Photon spin-orbit coupling in fiber-based optical cluster states

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Outline

- Review of optical cluster states
- Discuss fiber-based cluster states entangled in spatial mode
- Introduce photon spin-orbit coupling (SOC)
- Discuss the role photon SOC plays in simplifying the construction of spatial mode entangled photonic cluster states

Cluster state review

Cluster state history

- R&B (2001): Introduced cluster states, circumvented highly controlled interactions between specific qubits, but required nonlinear interactions
- KLM (2001): (LOQC) Replaced nonlinear interactions by linear optics and measurement, but still required Mach-Szender stability, quantum teleportation, quantum error correction, and photon number indistinguishability.

Cluster state advantages

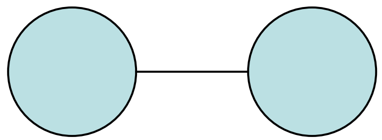
- Browne & Rudolph (‘05), Zhang et al. (‘06)
- Combined cluster states with LOQC
- Mach-Zehnder stability no longer required
- Quantum error correction no longer required
- Photon number distinguishability no longer required
- Only Hong-Ou-Mandel stability and Bell states as an initial resource are required
- There are some caveats to the above

What is a cluster state?

- An optical cluster state is an entangled quantum state involving multiple photonic qubits $\hat{S}_z$
- Building them is most of the work
- Once constructed, the quantum computation proceeds completely by making various measurements on photons in the cluster state

Cluster states: the simplest example

The two qubit cluster state:



The complete (entangled) quantum state of two distinct two-state quantum systems as constructed (theoretically) through a two step process:

The two qubit cluster state

1.) Each qubit is initially prepared in a balanced superposition $\frac{1}{\sqrt{2}}\left(|0\rangle_i + |1\rangle_i\right)$ so the complete state is

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 + |1\rangle_1\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_2 + |1\rangle_2\right)$$

$$= \frac{1}{2}\left(|0\rangle_1 \otimes |0\rangle_2 + |0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right)$$

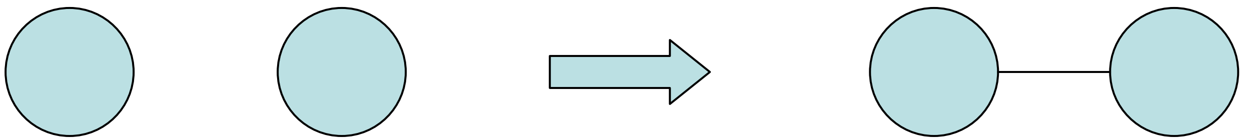
$$\equiv \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Graphical representation:

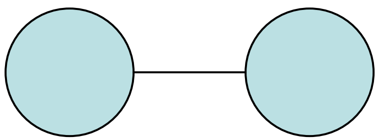
Two qubit cluster state

2.) A “controlled phase” (C-Phase) operation is then performed on one qubit with respect to the other qubit

$$\tfrac{1}{2}\left(|00\rangle+|01\rangle+|10\rangle+|11\rangle\right) \overset{C-Phase}{\overbrace{\rightarrow}} \tfrac{1}{2}\left(|00\rangle+|01\rangle+|10\rangle-|11\rangle\right)$$

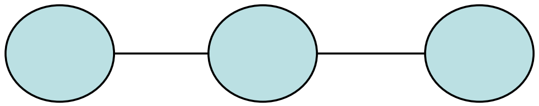


Note that the state is no longer factorizable, and is equivalent (up to local unitaries) to a “Bell” state:



$$\underset{LU}{\approx} \frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)$$ **(Bell)**

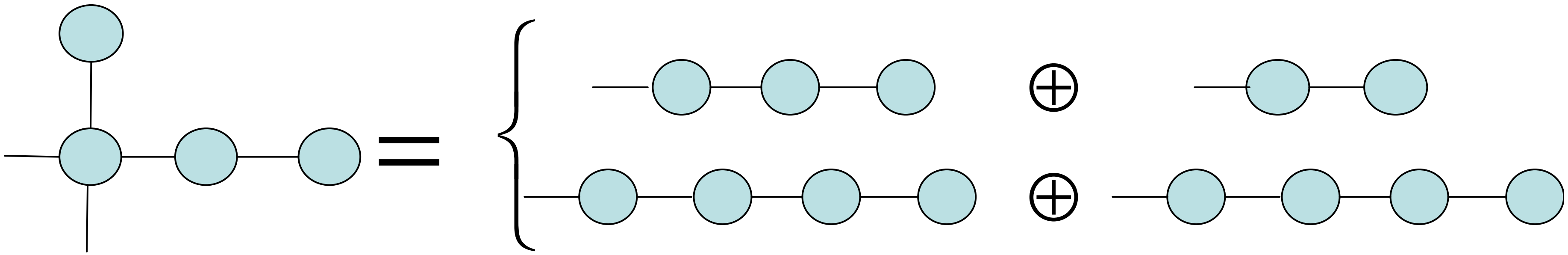
More general cluster states



$$\frac{1}{4}\left(\left|+0+\right\rangle+\left|-1-\right\rangle\right)$$

$$\left|\pm\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|0\right\rangle \pm \left|1\right\rangle\right) \qquad \underset{LU}{\approx} \frac{1}{\sqrt{2}}\left(\left|000\right\rangle+\left|111\right\rangle\right)$$ **(GHZ)**

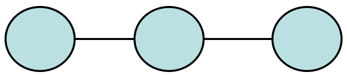
Above is the most complicated state in this talk



The above "L" shape is a universal resource for quantum computation

Optical cluster states

- The two-state quantum variable for each qubit involves a particular photonic degree of freedom
- If polarization is chosen, then



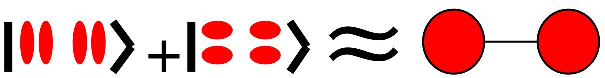
$$= \frac{1}{4}\left(\left|+H+\right\rangle + \left|-V-\right\rangle\right) \text{ where } \left|\pm\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|H\right\rangle \pm \left|V\right\rangle\right)$$

$$= \frac{1}{8}\left(\left|HHH\right\rangle + \left|HHV\right\rangle + \left|VHH\right\rangle + \left|VHV\right\rangle\right.$$

$$\left. + \left|HVH\right\rangle - \left|HVV\right\rangle - \left|VVH\right\rangle + \left|VVV\right\rangle\right)$$

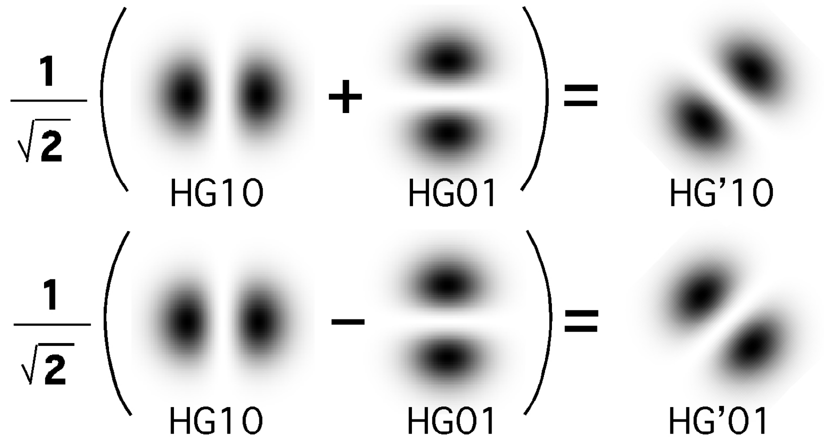
$$\underset{LU}{\approx} \frac{1}{\sqrt{2}}\left(\left|HHH\right\rangle + \left|VVV\right\rangle\right)$$

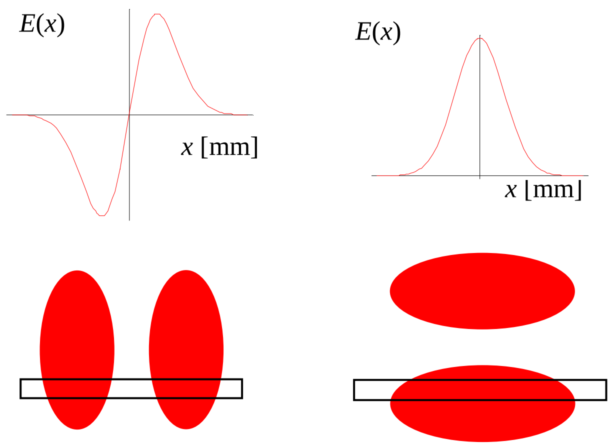
HG mode entangled cluster states

- Polarization is the common choice of photonic qubit variable
- For (approximately) paraxial photons, transverse spatial mode is a distinct photonic degree of freedom– we choose these as qubits
- With HG modes, a two photon cluster state (Bell state) is represented as , while a three photon (GHZ) state is 

Two useful properties of first order HG modes

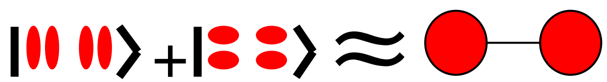
- The sum and difference of their fields result in identical, 45 degree rotated intensity distributions
- The modes are respectively odd and even upon reflection about the y axis: parity distinguishable!

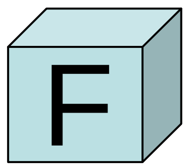




Creating spatial mode entangled cluster states in free space

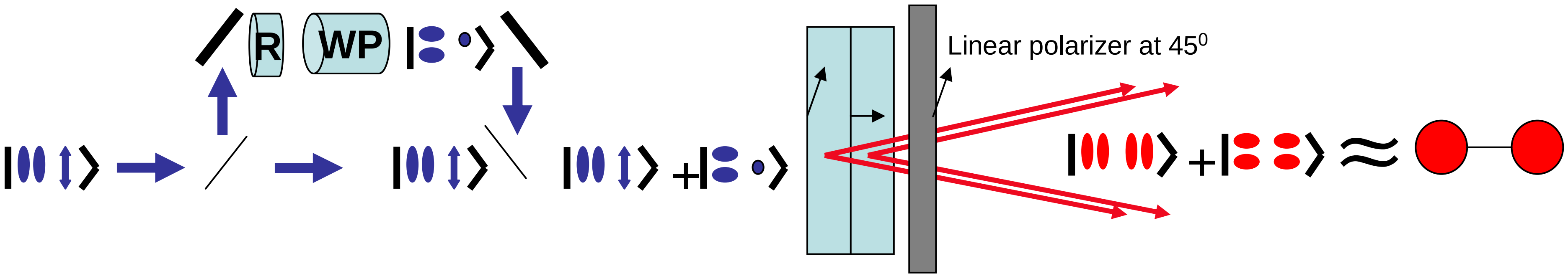
- To create multi qubit spatial mode entangled cluster states we need two things:

- A source of Bell states 

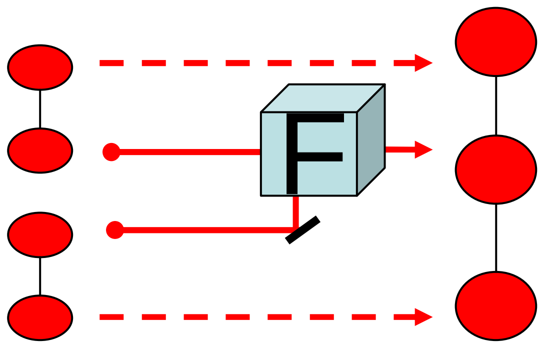
- A “fusion” gate 

Creating a cluster state from a Bell state source: Type-I fusion

- Down conversion setup creates Bell states

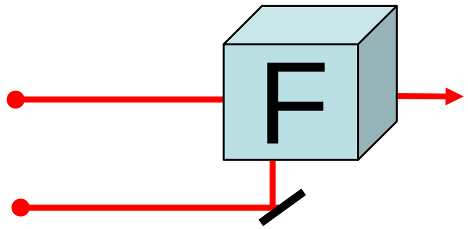


- Type one fusion gate converts two Bell states into a three cubit cluster state (GHZ state)

