near cutoff”– that is, their wavefunction “peak” is near the fiber radius:

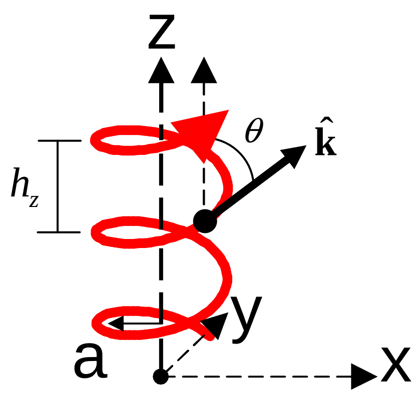
$$\gamma = \delta\beta z = \sigma m_{\ell} \frac{\Delta}{2\beta_0 a^2} z$$

- In this case the two approaches agree exactly.









Conclusions

- The spin-orbit interaction dynamics of the electron and photon are identical to first order in perturbation theory
- They have a common geometric origin
- The role of the electron's potential energy is played by the permittivity in the photon case
- Any particle with spin (i.e. neutrons) will also undergo SOI
- The SOI allows for the construction of both spin and orbital "gates", which may enable reversible transfer of entanglement between SAM and OAM
- Experiments are underway to observe these effects
- **Come down to the U of Oregon for summer research!**