

Spin and Orbital Rotation of Electrons and Photons via Spin-Orbit Interaction

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Portland, Oregon
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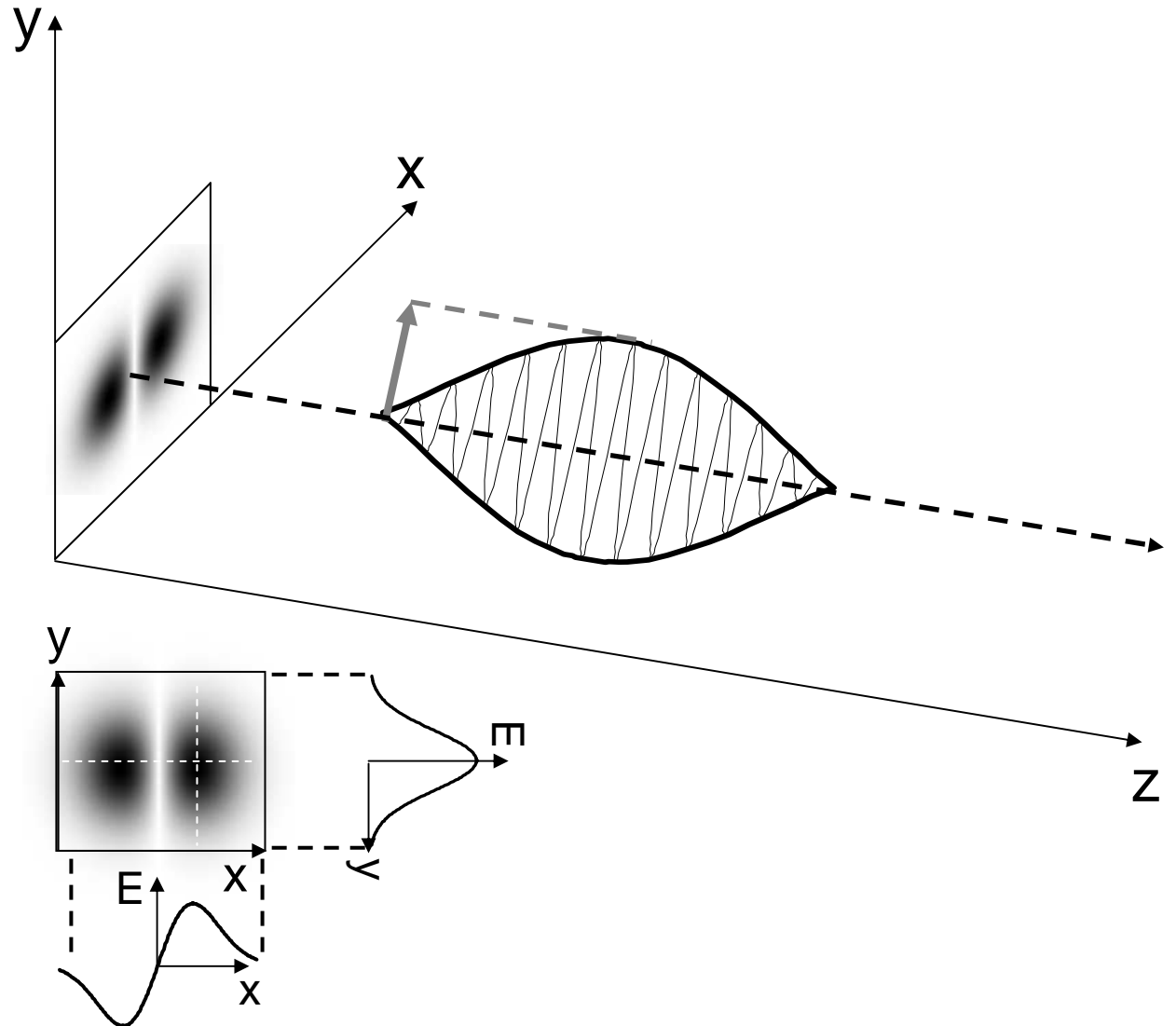


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Paraxial Beams and Transverse Spatial Modes

- Photons & electrons have four degrees of freedom
- For paraxial beams, they may be taken as:
 - 1.) Energy (frequency)
 - 2.) Spin/Polarization (SAM)
 - 3.) Transverse orbital angular momentum (OAM)
 - 4.) Radial quantum number