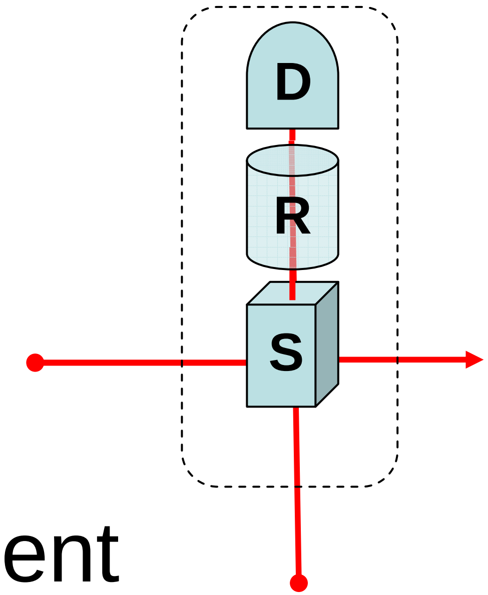


Type-I fusion gate

- Composed of three elements:



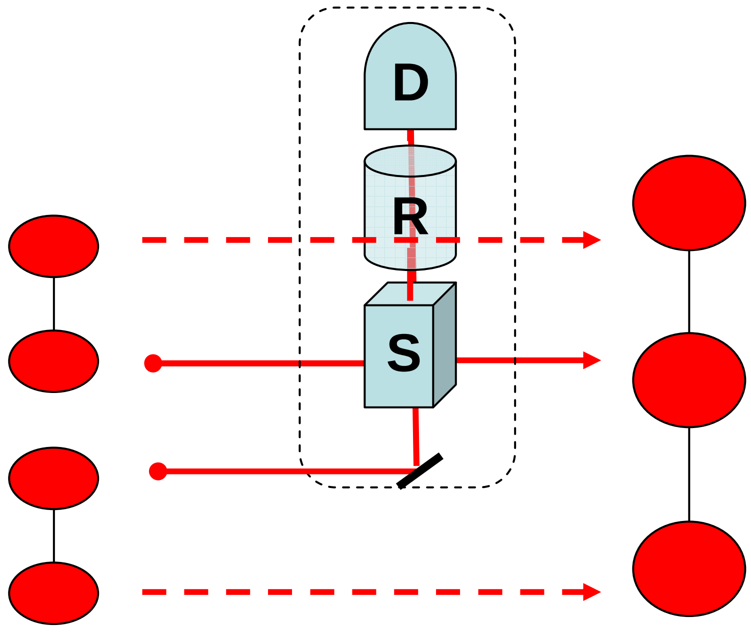
- **S**orter
- **R**otator (Hadamard gate)
- **D**etector (Sorter plus PMTs)

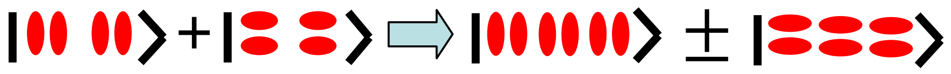


- Two photon input, one photon output: measurement induced nonlinearity provides entanglement

Type-I fusion gate operation

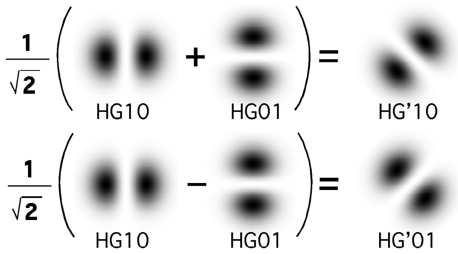
- Initial resource: two Bell states
- Input one photon from each Bell state
- The remaining two photons propagate freely
- One of the input photons is measured, the other exits the gate as output
- The result is an entangled three-qubit cluster state





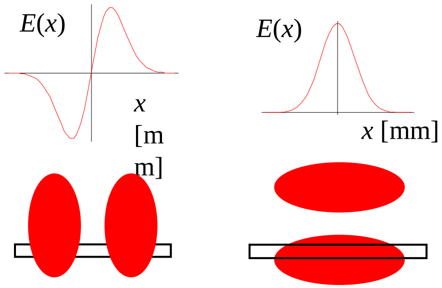
First order Hermite-Gauss spatial mode fusion gate elements

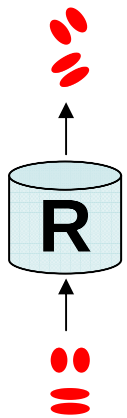
Use transverse spatial modes as qubits



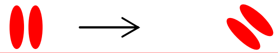
Sums/differences make rotations

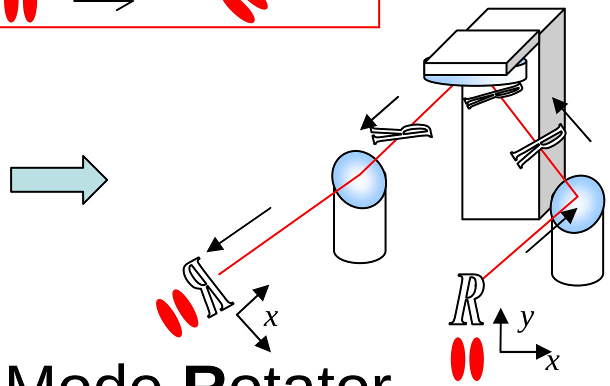
Odd/even upon y axis reflection



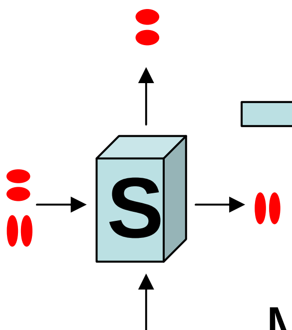


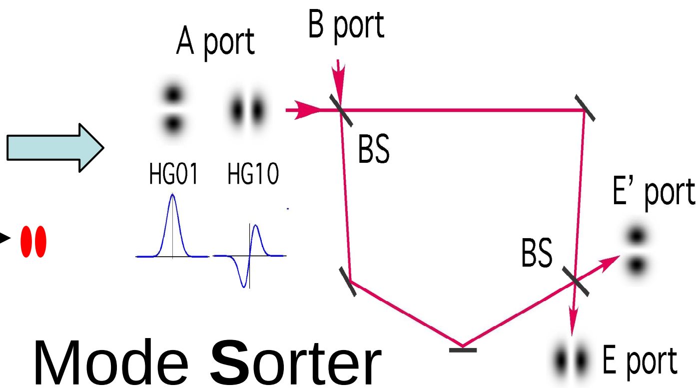
$$|0\rangle \rightarrow |0\rangle + |1\rangle$$



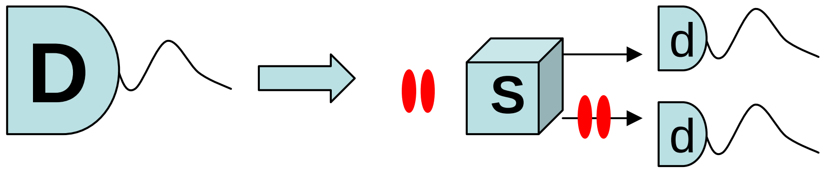


Mode **R**otator (Hadamard)





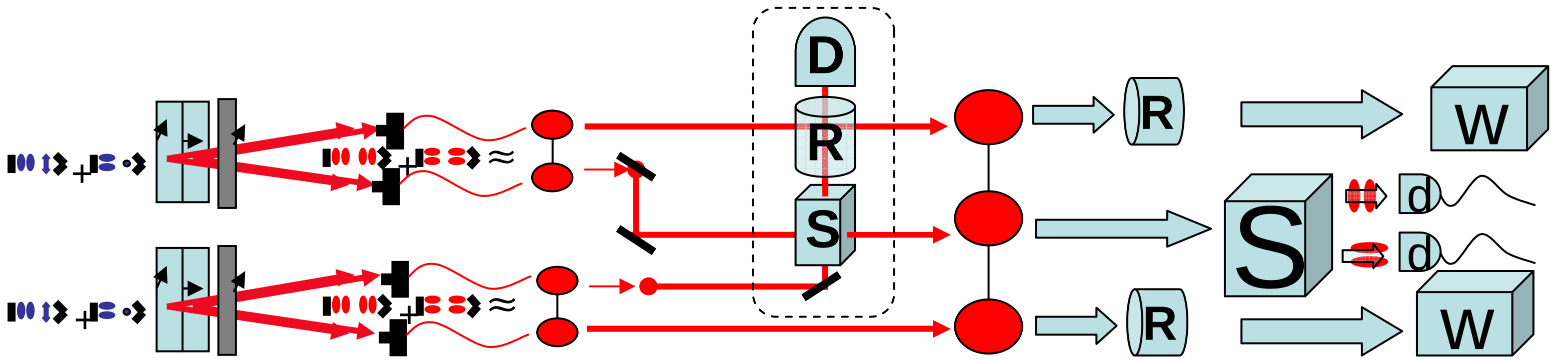
Mode **S**orter



Mode **D**etector

Conclusions so far

- One could make spatial mode cluster states
- But golly that's a lot of mirrors and beam splitters!



- **Solution: We do all of this in optical fibers!**
- Coupling photons into fibers not too difficult
- Fibers make alignment and detection easier
- **Photon spin-orbit coupling manifest in fibers**

Conclusions so far

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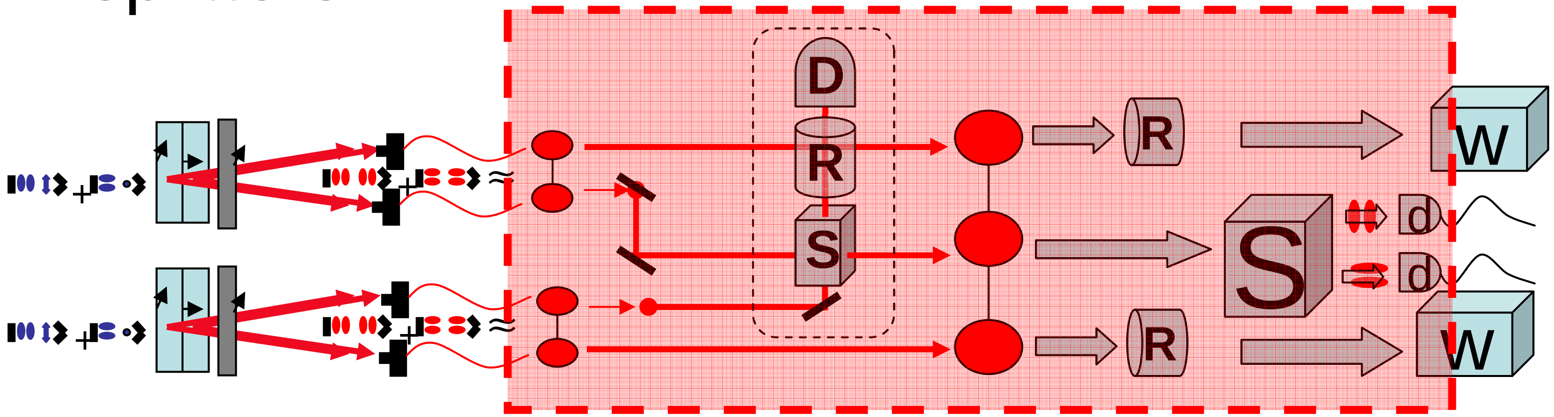
- But golly that's a lot of mirrors and beam splitters!



