Single-Photon Spin-Orbit Interaction for Cluster State Quantum Computation

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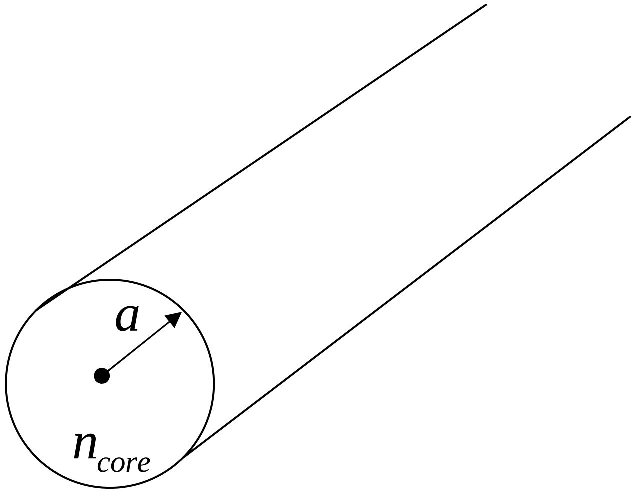
Overview of photon SOI

- Ingredients:
  - Inhomogeneous medium
  - Cylindrical symmetry
  - Quasi-paraxial photon:

    $\kappa << |\mathbf{k}|$, where $\kappa \equiv \sqrt{k_x^2 + k_y^2}$
  - Quasi-monochromatic

- Simplest example:

  step-index optical fiber
  - $n_{core} > n_{cladding}$
  - Infinite cladding radius



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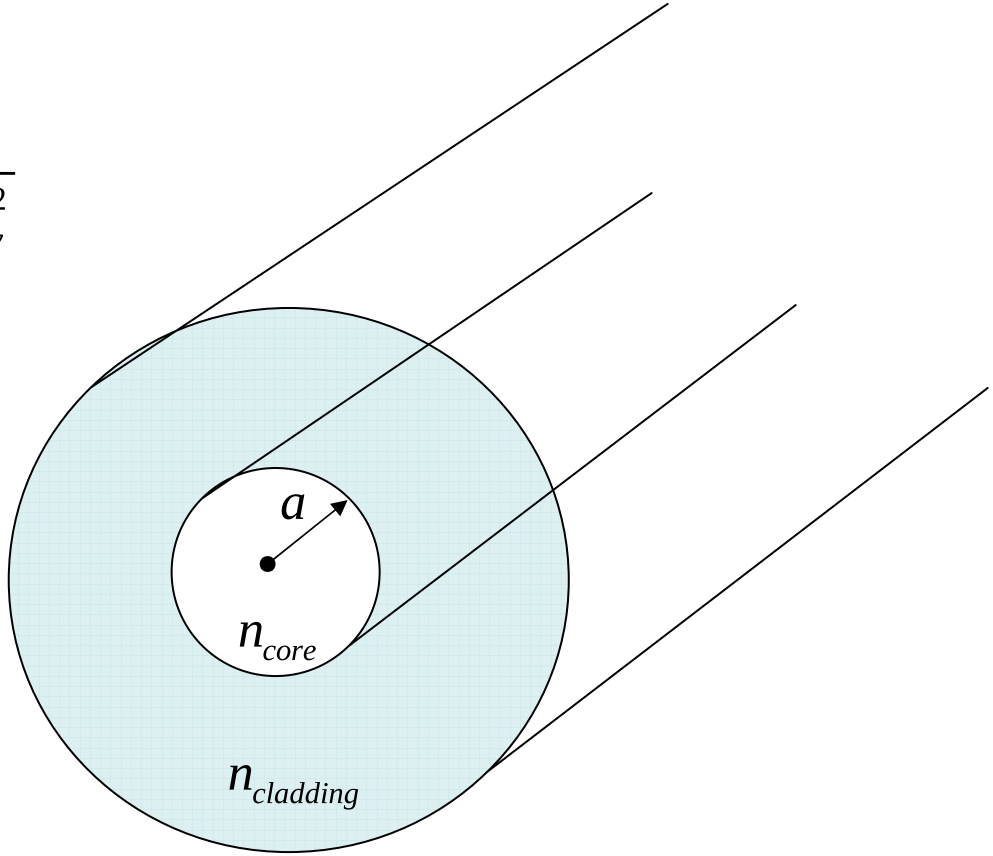


Overview of photon SOI

- Field in fiber proportional to $\mathbf{E}(\mathbf{r},t) \propto e^{i(\beta z-\omega t)}$

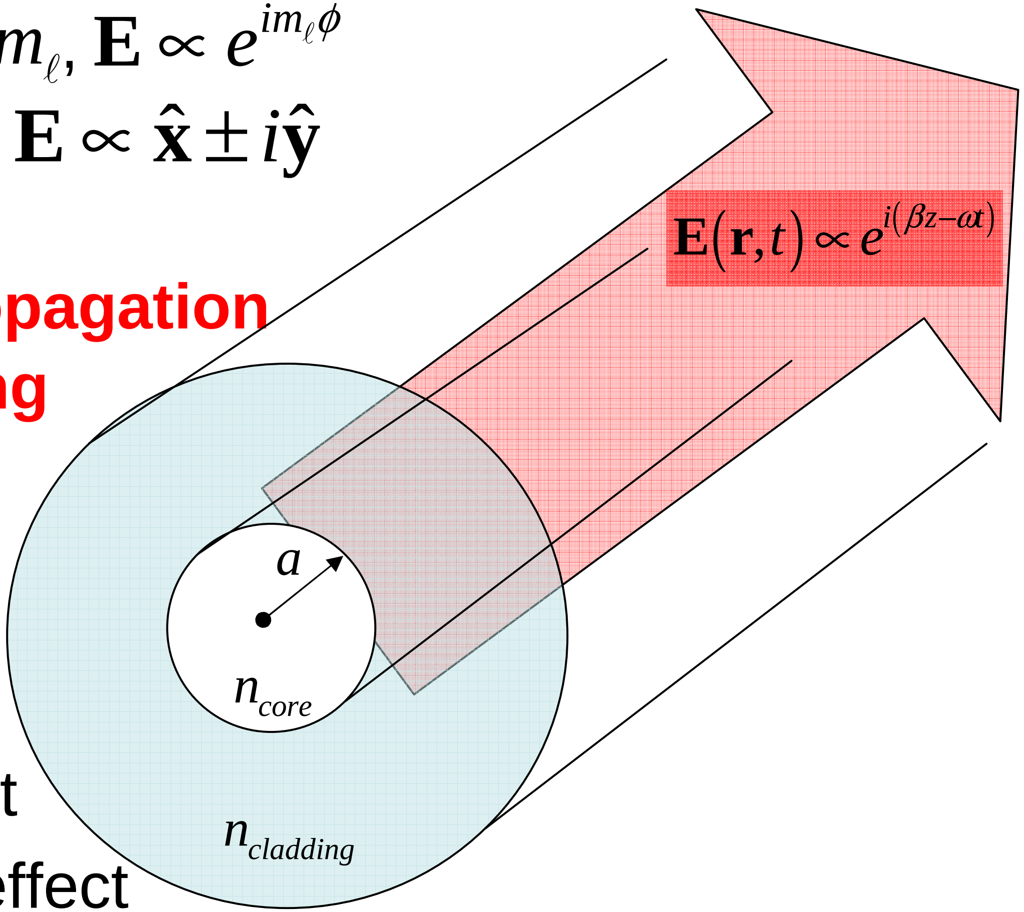
- Fiber modes can have well-defined:

  Orbital angular momentum m_ℓ, $\mathbf{E} \propto e^{im_\ell\phi}$

  Spin angular momentum σ, $\mathbf{E} \propto \hat{\mathbf{x}} \pm i\hat{\mathbf{y}}$

- **The effect: Eigenmode propagation constants β split according to the *product* of σ and m_ℓ**

- The above statement is valid for $n_{core} \approx n_{cladding}$ and for modes far from cutoff, but SOI is also a more general effect



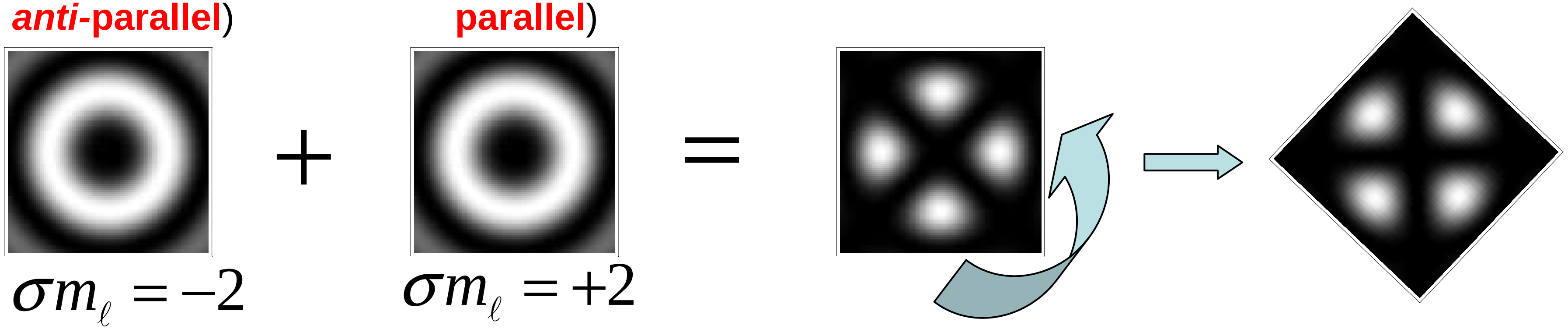
Observable consequence of SOI

- If two fiber photon eigenmodes with opposing values of the quantity σm_ℓ propagate in superposition, the relative phase between them will vary as the photon field propagates, due to their velocity mismatch.
- This phase shift gives rise to a σ-dependent rotational effect. For example:

$\mathbf{E} \propto e^{-i2\phi}\left(\hat{\mathbf{x}}+i\hat{\mathbf{y}}\right)$ (σ and m_ℓ ***anti*-parallel**)

$\mathbf{E} \propto e^{+i2\phi}\left(\hat{\mathbf{x}}+i\hat{\mathbf{y}}\right)$ (σ and m_ℓ **parallel**)

The superposition mode field rotates **clockwise** or **counterclockwise**, as determined by the sign of σ



$\sigma m_\ell = -2$ $\sigma m_\ell = +2$

Observable consequence of SOI

This is a spin-controlled spatial-mode Hadamard gate: a cluster state requirement

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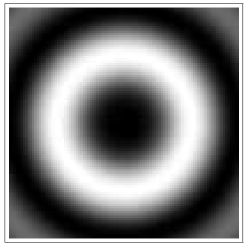
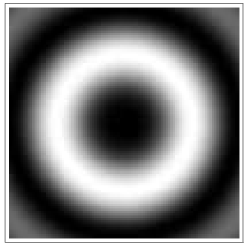
(σ and m_ℓ ***anti*-parallel**)



$\sigma m_\ell = -2$

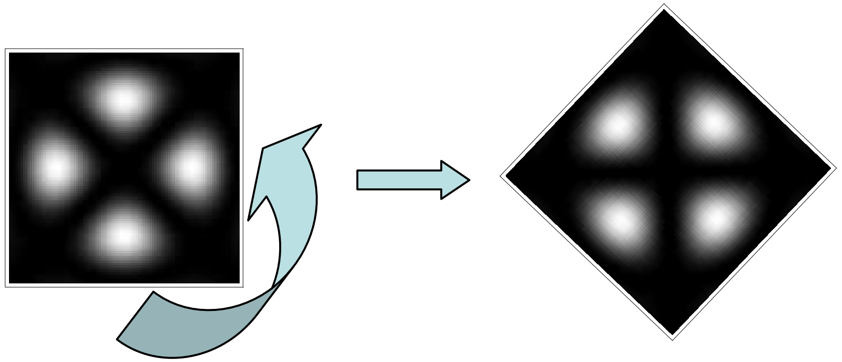
$+$

(σ and m_ℓ **parallel**)