- **Solution: We do all of this in optical fibers!**
- Coupling photons into fibers not too difficult
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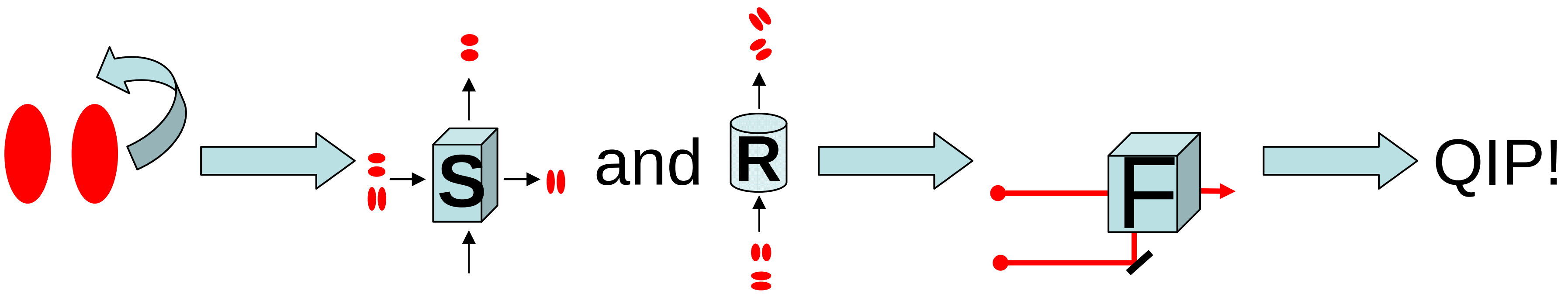
- **Photon spin-orbit coupling manifest in fibers**

Photon spin-orbit coupling

- The effect: photons with well defined helicity (circular polarization) will exhibit rotating transverse spatial intensity profiles while propagating though an optical fiber (Zeldovich '92)
- A fusion gate requires both 45 degree rotations and mirror reflections to build **R**otator, **S**orter.
- But--a 180 degree rotation is like a reflection!
- Therefore, mode rotation control is all we need to build a fusion gate for multiphoton cluster state



Spin-orbit coupling experiment

- We coupled a HG mode from a Helium Neon laser into an optical fiber
- Fiber length determines rotation

$$\mathbf{E} \propto \cos[l\phi] \Rightarrow \cos[l(\phi + \sigma B z)]$$

- B a function of λ , n
- Result not "clean"

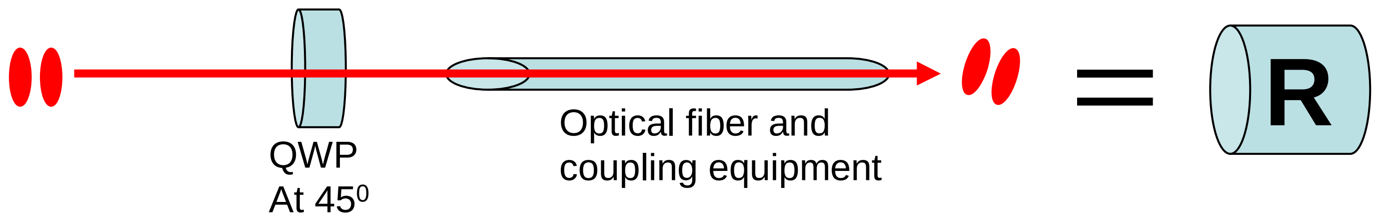
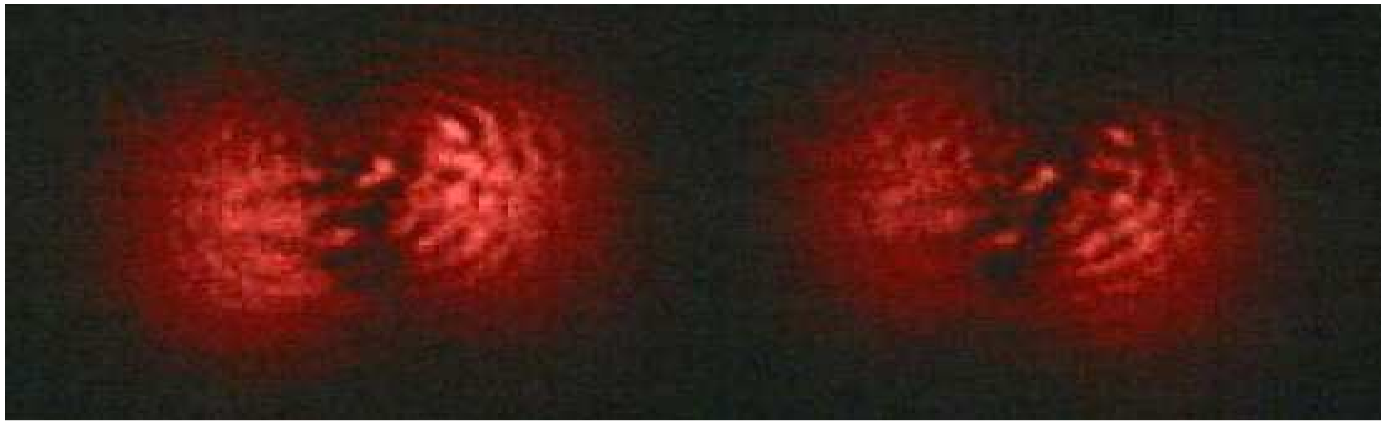


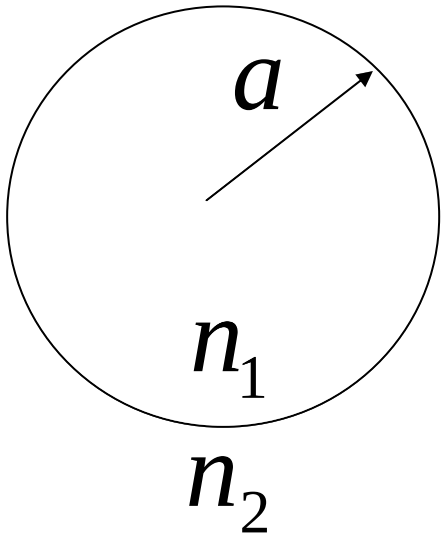
Fig. 1: Linearly Polarized

Fig. 2: Circularly Polarized



Spin-orbit coupling (SOC) theory

- The inhomogeneous nature of the medium gives rise to the effect--no strong free-space SOC
- Zeldovich treated a parabolic index of refraction profile
- We treat a step index profile for a stronger effect
- We first solve Maxwell's equations *exactly* for a **homogeneous** medium
- Boundary matching gives SOC



$$e^{il\phi} \cdot e^{il\sigma Bz} = e^{il[\phi + \sigma Bz]}$$

Transverse electric field in the circularly polarized basis:

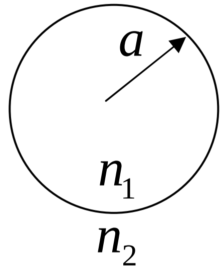
$n \geq 0$

$r < a$

$\Delta n \equiv \frac{1}{2}\left(n_1^2 - n_2^2\right)$

$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$

$$\mathbf{E}_T(\mathbf{r},t) = \left\{E_+\mathbf{e}_+ + E_-\mathbf{e}_-\right\} e^{i\omega t - ik_z z}$$



$$E_\pm = \pm A\left(1 \pm \alpha_{n,m}\right) J_{(n\pm1)}\left(\frac{u_{\pm n,m}}{a} r\right) e^{i(n\pm1)\phi}$$

$HE_{n,m}$

$\alpha_{n,m} \approx -1 - \frac{1}{n}\frac{\Delta n}{n_1^2}\frac{u_{-n,m}^2}{V} \quad n > 0$

$\alpha_{0,m} = 0$

$(k_z)_{n,m} = k_1\left(1 - \frac{\Delta n}{n_1^2}\left(\frac{u_{-n,m}}{V}\right)^2\right)$

$k_1 \equiv n_1 \frac{\omega}{c}$

$$\alpha_{-n,m} = -\alpha_{n,m}$$

$EH_{n,m}$

$\alpha_{n,m} \approx +1 - \frac{1}{n}\frac{\Delta n}{n_1^2}\frac{u_{+n,m}^2}{V} \quad n > 0$

$\alpha_{0,m} = \infty$

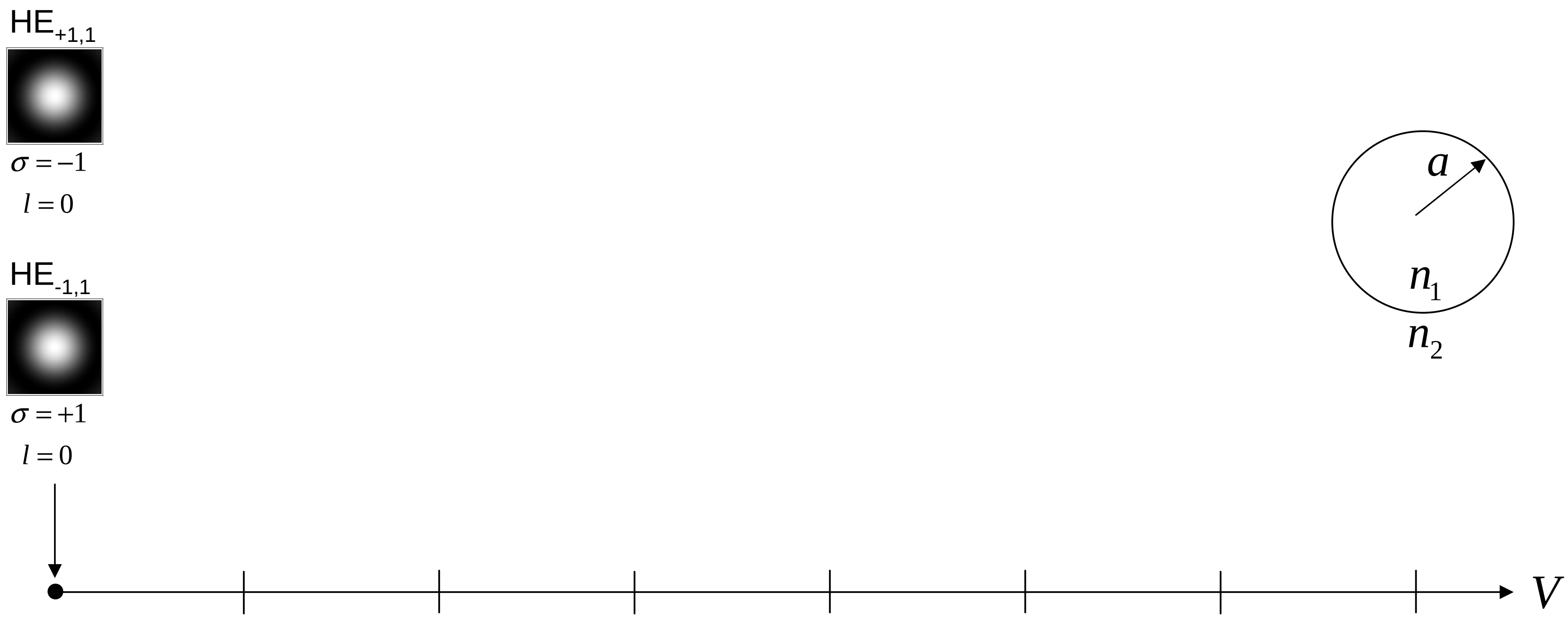
$(k_z)_{n,m} = k_1\left(1 - \frac{\Delta n}{n_1^2}\left(\frac{u_{+n,m}}{V}\right)^2\right)$

$k_1 \equiv n_1 \frac{\omega}{c}$

These solutions valid in the paraxial, far-from-cutoff limits:

$U_{n,m} << \frac{2\pi}{\lambda} n_1 a$

$1 << \left(\frac{2\pi}{\lambda} a\right)\sqrt{n_1^2 - n_2^2}$



$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$$

$HE_{n,m}$

	m = 1	*m = 2*
n = 0	2.405	5.520
n = 1	0	3.832
n = 2	2.445	5.538
n = 3	3.841	
n = 4	5.146	
n = 5	6.392	

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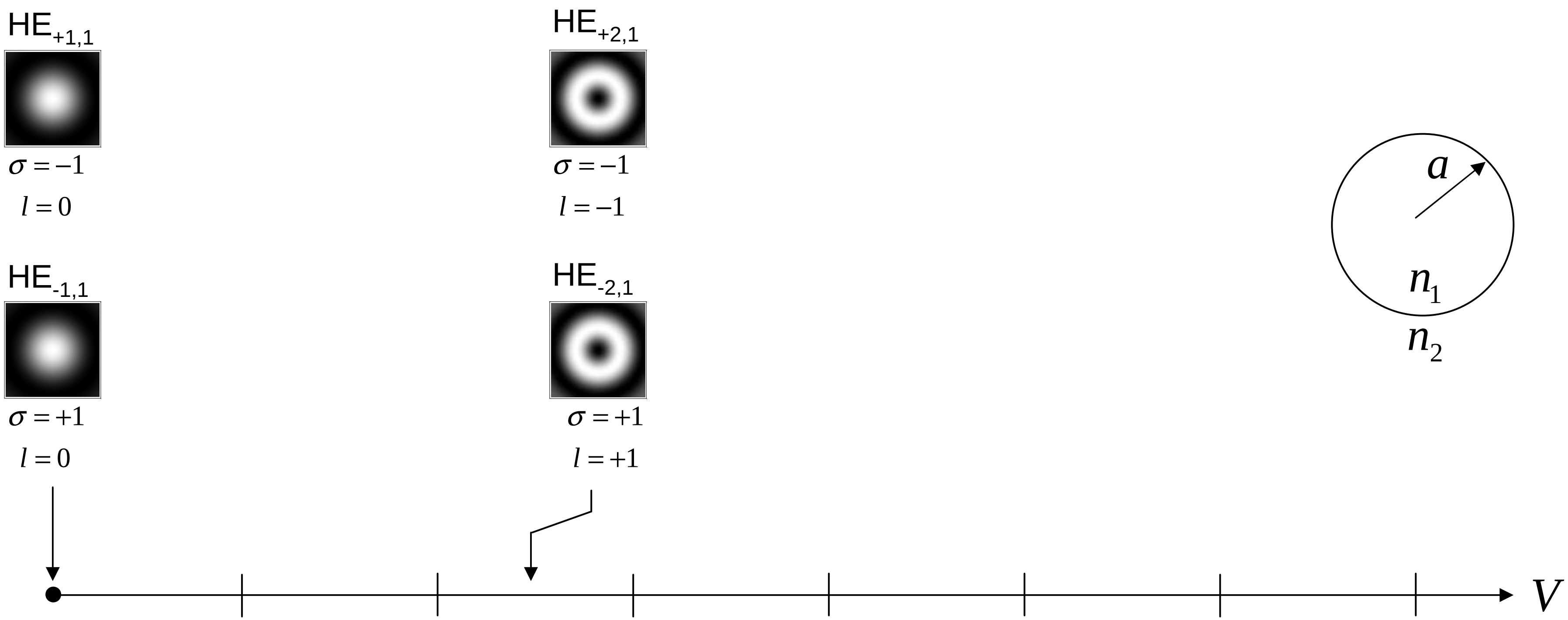

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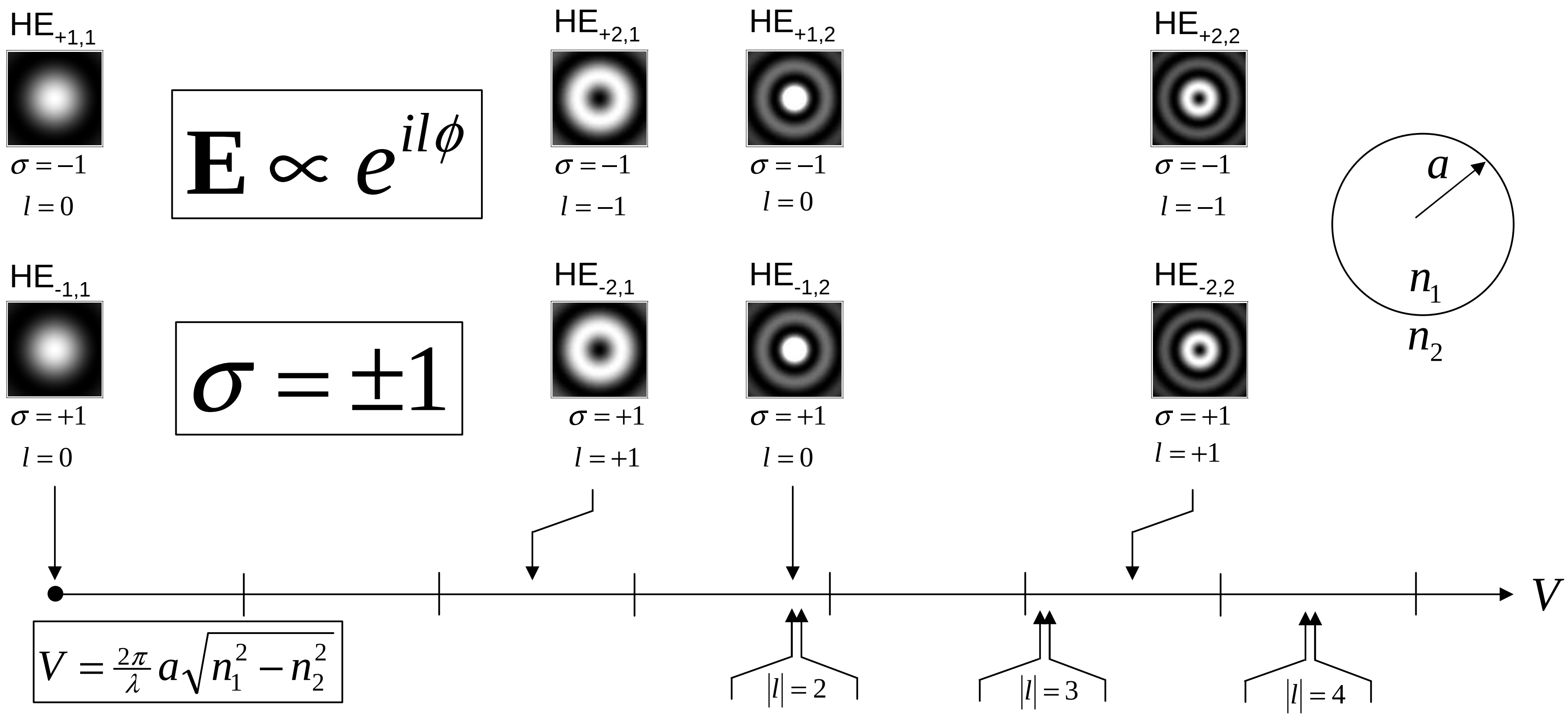
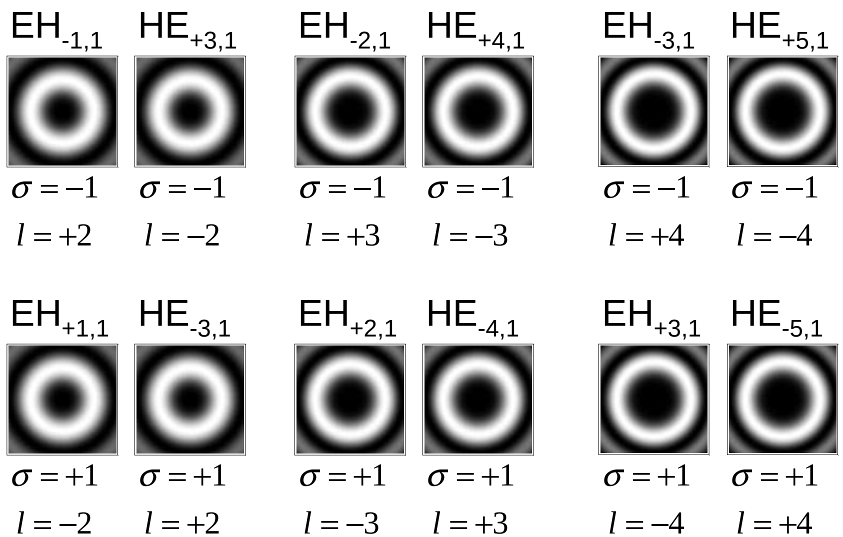
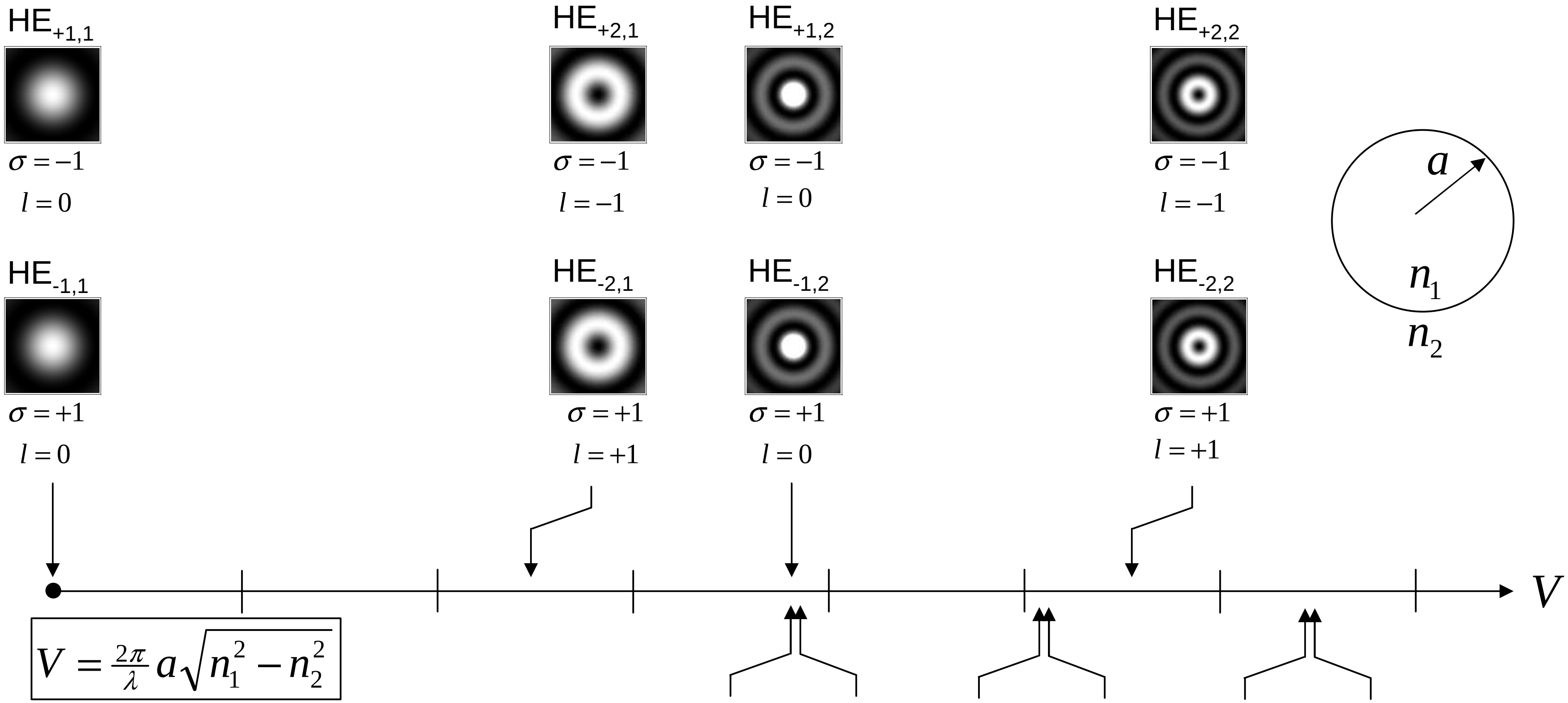
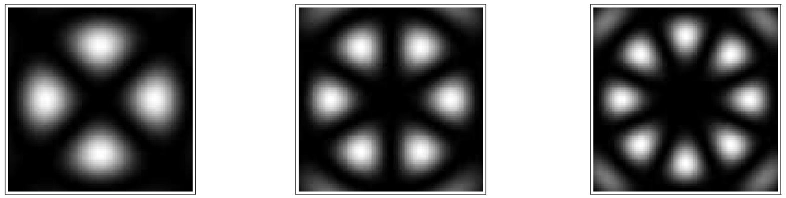