$\sigma m_\ell = +2$

$=$

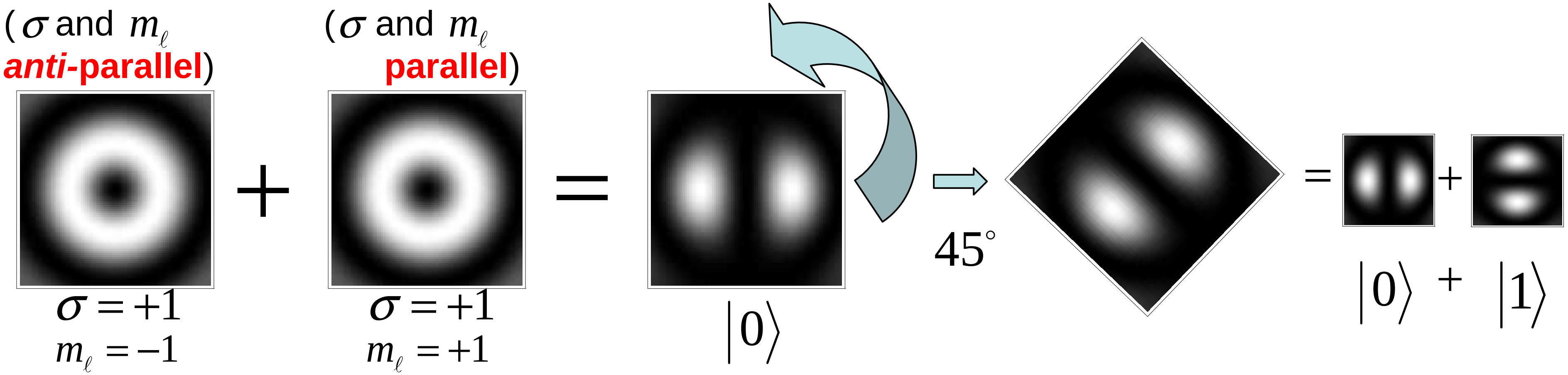


Spin-controlled Hadamard gate: simplest example

$$\mathbf{E} \propto \cos\left[m_\ell \varphi\right] \Longrightarrow \cos\left[m_\ell\left(\phi + \sigma \Delta\beta z\right)\right]$$

(σ and m_ℓ ***anti*-parallel**)

(σ and m_ℓ **parallel**)



$\sigma = +1$
$m_\ell = -1$

$\sigma m_\ell = -1$

$\sigma = +1$
$m_\ell = +1$

$\sigma m_\ell = +1$

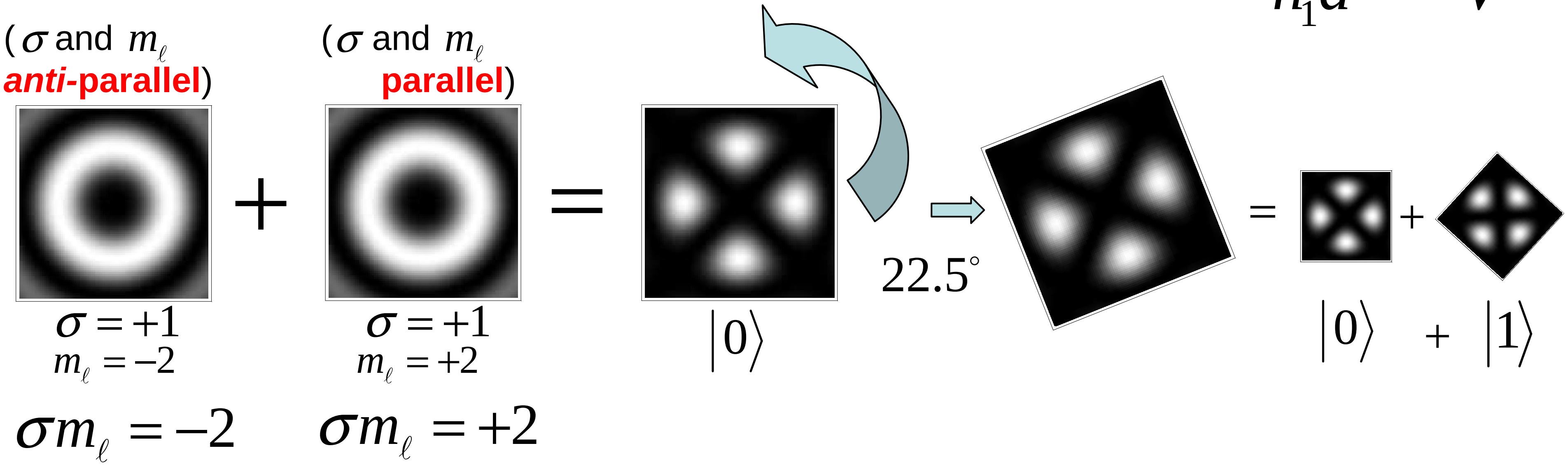
Flipping the photon spin (circular polarization) also flips the direction of rotation of the superposition spatial mode.

Spin-controlled Hadamard gate: generalized example

$$\mathbf{E} \propto \cos\left[m_\ell \varphi\right] \longrightarrow \cos\left[m_\ell\left(\phi + \sigma \Delta\beta z\right)\right]$$

$$V \equiv \frac{\omega a}{c}\sqrt{n_{core}^2 - n_{cladding}^2}$$

$$\boxed{\text{Mode rotation rate:}} \quad \Delta\beta \approx \left|m_\ell\right| \frac{\sqrt{n_1^2 - n_2^2}}{n_1 a} \frac{3U_m^2}{V^3}$$



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$\sigma m_\ell = +2$

$|0\rangle$

22.5°

$= |0\rangle + |1\rangle$

This gate can in principle be realized for arbitrary OAM, provided that the fiber mode remains far from cutoff.

Spin-controlled Hadamard gate: generalized example

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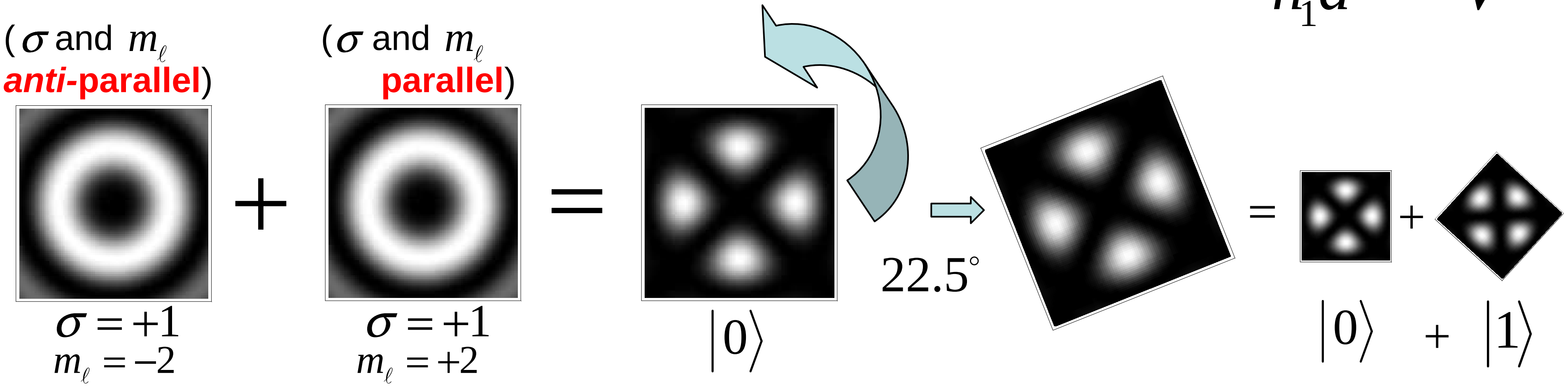

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A spatial-mode Hadamard gate is one of two basic operations required in order to prepare multi-photon cluster states entangled in spatial mode

How does it work?

Physics of the photon spin-orbit interaction

- It is the inhomogeneity of the medium gives rise to the effect– there is no paraxial free-space SOI

- B. Zeldovich (PRA ’91) first mentioned photon SOI:
  - He treats a many-mode fiber with a parabolic index profile
  - Rotation of speckle pattern observed, but not of single modes
  - Kapany and Burke (’72) mention mode rotation, but not in the context of single photons or of spin-orbit interaction.

- We consider individual fiber modes propagating in a step-index fiber
  - The SOI effect is strongest for the step-index case
  - This is due to the $-\nabla\left[\mathbf{E}\cdot\nabla\left(\ln\varepsilon(\mathbf{r})\right)\right]$ term in the Helmholtz eqn

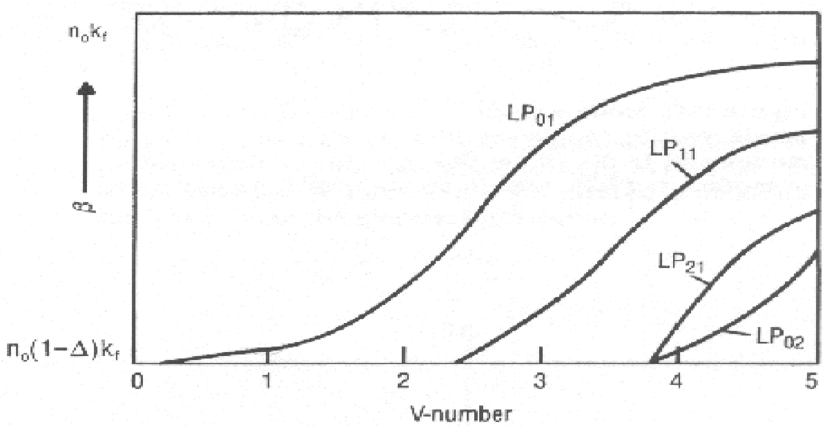
Physics of photon SOI

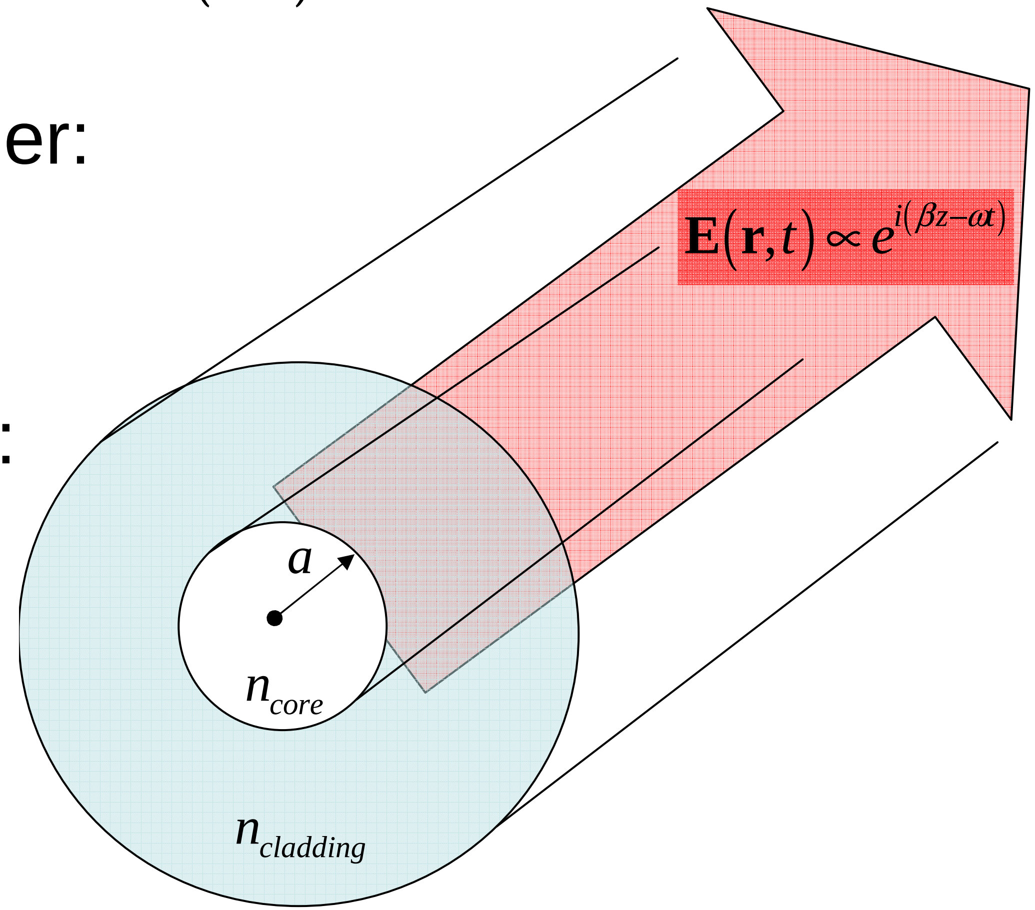
- Field in fiber proportional to $\mathbf{E}(\mathbf{r},t) \propto e^{i(\beta z-\omega t)}$