Note that the orbital angular momentum is aligned anti-parallel to the spin angular momentum for all EH modes, but is aligned parallel to the spin for all HE modes. This property applies to any fiber modes that are sufficiently far from cutoff.

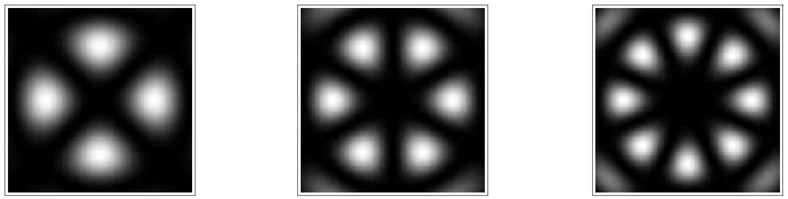




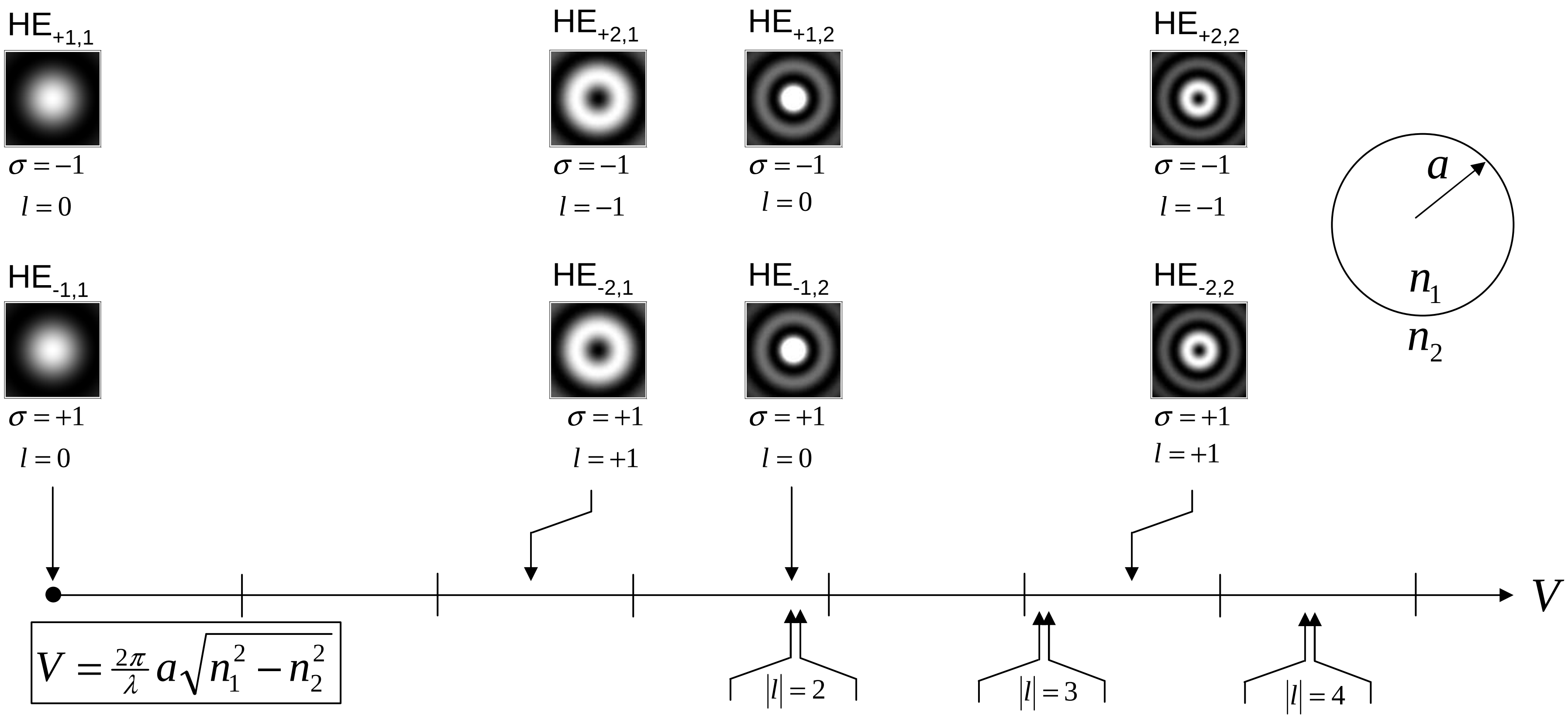
The fields of the respective mode pairs have slightly different propagation constants: for example, $\vec{E}_{EH_{-1,1}} \propto e^{i\beta_1 z}$ and $\vec{E}_{HE_{+3,1}} \propto e^{i\beta_2 z}$. This causes the resultant mode to rotate either clockwise or counterclockwise, depending on the field helicity.



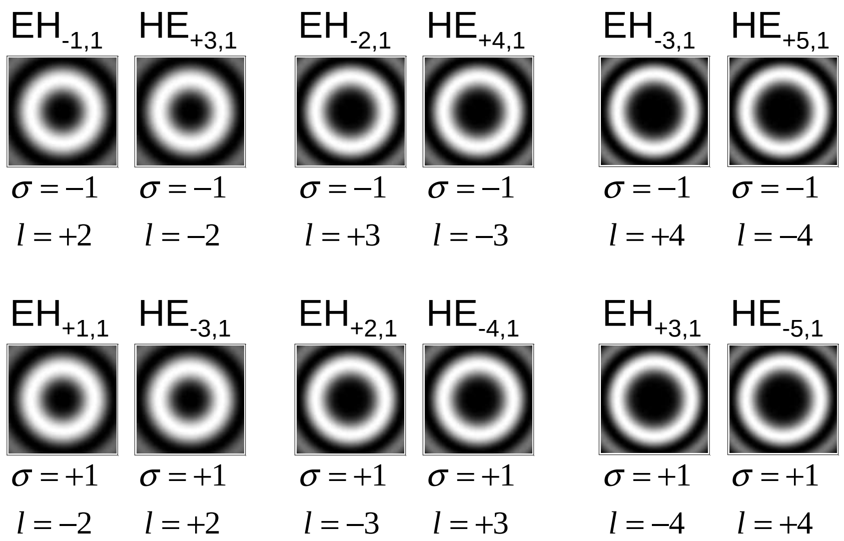
Above mode combinations rotate clockwise