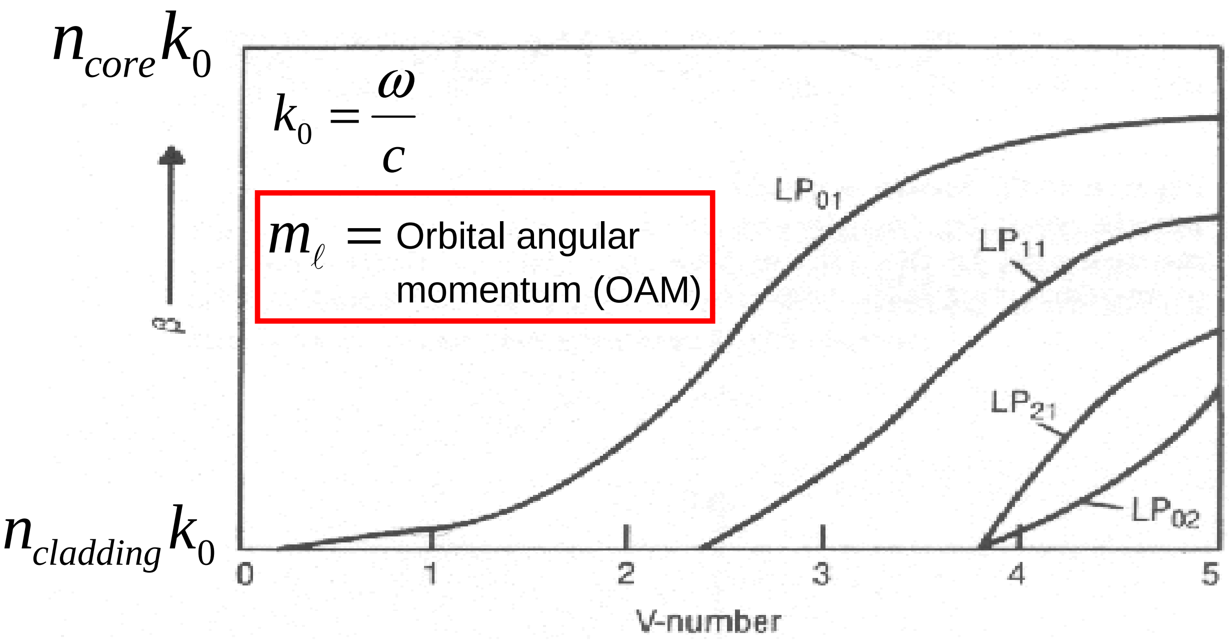
- Define the fiber V-number:

$$V \equiv \frac{\omega a}{c}\sqrt{n_{core}^2 - n_{cladding}^2}$$

- Standard “LP” solutions:



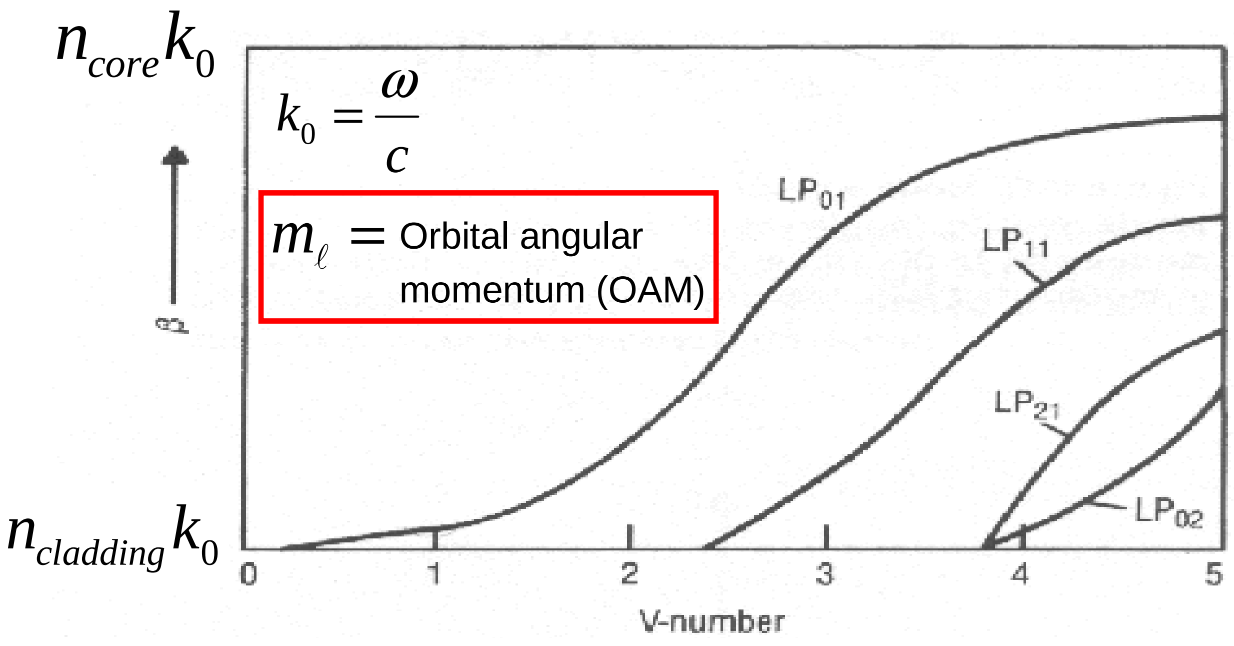




$k_0 = \frac{\omega}{c}$

$m_\ell =$ Orbital angular momentum (OAM)

- Standard “LP” approximation: modes have well-defined orbital angular momentum (OAM)



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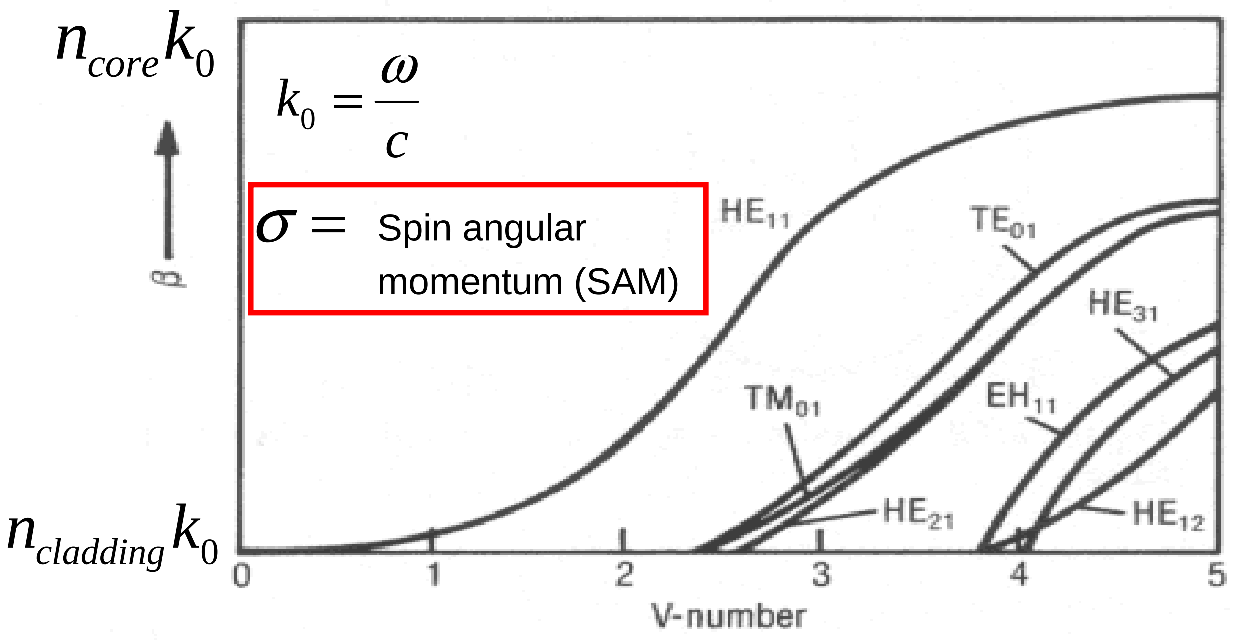
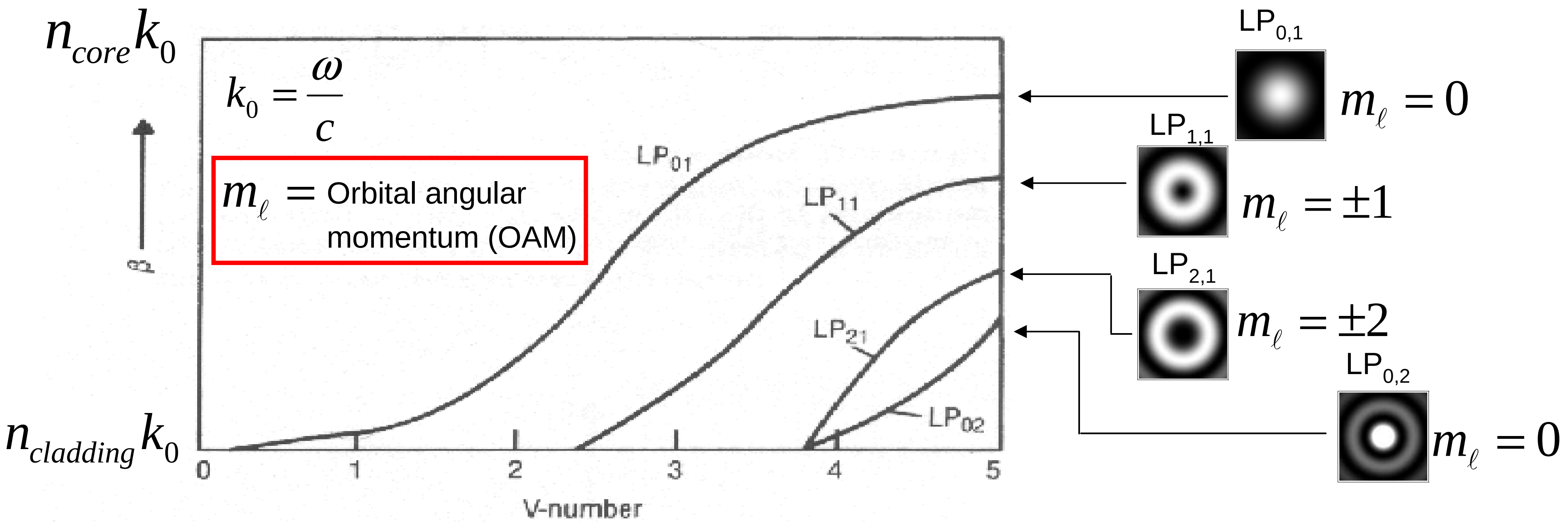


- Exact solutions of Maxwell’s equations: modes have well-defined OAM **and** well-defined spin angular momentum (SAM)–assuming the far-from-cutoff condition $V >> 1$

Source of figures: http://electron9.phys.utk.edu/optics507/modules/m5/single.htm



$n_{core}k_0$

$k_0 = \frac{\omega}{c}$

$m_\ell =$ Orbital angular momentum (OAM)