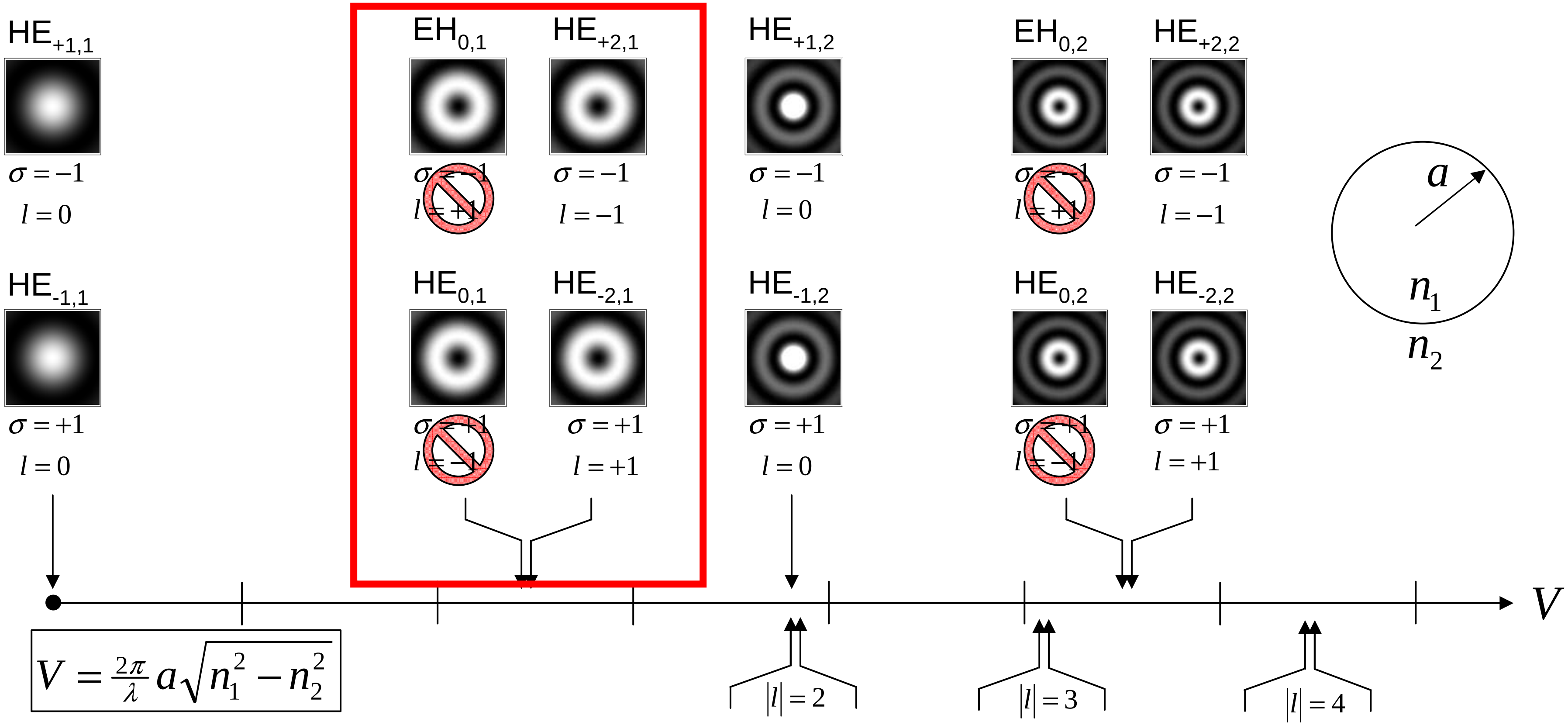


Above mode combinations rotate counterclockwise

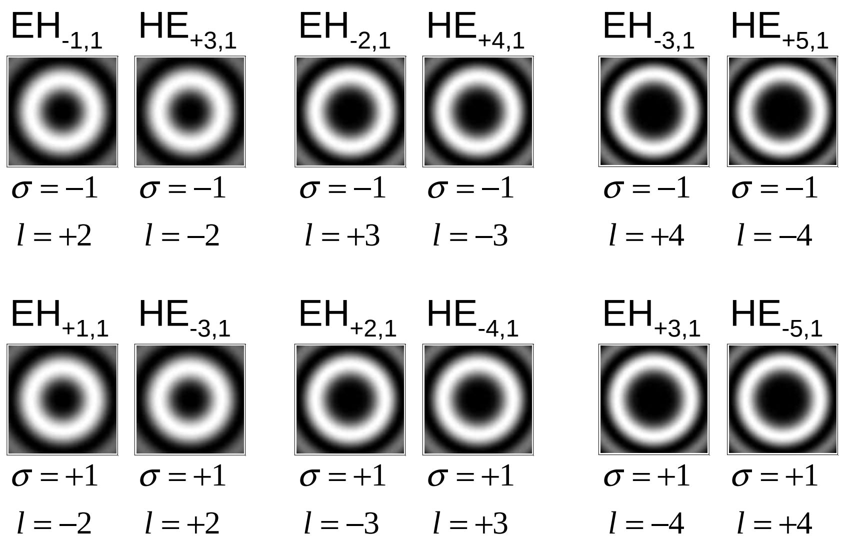


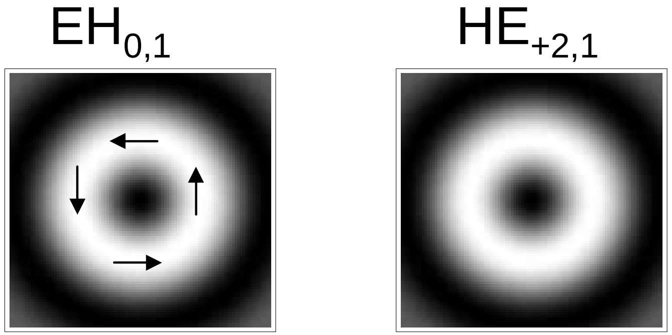
One might expect this pattern to continue, so that the above HE modes might also have nearly degenerate pairs. For example, we might expect the $HE_{+2,1}$ mode to pair up with the $EH_{0,1}$ mode in order to form a rotating, two-lobed resultant mode.





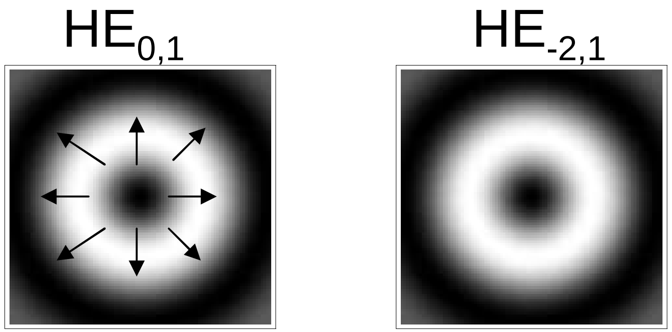
The modes added here *do* have nearly the same propagation constants as their HE$_{+2,m}$ and HE$_{-2,m}$ counterparts, and their spatial intensity distributions are the same, but their *polarization properties* are quite different.





$$\sigma = -1$$

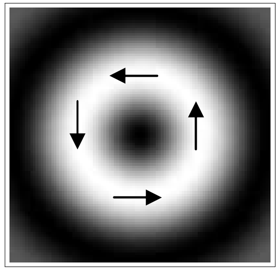
$$l = -1$$

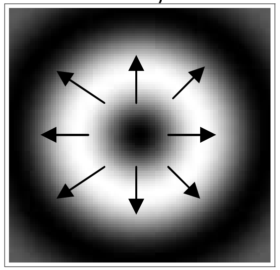


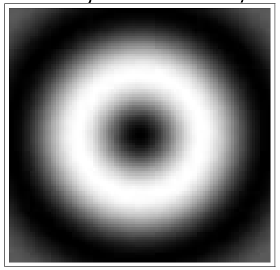
$$\sigma = +1$$

$$l = +1$$

$EH_{0,1}$ $-$ $HE_{0,1}$ $\approx$ $EH_{0,1} - HE_{0,1}$

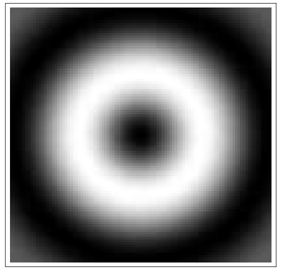






$$\sigma = -1 \qquad l = +1$$

$HE_{+2,1}$

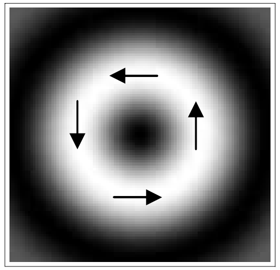


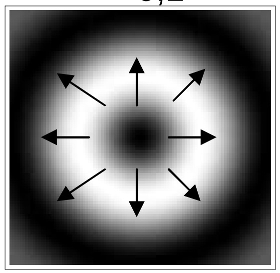
$$\mathbf{E}(\mathbf{r},t) = AJ_1(\tfrac{u_0}{a}r)\left[e^{i\phi}\left(e^{-ik_{z(+)}z} - e^{-ik_{z(-)}z}\right)\mathbf{e}_+ - e^{-i\phi}\left(e^{-ik_{z(+)}z} + e^{-ik_{z(-)}z}\right)\mathbf{e}_-\right]e^{i\omega t}$$

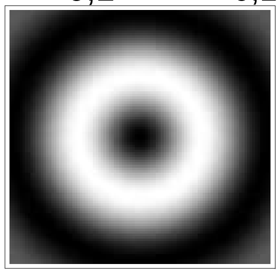
$$\approx -2AJ_1(\tfrac{u_0}{a}r)e^{-i\phi}e^{-i\frac{1}{2}\left(k_{z(+)}+k_{z(-)}\right)z}e^{i\omega t}\mathbf{e}_- \quad \text{for} \quad k_{z(+)} = k_{z(-)} + \frac{2\pi N}{z}$$

$$\sigma = -1 \qquad l = -1$$

$EH_{0,1}$ $+$ $HE_{0,1}$ $\approx$ $EH_{0,1} + HE_{0,1}$

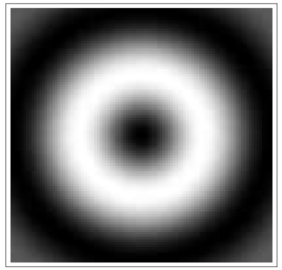






$$\sigma = +1 \qquad l = -1$$

$HE_{-2,1}$

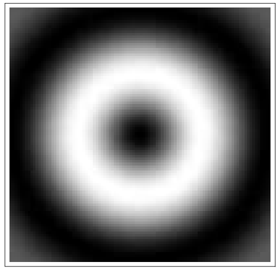


$$\mathbf{E}(\mathbf{r},t) = AJ_1(\tfrac{u_0}{a}r)\left[e^{i\phi}\left(e^{-ik_{z(+)}z} + e^{-ik_{z(-)}z}\right)\mathbf{e}_+ - e^{-i\phi}\left(e^{-ik_{z(+)}z} - e^{-ik_{z(-)}z}\right)\mathbf{e}_-\right]e^{i\omega t}$$

$$\approx 2AJ_1(\tfrac{u_0}{a}r)e^{i\phi}e^{-i\frac{1}{2}\left(k_{z(+)}+k_{z(-)}\right)z}e^{i\omega t}\mathbf{e}_+ \quad \text{for} \quad k_{z(+)} = k_{z(-)} + \frac{2\pi N}{z}$$