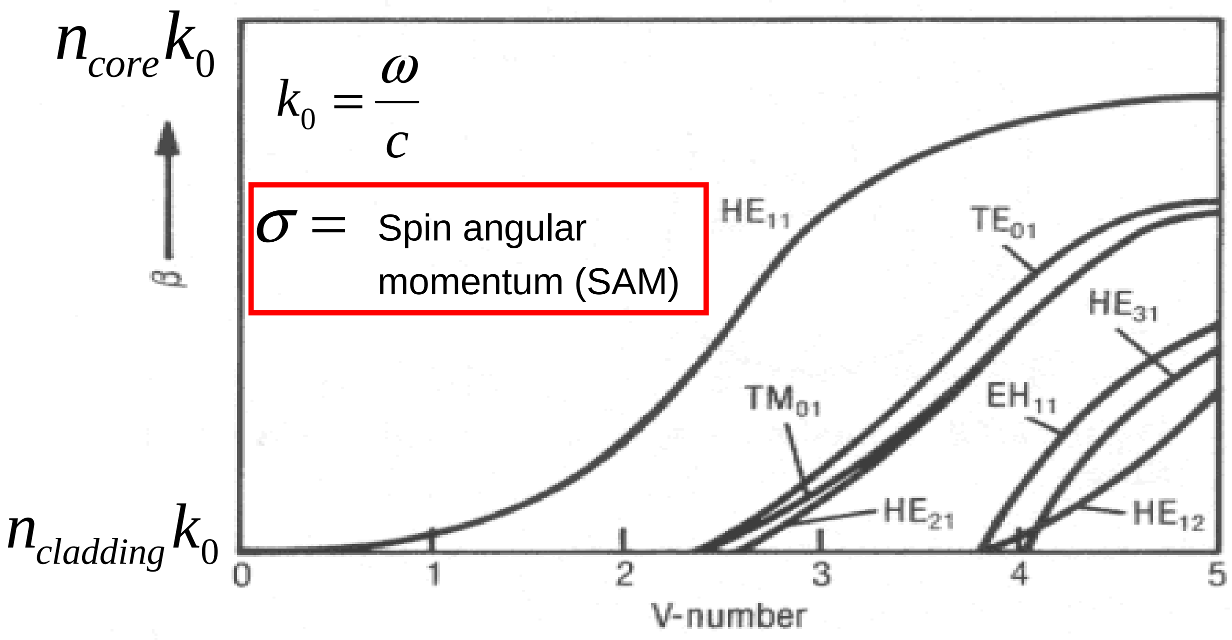
$n_{cladding}k_0$

$LP_{0,1}$ $m_\ell = 0$

$LP_{1,1}$ $m_\ell = \pm 1$

$LP_{2,1}$ $m_\ell = \pm 2$

$LP_{0,2}$ $m_\ell = 0$

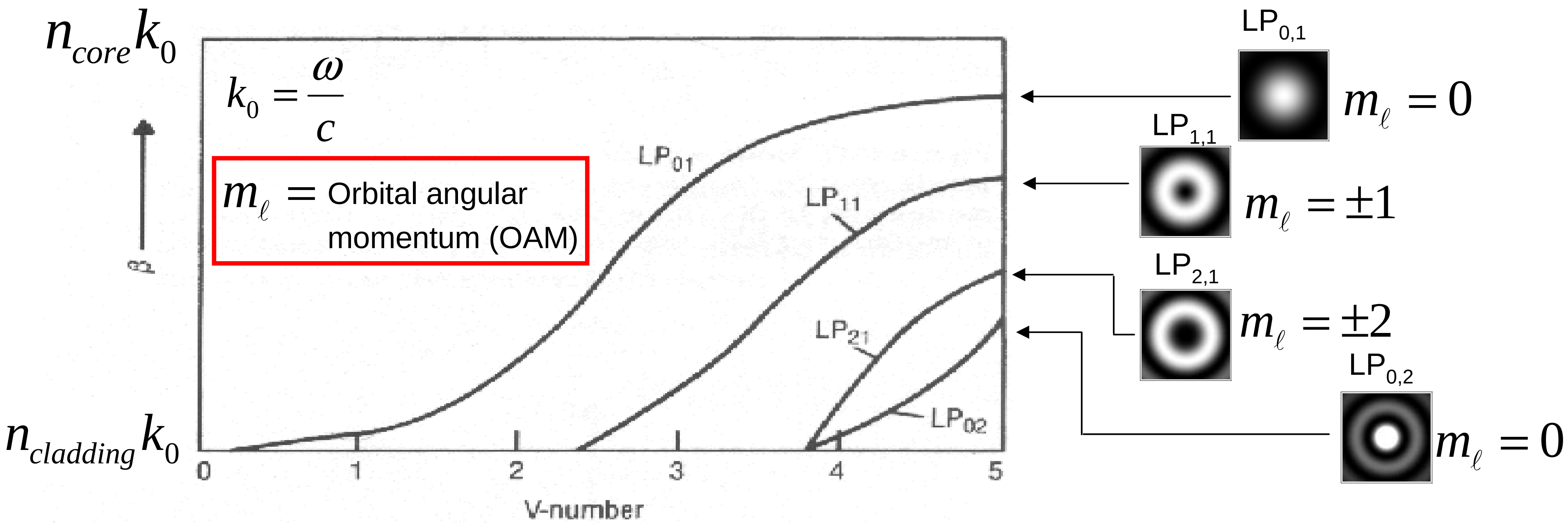


$n_{core}k_0$

$k_0 = \frac{\omega}{c}$

$\sigma =$ Spin angular momentum (SAM)

$n_{cladding}k_0$



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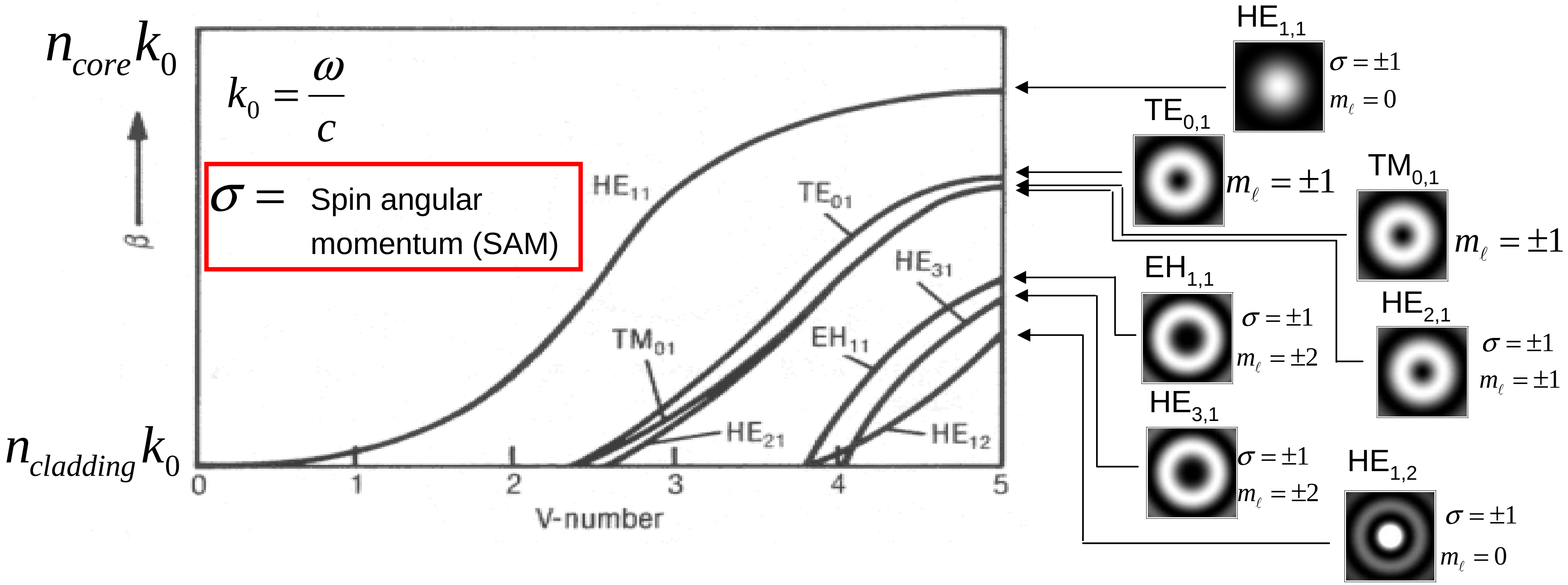

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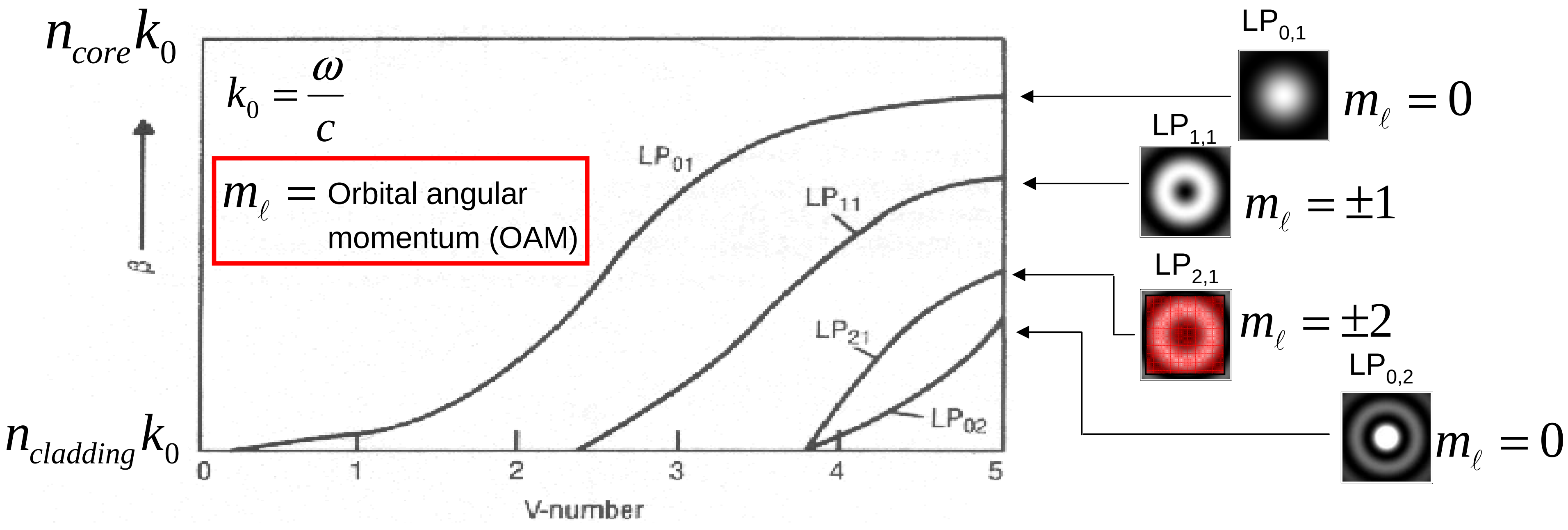

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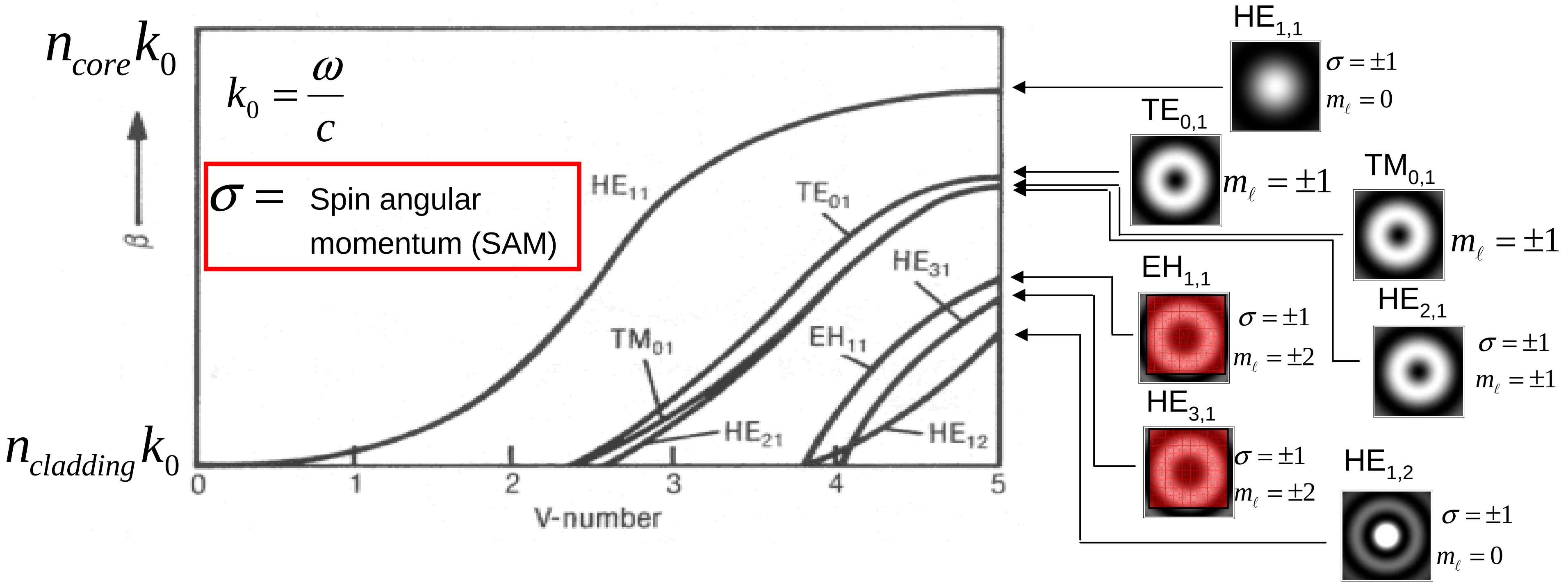