$$\sigma = +1 \qquad l = +1$$

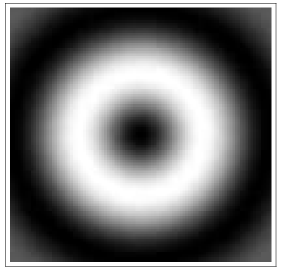
$EH_{0,1}$ - $HE_{0,1}$



$$\sigma = -1$$

$$l = +1$$

$HE_{+2,1}$

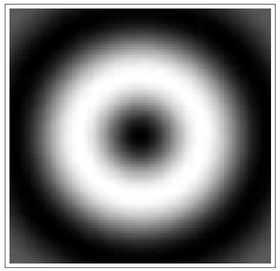


$$\sigma = -1$$

$$l = -1$$

$$k_{z(+)} = k_{z(-)} + \frac{2\pi N}{z}$$

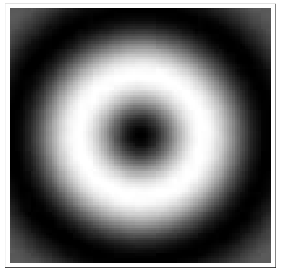
$EH_{0,1}$ + $HE_{0,1}$



$$\sigma = +1$$

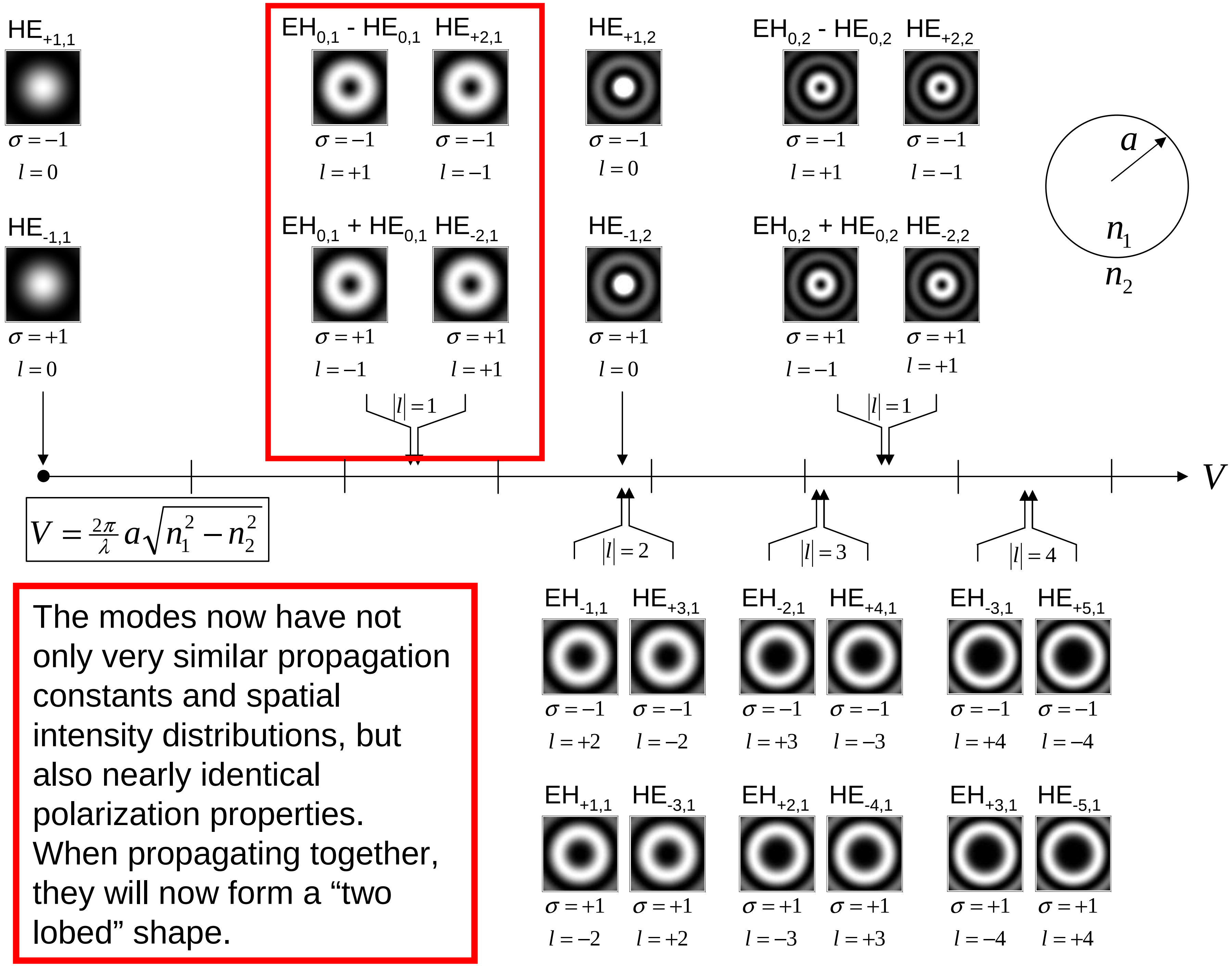
$$l = -1$$

$HE_{-2,1}$

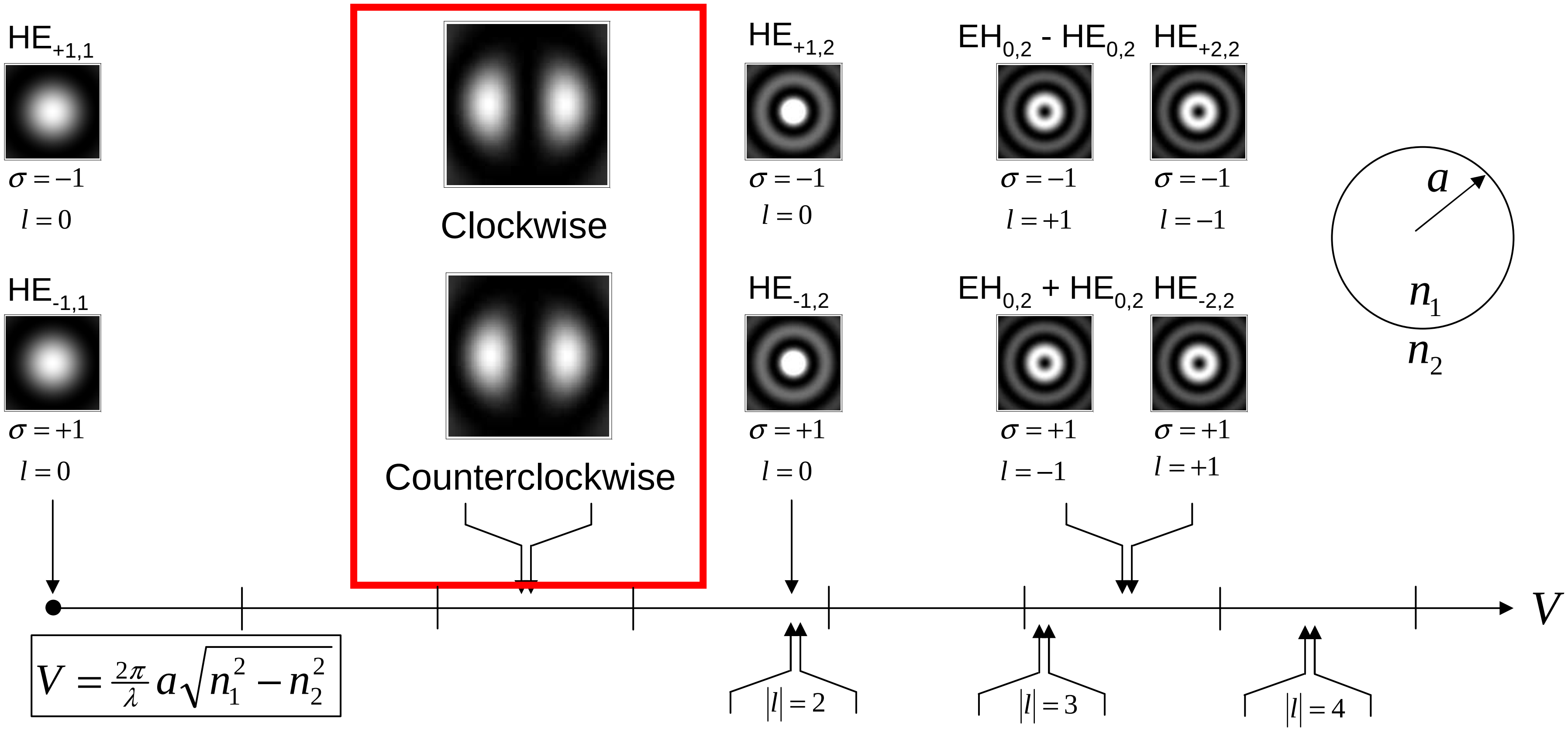


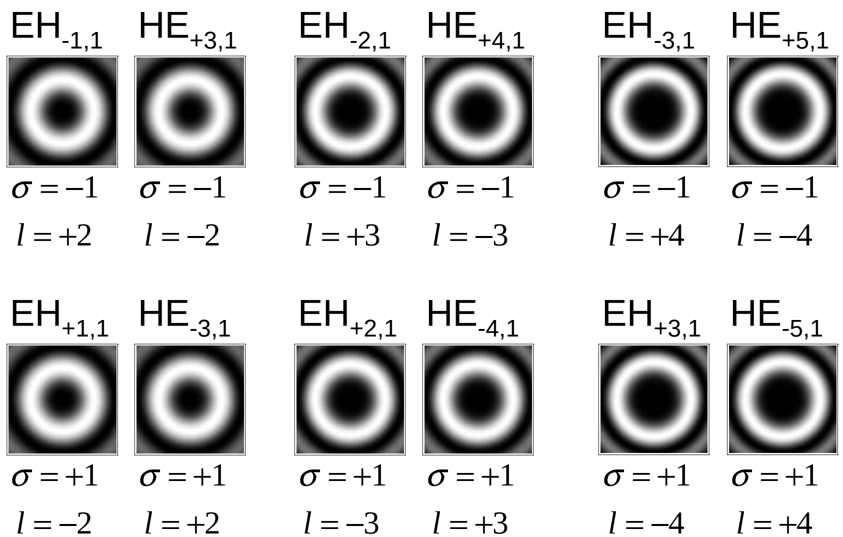
$$\sigma = +1$$

$$l = +1$$

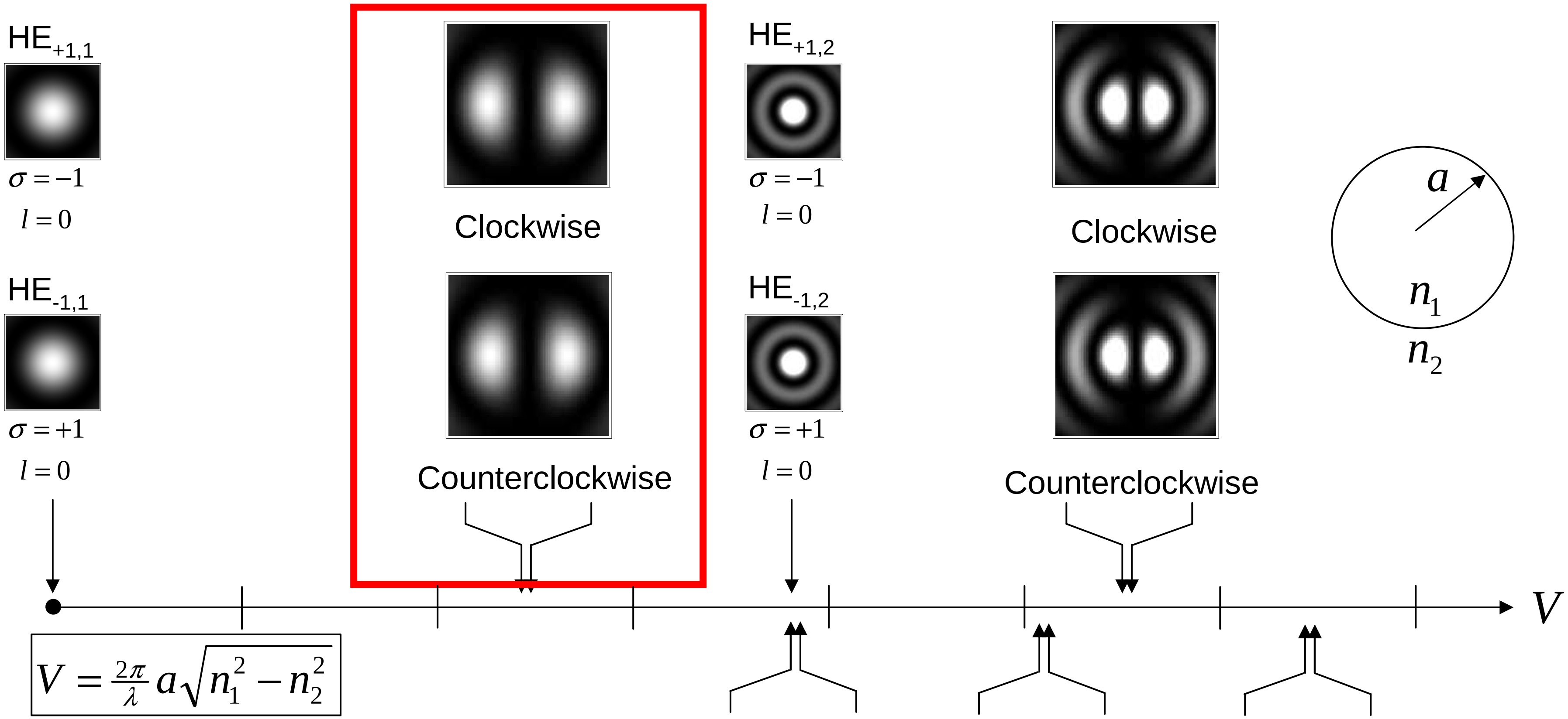


The modes now have not only very similar propagation constants and spatial intensity distributions, but also nearly identical polarization properties. When propagating together, they will now form a “two lobed” shape.

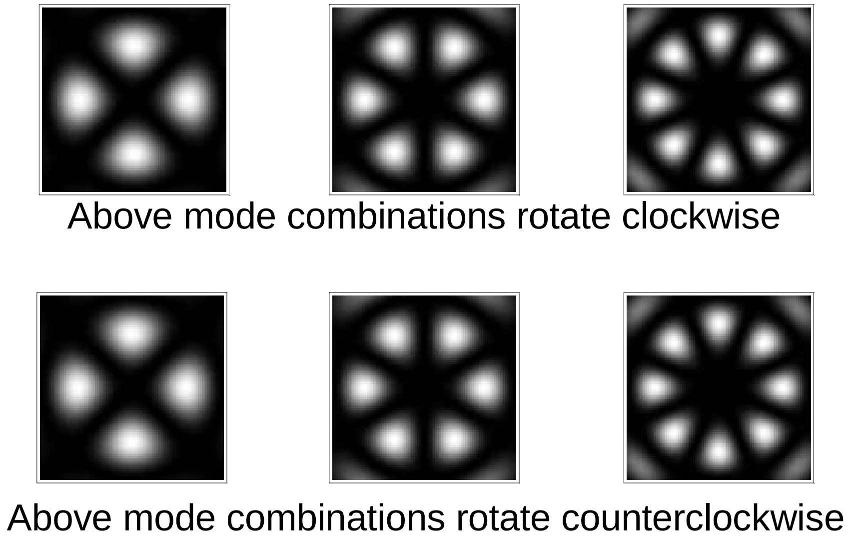


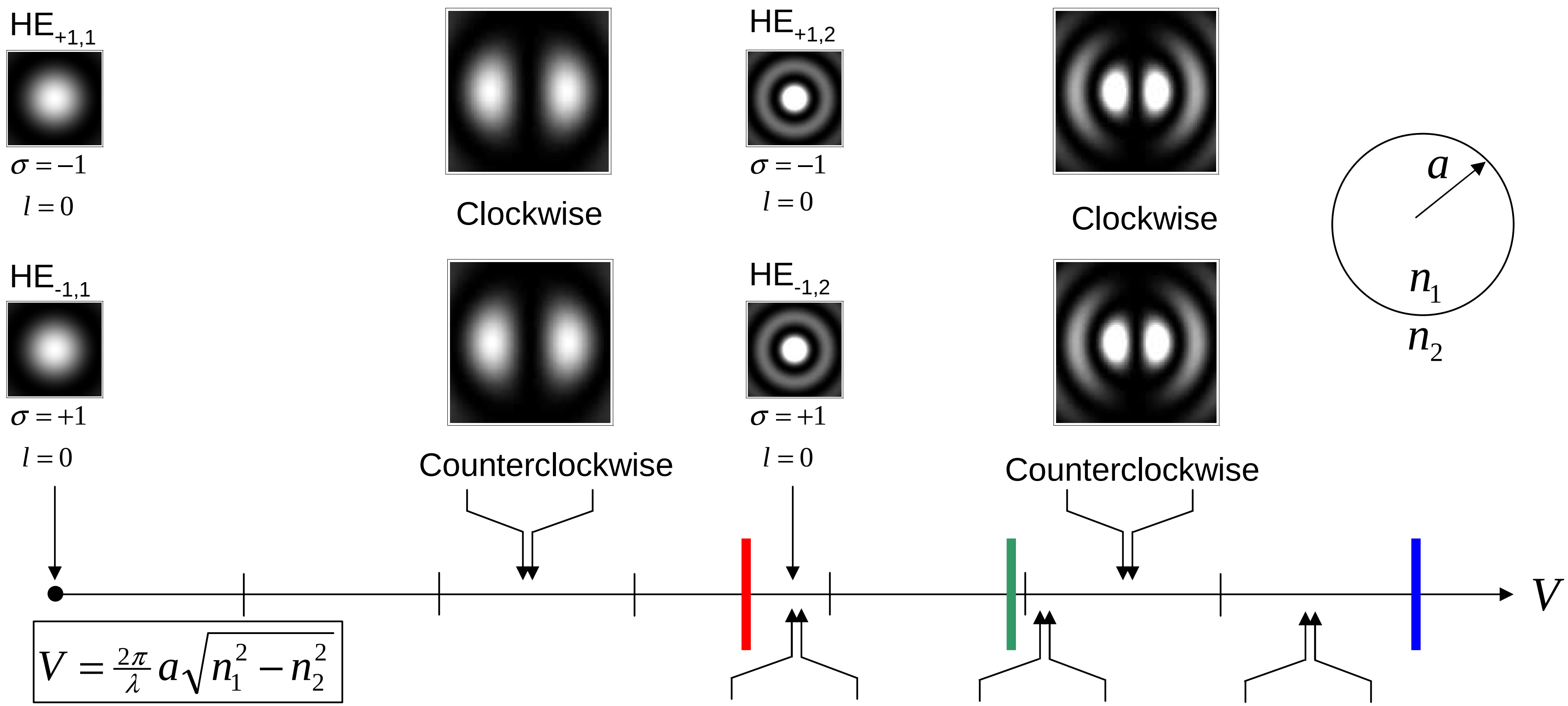


The modes now have not only very similar propagation constants and spatial intensity distributions, but also nearly identical polarization properties. When propagating together, they will now form a “two lobed” shape.

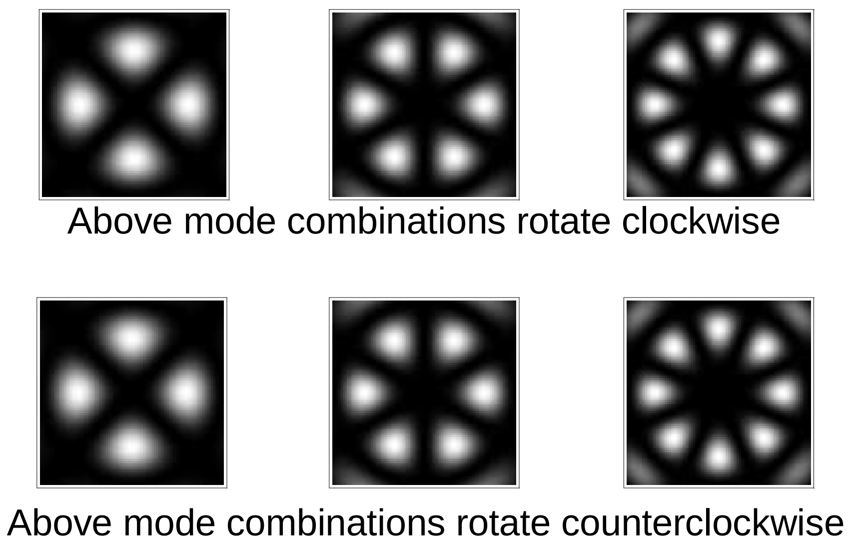


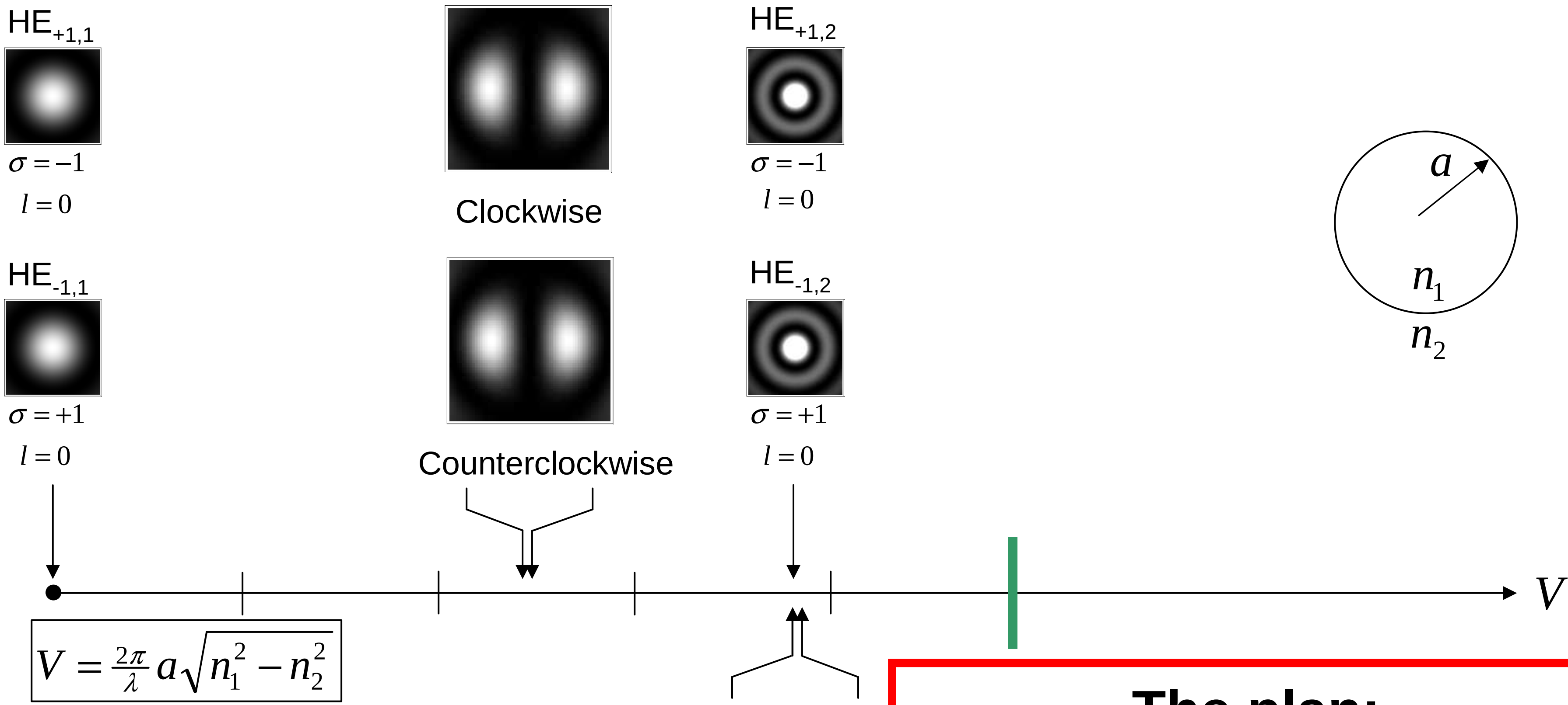
The pattern is complete– every EH mode pairs with a corresponding HE mode in order to form a lobed intensity distribution, while the HE$_{-1,m}$ modes are left unpaired.

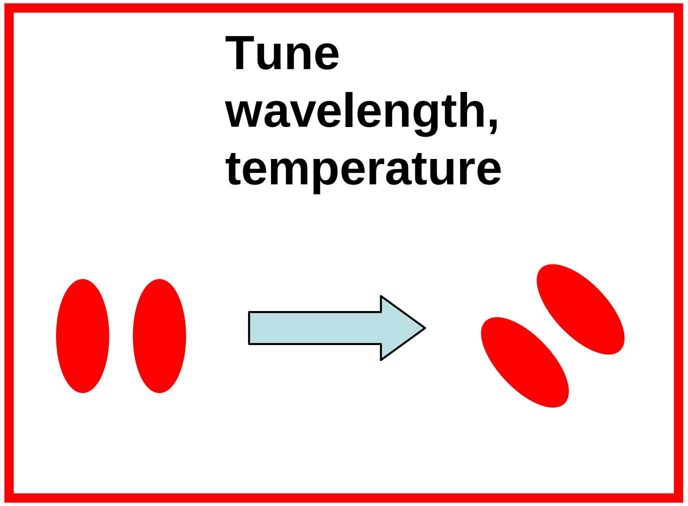


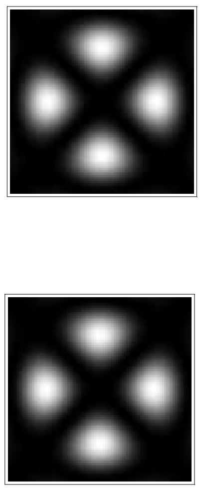


As the fiber V parameter moves from the blue to green to red, the rotational effect is destroyed because the far-from-cutoff approximation loses its validity.









The plan:

- Cut fiber to optimal length
- Work near green line
- Don't excite other modes
- Tune wavelength and temperature to observe real time rotation

Issues/Observations Since May

- The higher order HG mode can scatter into the fundamental mode, but we can filter this using a Sagnac interferrometer
- Wavelength tuning superior to temperature tuning, but can still introduce coupling issues
- A Ti:Saph laser will not lase with a symmetric HG mode, so we use holograms to obtain a tunable HG mode
- Circular symmetry is not broken between the HE and EH "twins", but they travel at different speeds through a step index fiber… ?

Thanks for the hospitality!