$n_{core}k_0$

$k_0 = \frac{\omega}{c}$

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$n_{cladding}k_0$

$HE_{1,1}$ $\sigma = \pm 1$ $m_\ell = 0$

$TE_{0,1}$ $m_\ell = \pm 1$

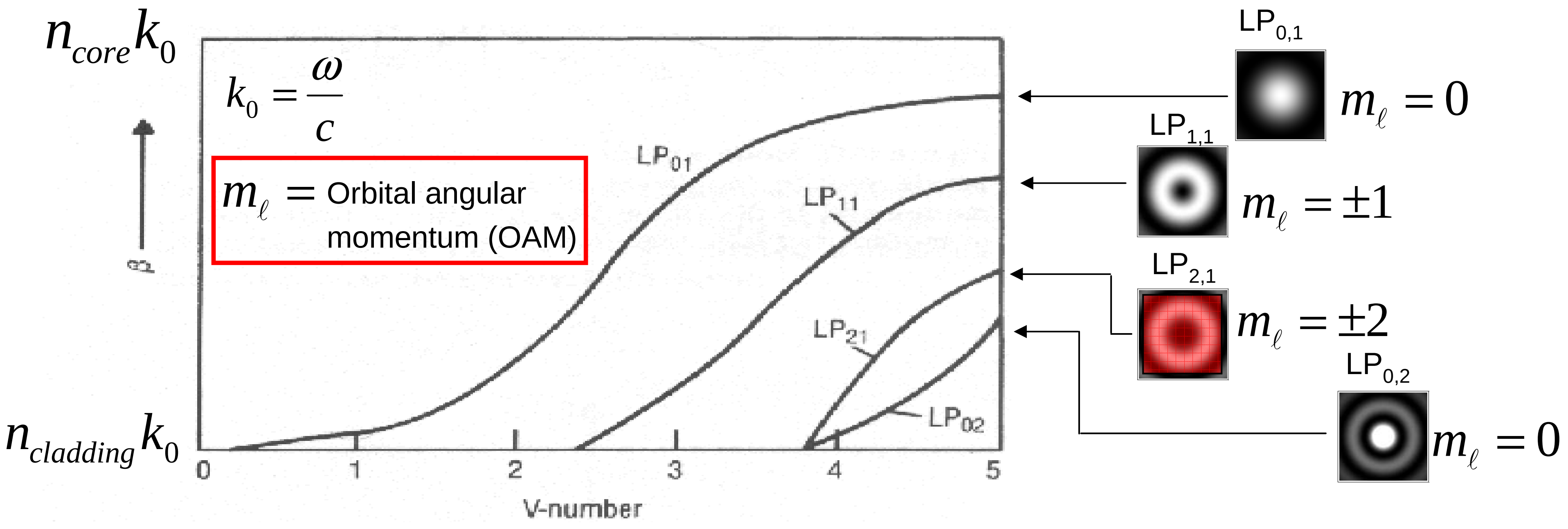
$TM_{0,1}$ $m_\ell = \pm 1$

$EH_{1,1}$ $\sigma = \pm 1$ $m_\ell = \pm 2$

$HE_{2,1}$ $\sigma = \pm 1$ $m_\ell = \pm 1$

$HE_{3,1}$ $\sigma = \pm 1$ $m_\ell = \pm 2$

$HE_{1,2}$ $\sigma = \pm 1$ $m_\ell = 0$



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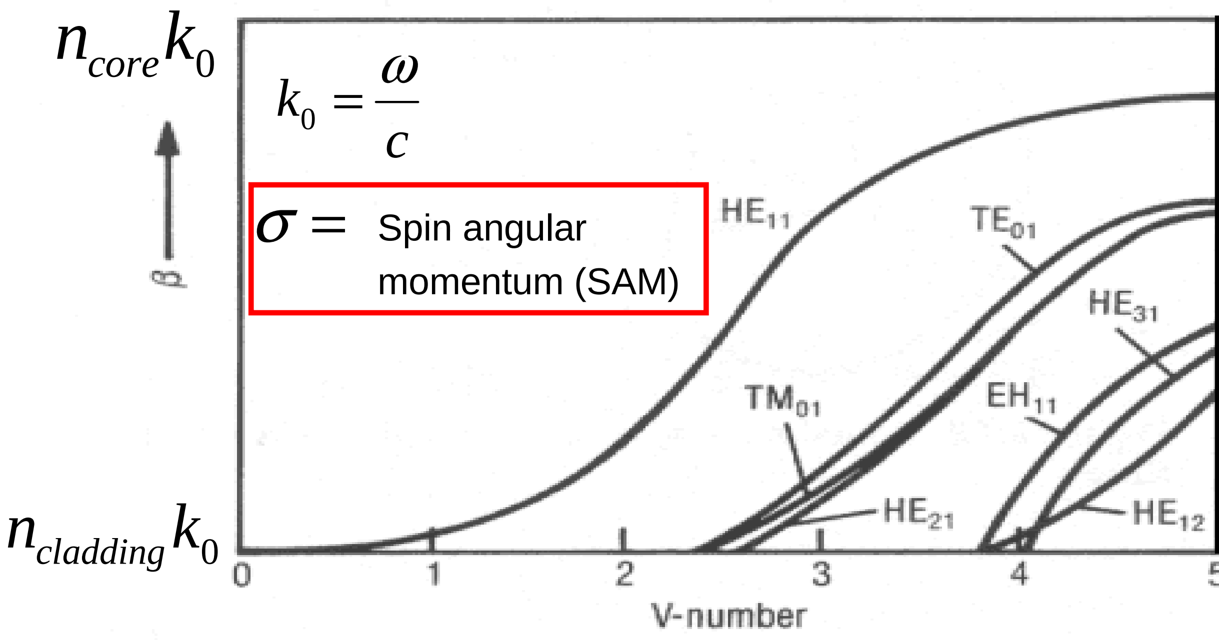

$k_0 = \frac{\omega}{c}$

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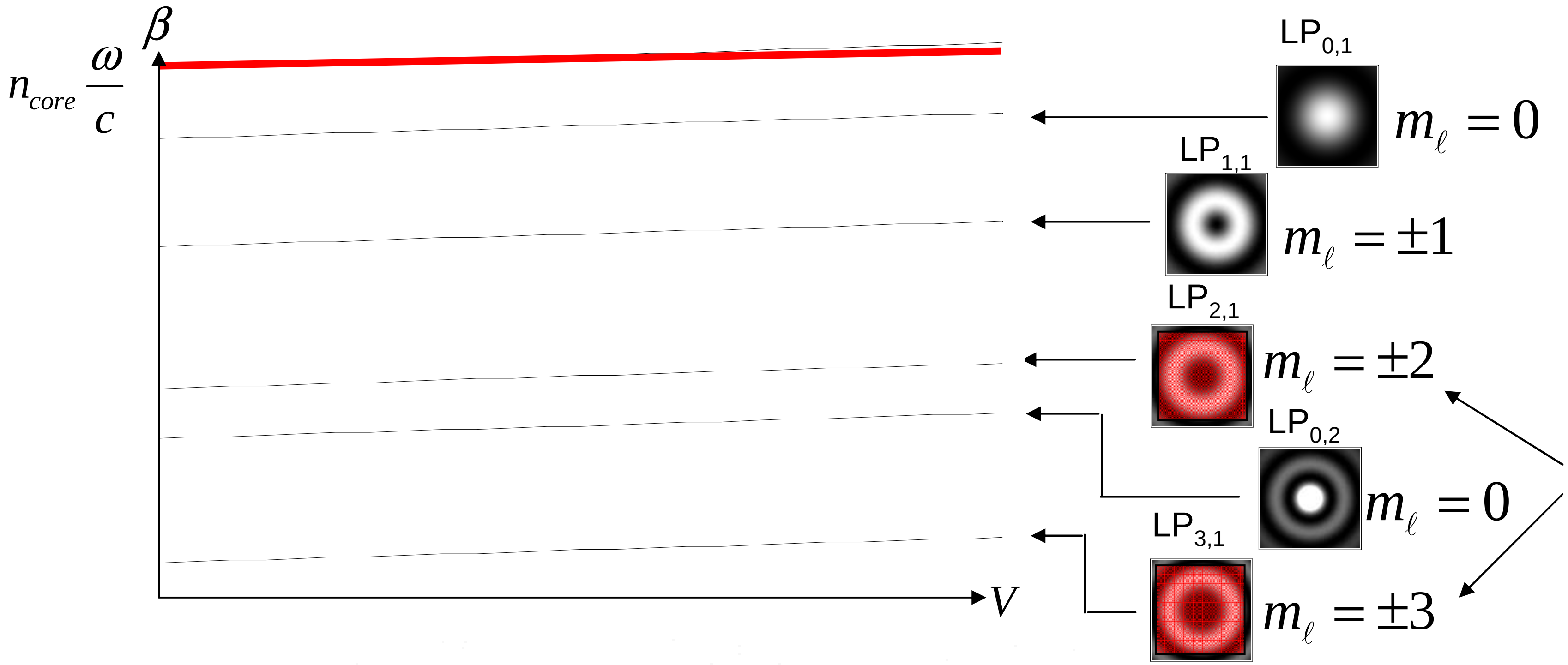
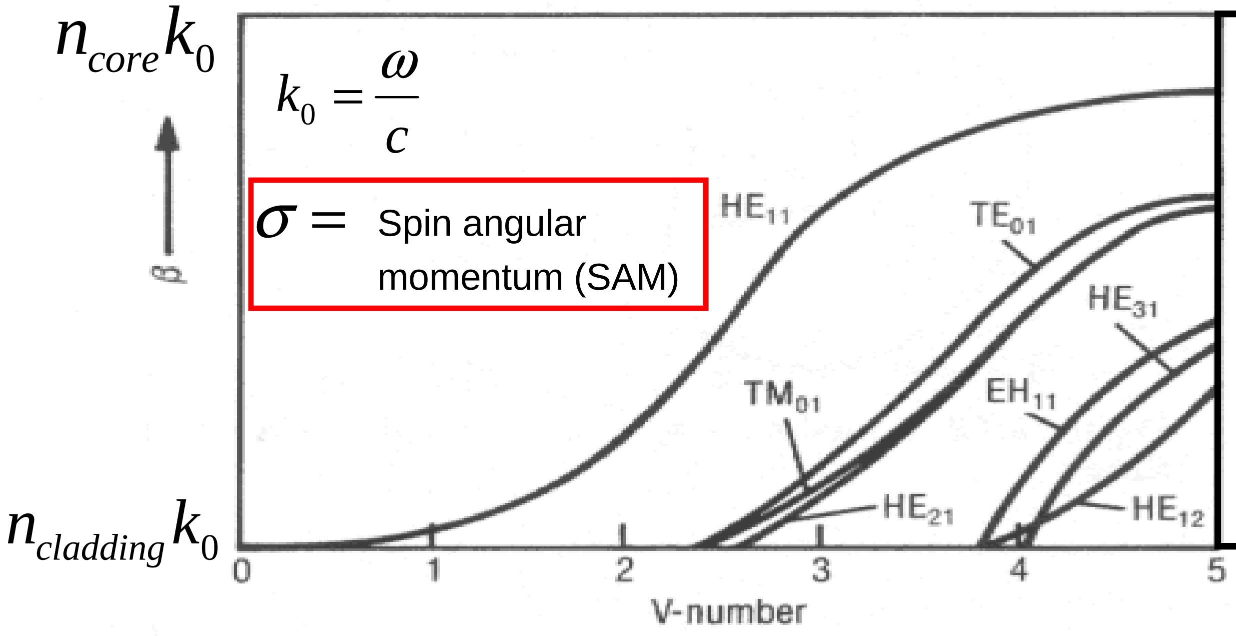
We focus on the far-from-cutoff regime, where

$V >> 1$



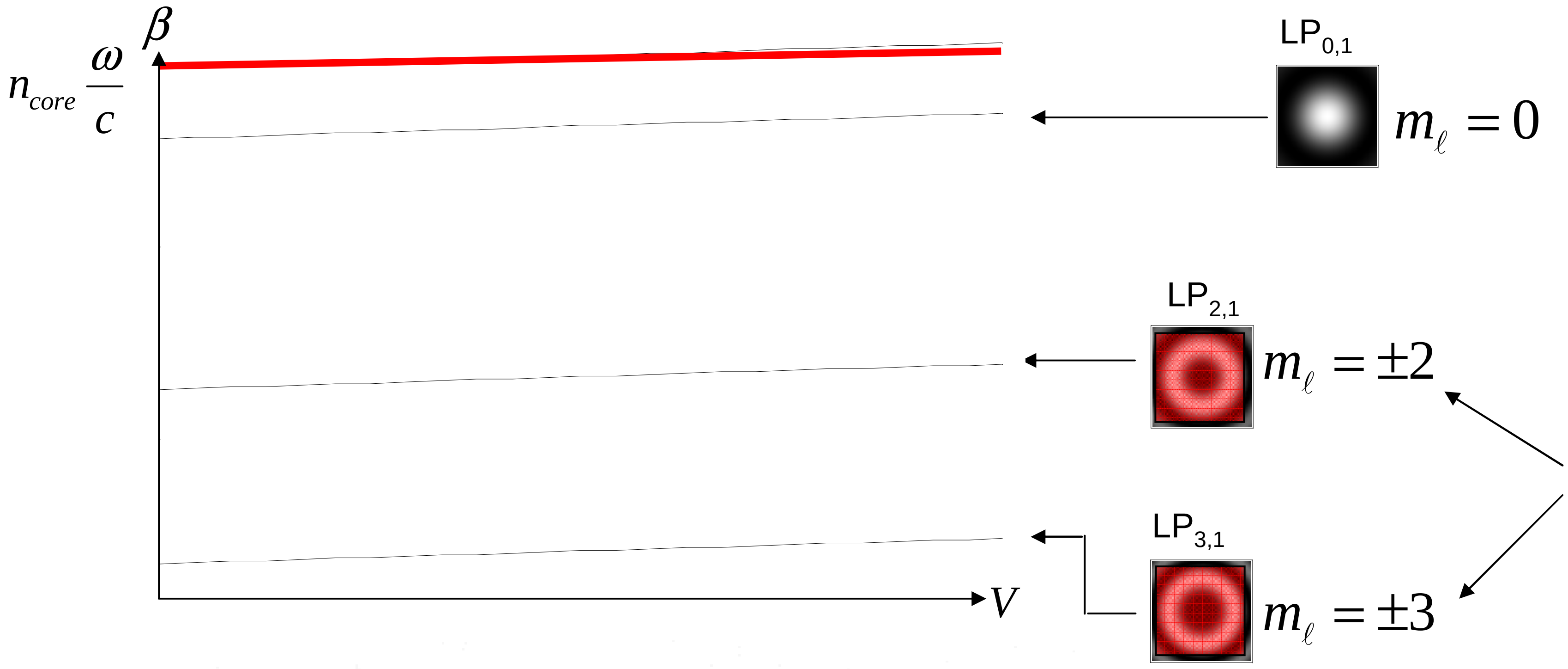
We focus on these modes



$\sigma =$ Spin angular momentum (SAM)

The above plot is for large values of the V-number

$V >> 1$



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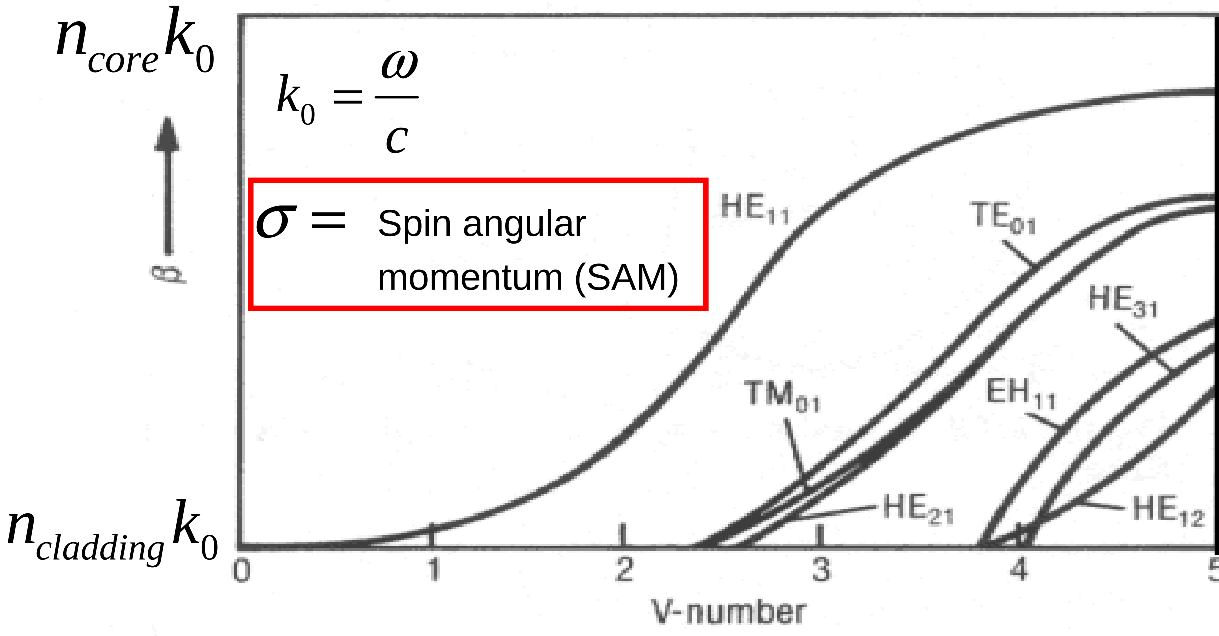
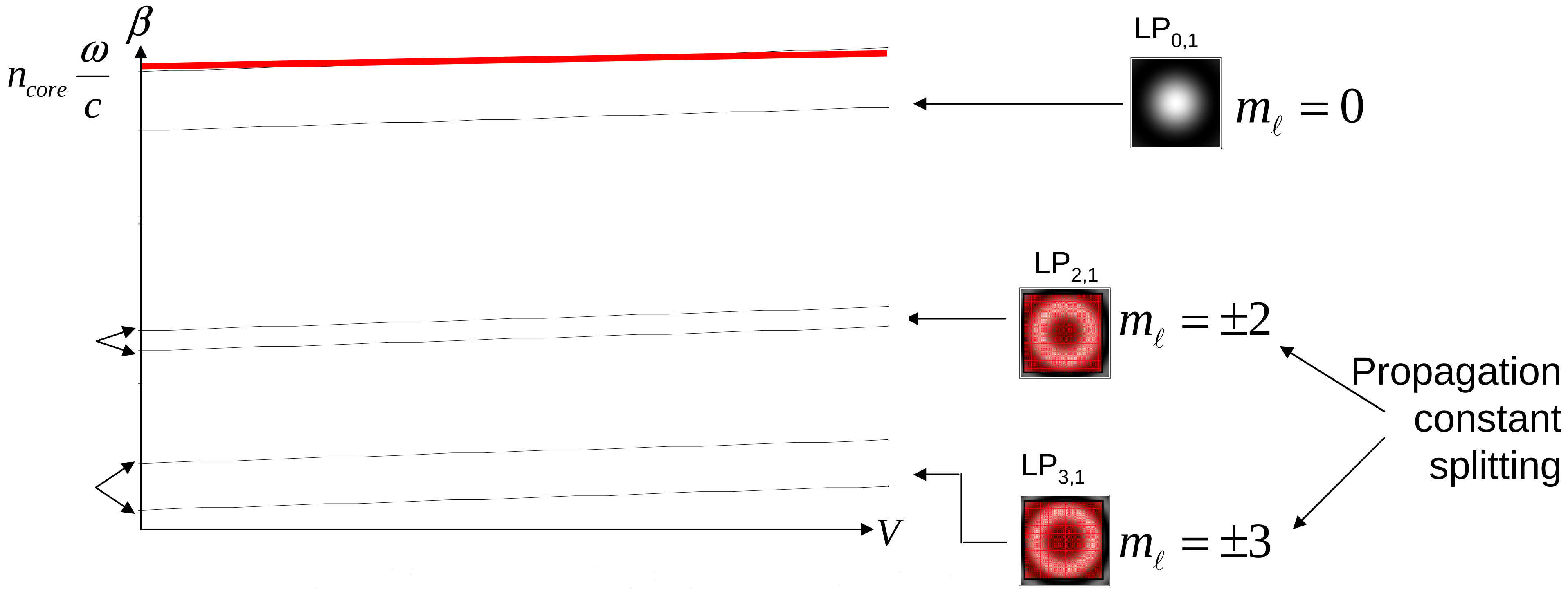


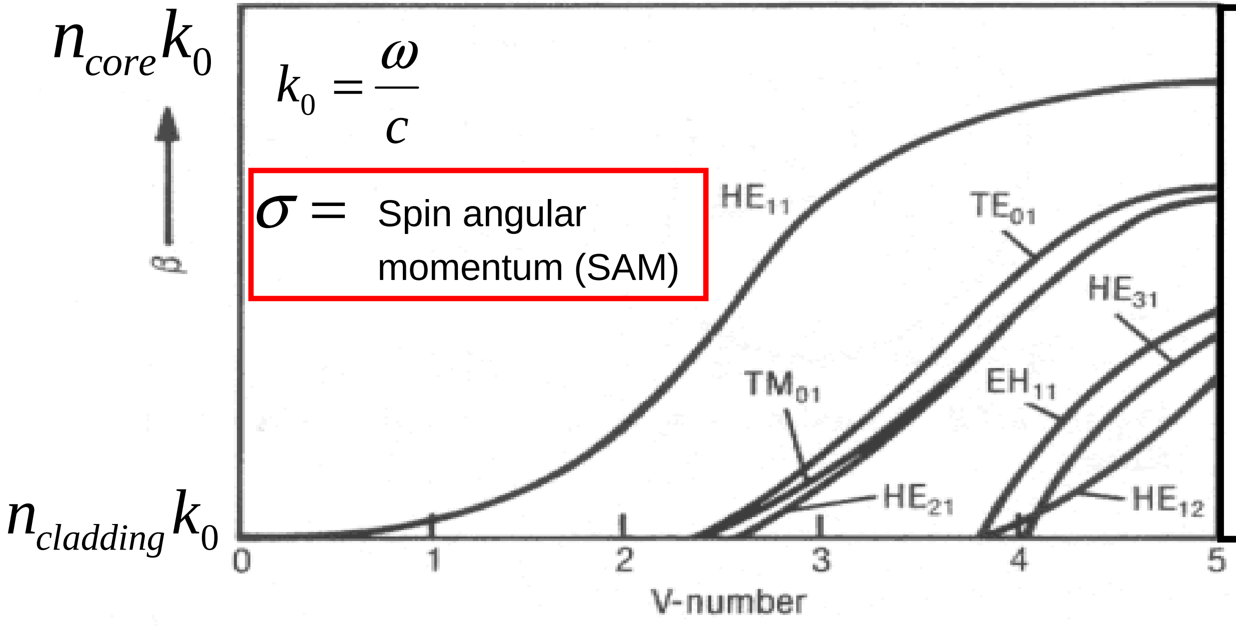
$k_0 = \frac{\omega}{c}$

$\sigma =$ Spin angular momentum (SAM)

We are interested in the modes with OAM greater than unity (highlighted in red)



Propagation constant splitting



$$k_0 = \frac{\omega}{c}$$

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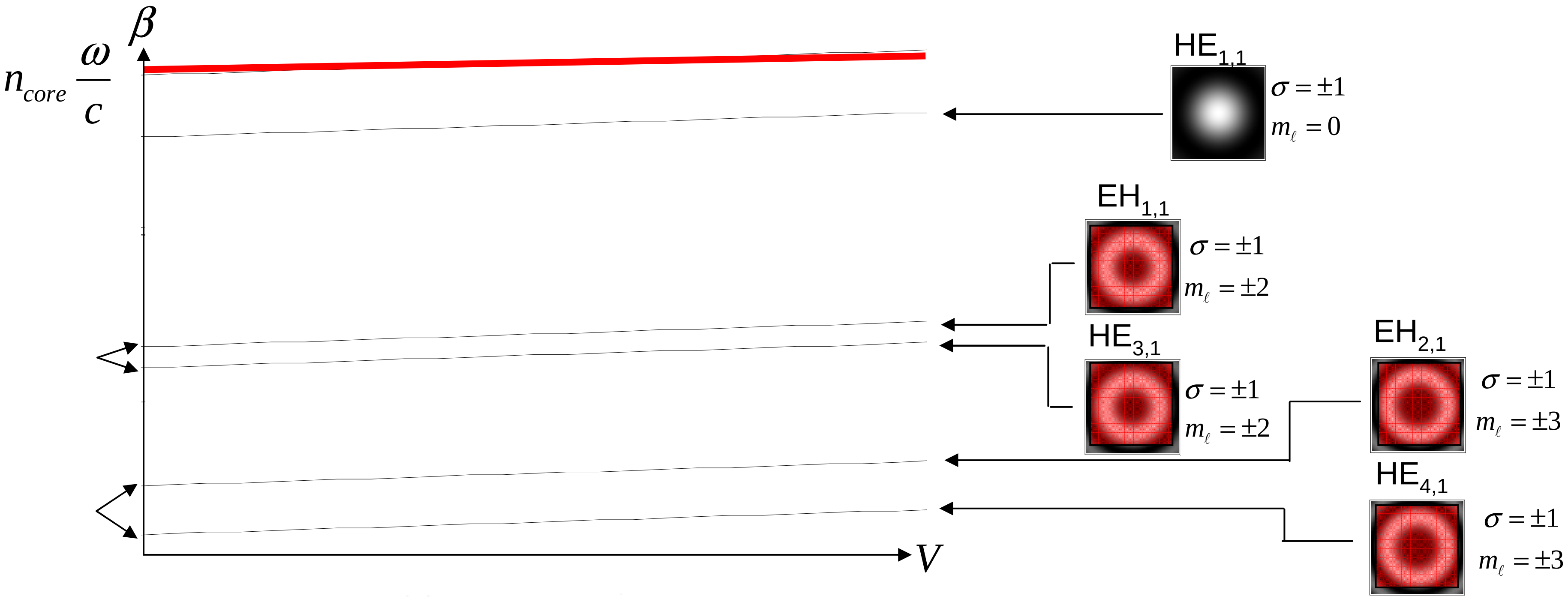
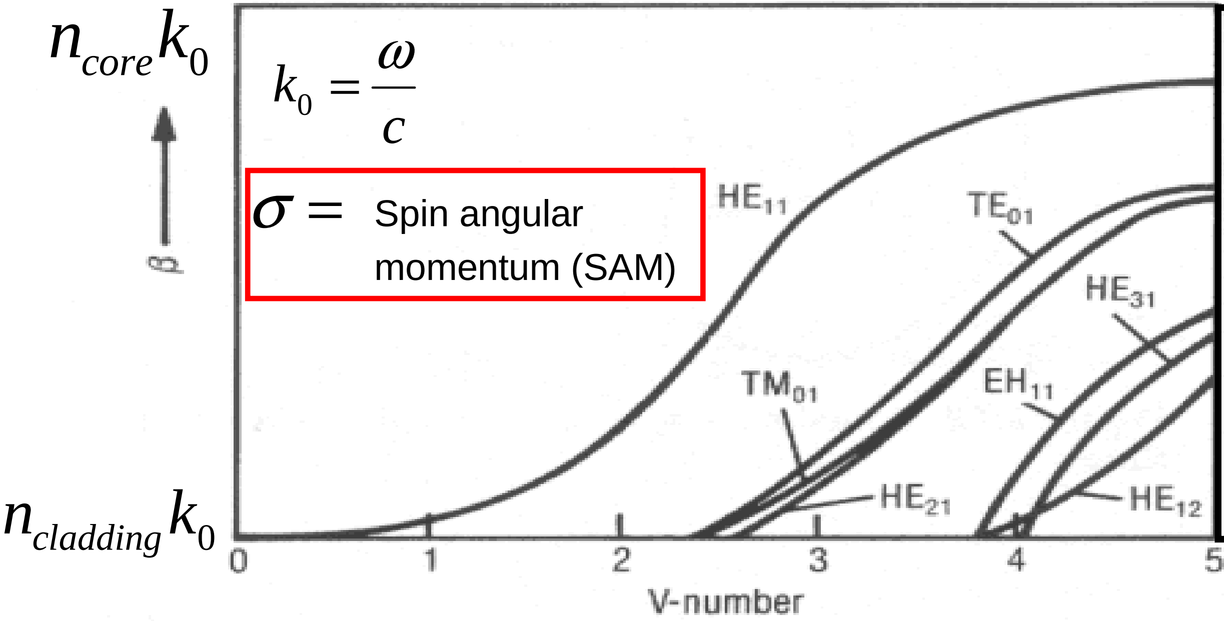
These modes undergo splitting in propagation constant under the SOI

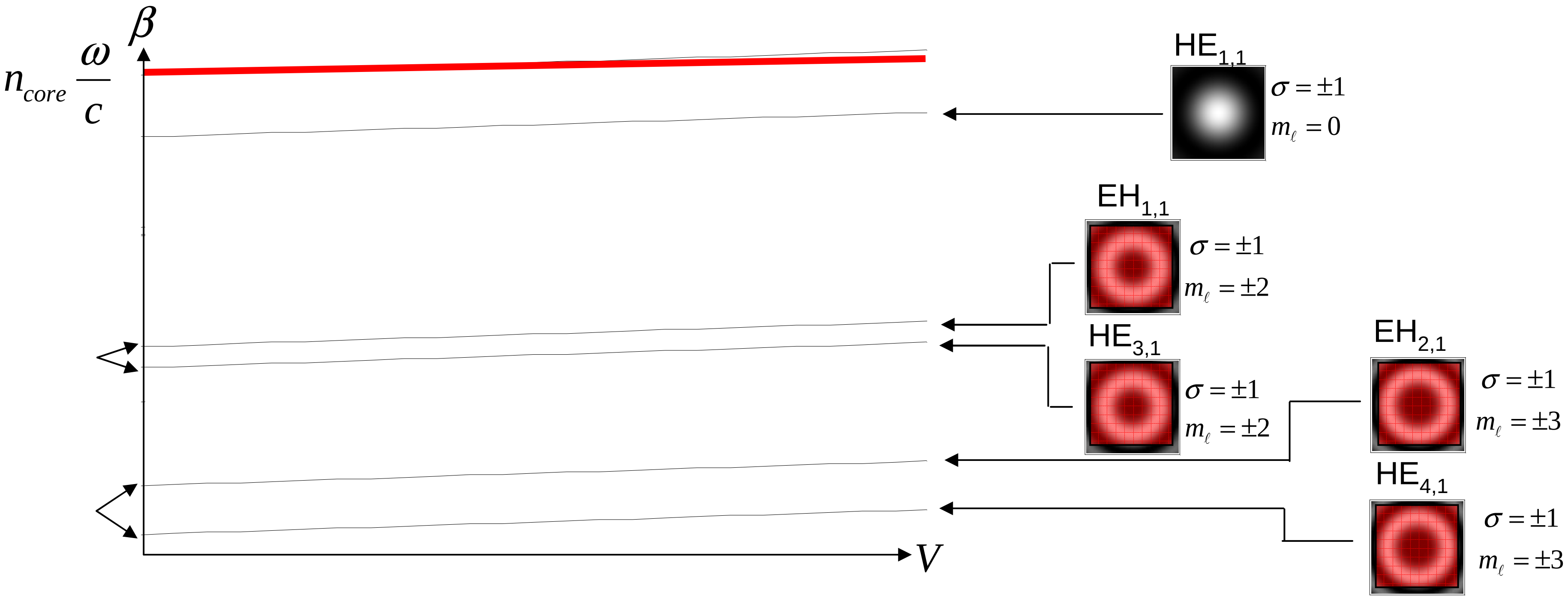




$$k_0=\frac{\omega}{c}$$

$\sigma=$ Spin angular momentum (SAM)

The “split” modes can be identified with the exact solutions as above



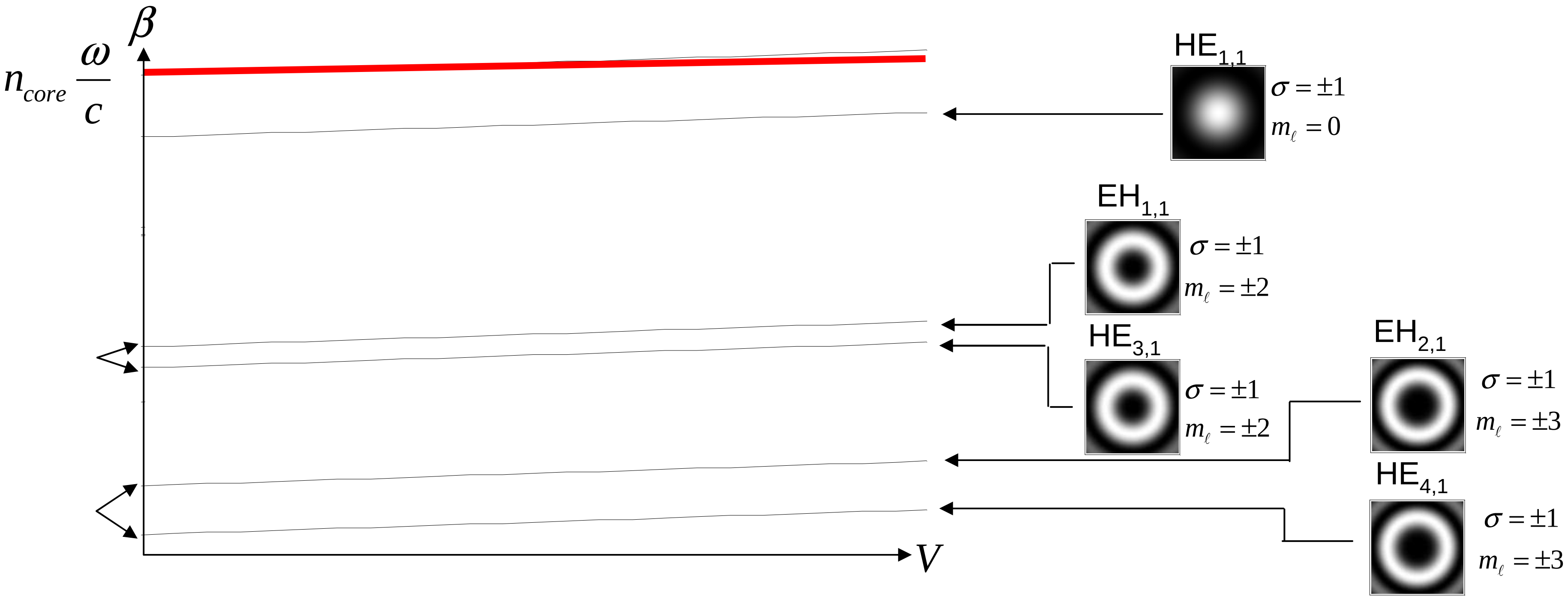
- **Quantitative spin-orbit interaction effect:**
  - **Eigenmode propagation constants β split according to**

$$\boxed{\beta \approx k\left\{1-\frac{f_1}{V^2}+\frac{f_2}{V^3}-\frac{f_3}{V^4}\left(\sigma m_\ell\right)\right\}}$$

(the "f" functions depend on simple fiber parameters)

For $n_{core} \approx n_{cladding}$ and $V >> 1$

The splitting depends upon the *product* of σ and m_ℓ

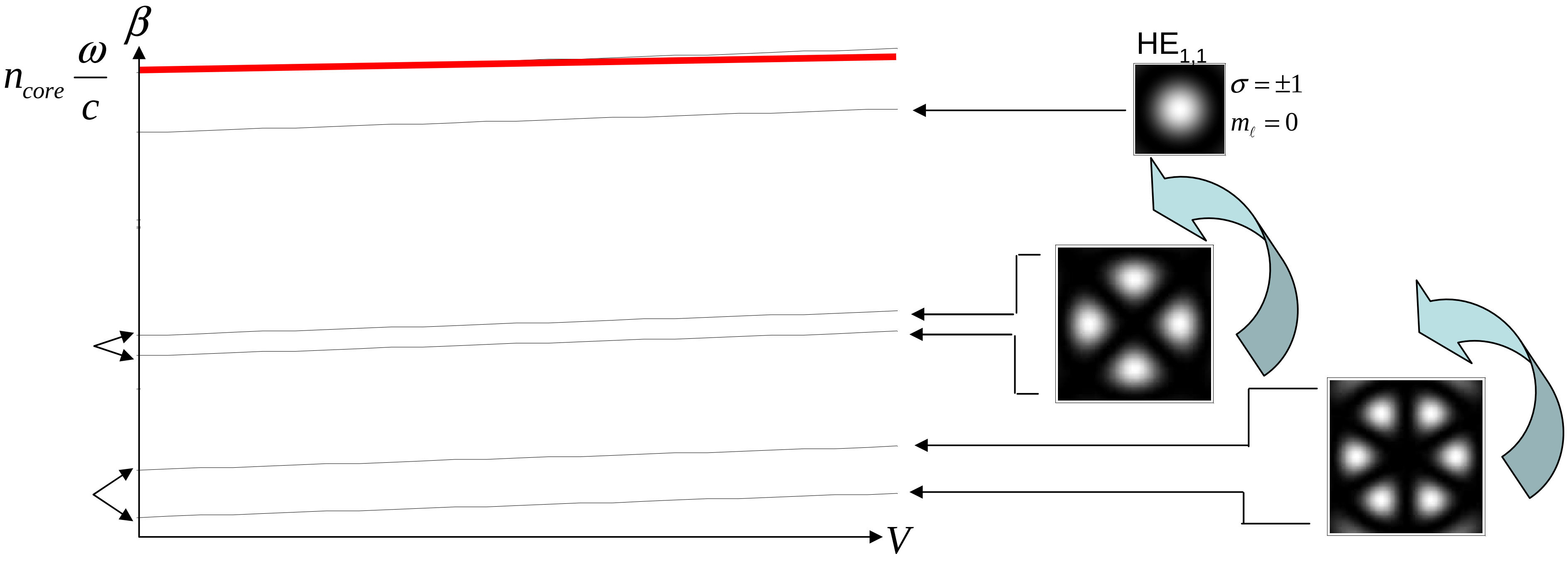


- **Quantitative spin-orbit interaction effect:**

The splitting depends upon the *product* of σ and m_ℓ

Therefore, the propagation speed of the mode differs depending on whether σ and m_ℓ are oriented **parallel** or **anti-parallel**.

- **Superposition modes exhibit spin-controlled rotation**



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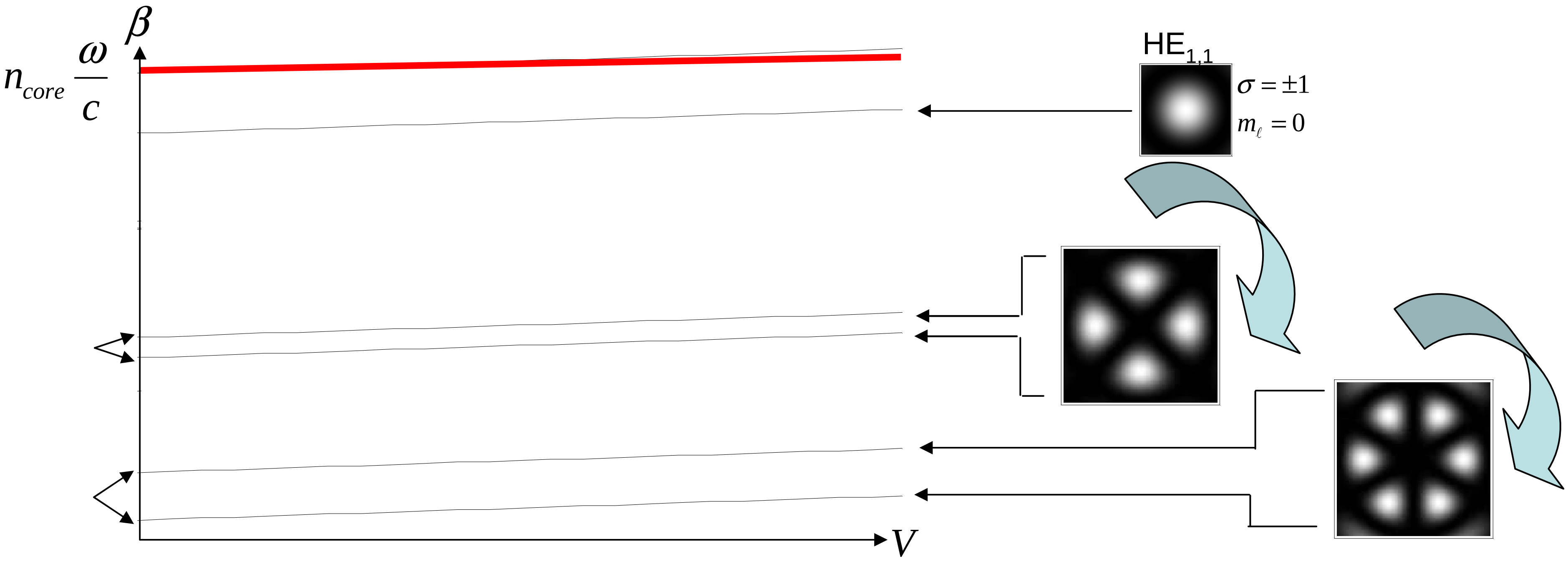

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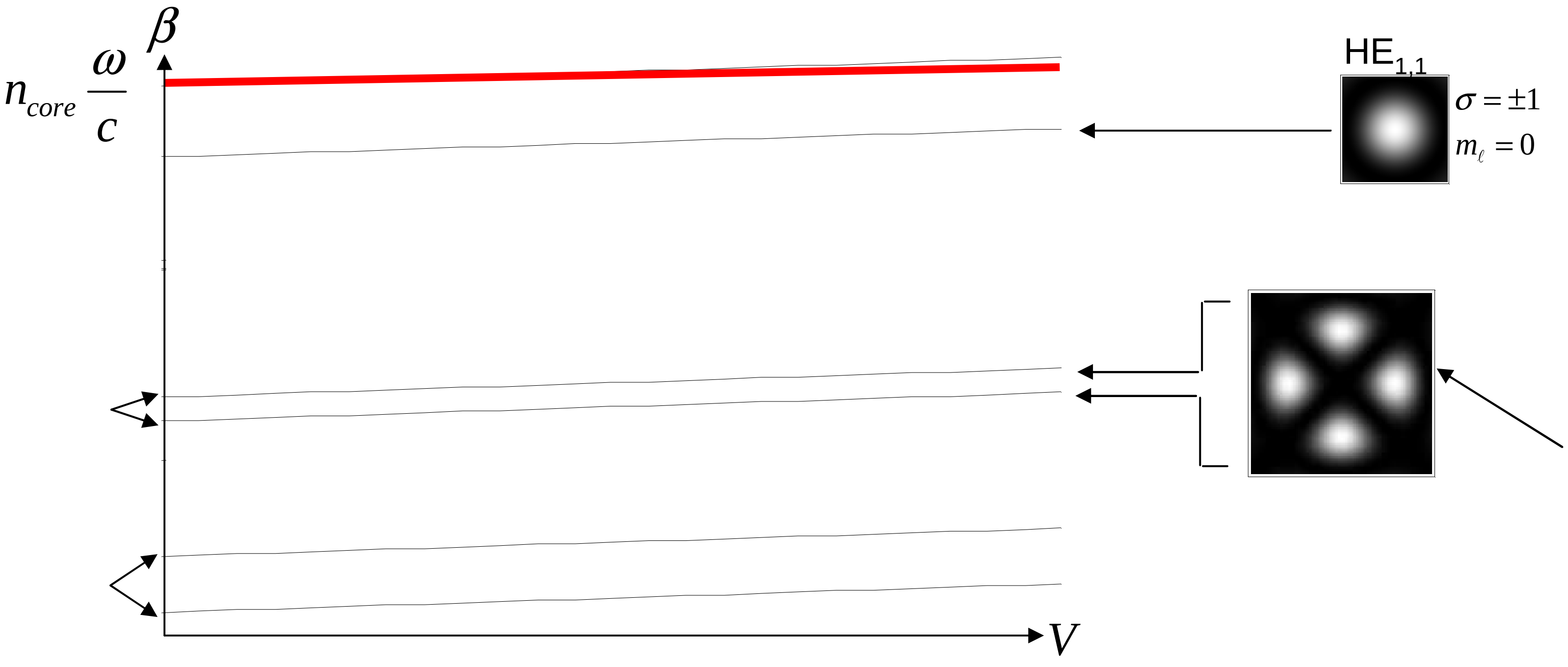
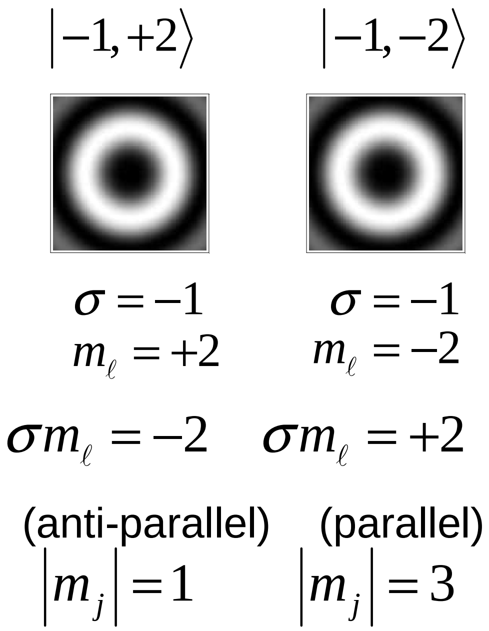


Focus on this superposition mode

- **Propagation depends upon absolute total AM:**
- The “parallel” photon wavefuctions have **larger** *total* angular momentum
- This gives rise to field distributions that penetrate slightly further into the fiber cladding, thereby causing the propagation constant shift



$|-1,+2\rangle$ $|-1,-2\rangle$

$\sigma=-1$ $\sigma=-1$

$m_\ell=+2$ $m_\ell=-2$

$\sigma m_\ell=-2$ $\sigma m_\ell=+2$

(anti-parallel) (parallel)

$|m_j|=1$ $|m_j|=3$

Comments and conclusions:

- Step-index fiber modes with SAM and OAM oriented anti-parallel to each other experience slightly differing dispersion relations, thus propagating at differing speeds
- This can be traced to the fact that there are slight corrections to the spatial eigenmode cladding penetration when one goes beyond the paraxial approximation
- This spin-orbit interaction gives rise to a rotational effect for spatial fiber modes, which can be used to implement a Hadamard gate toward cluster state LOQC.
- We also predict that electron wavefunctions undergo a completely analogous effect when propagating down a hollow, charged cylinder
- Experiments are underway to characterize this effect