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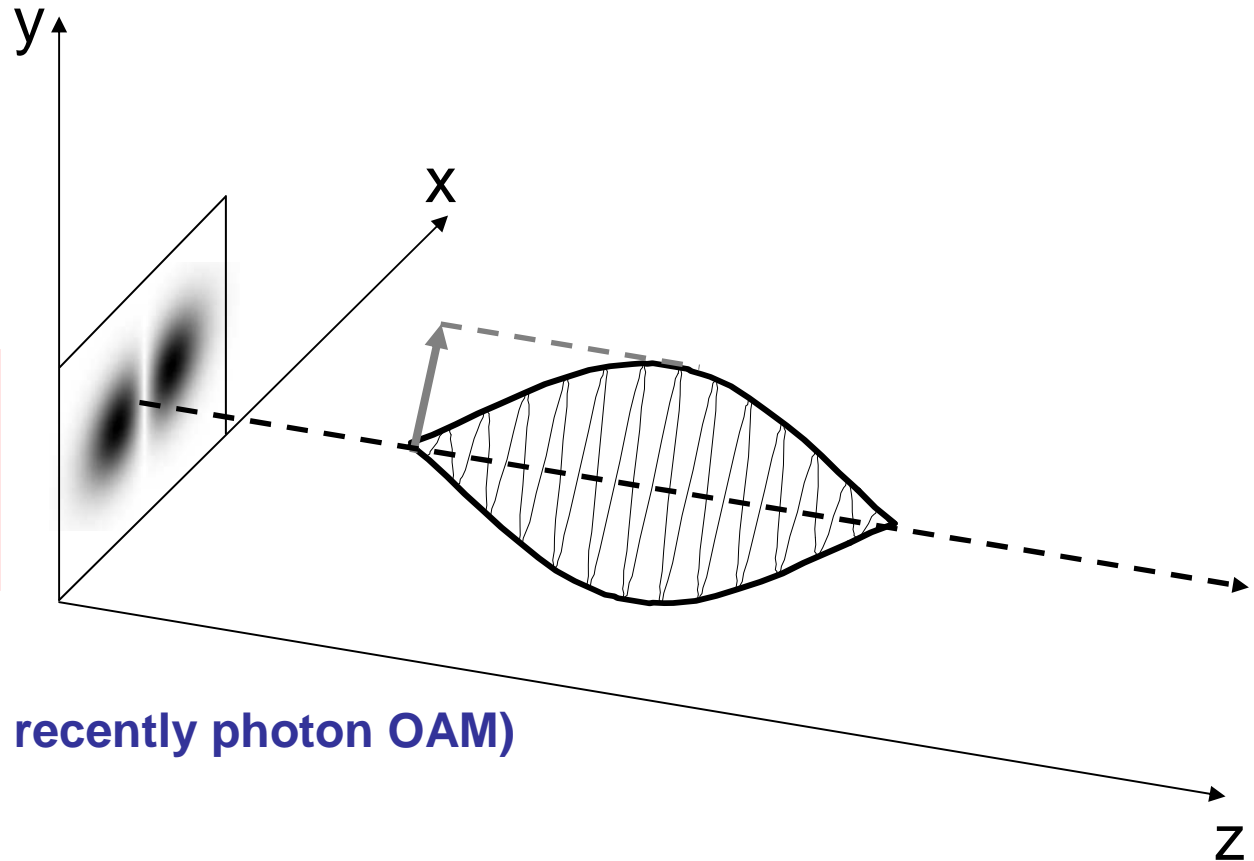
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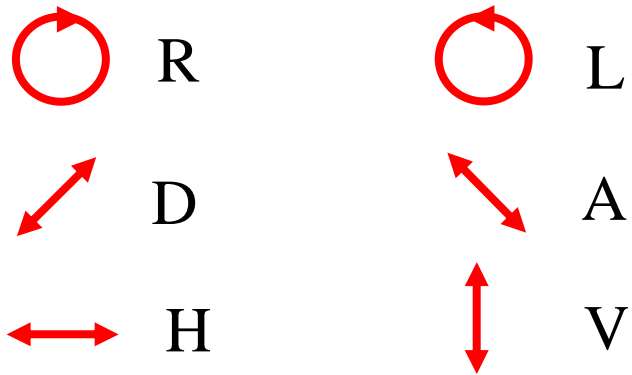


All previously studied (most recently photon OAM)

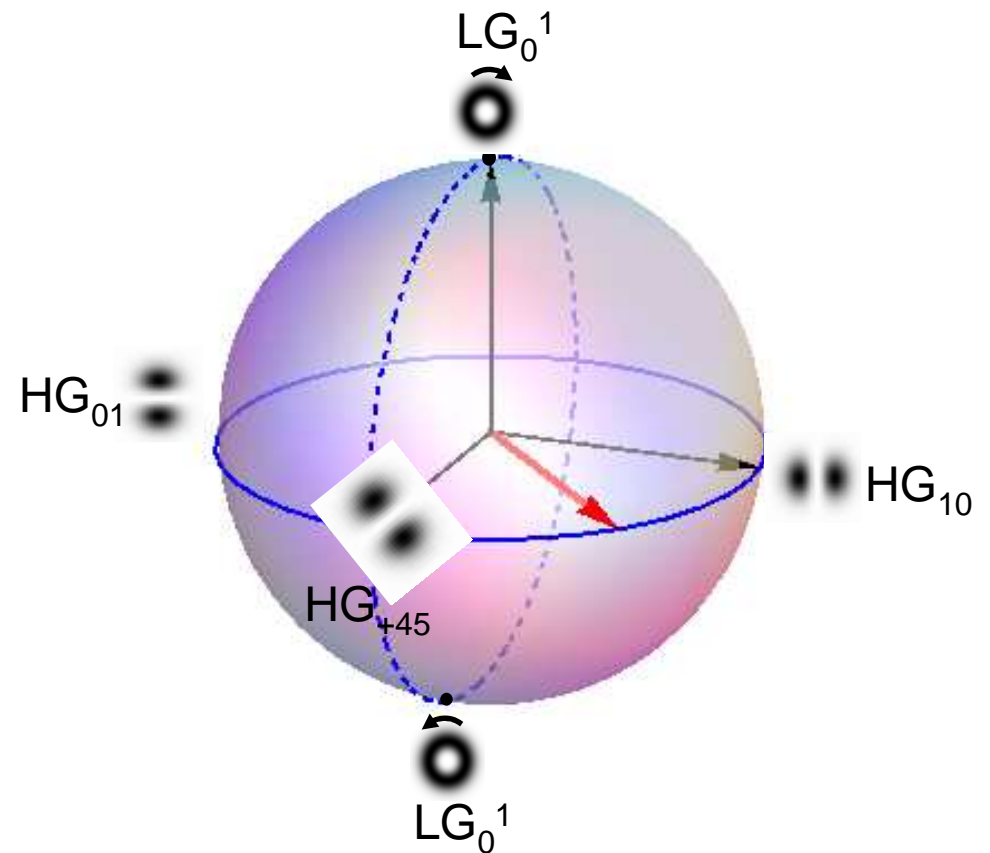
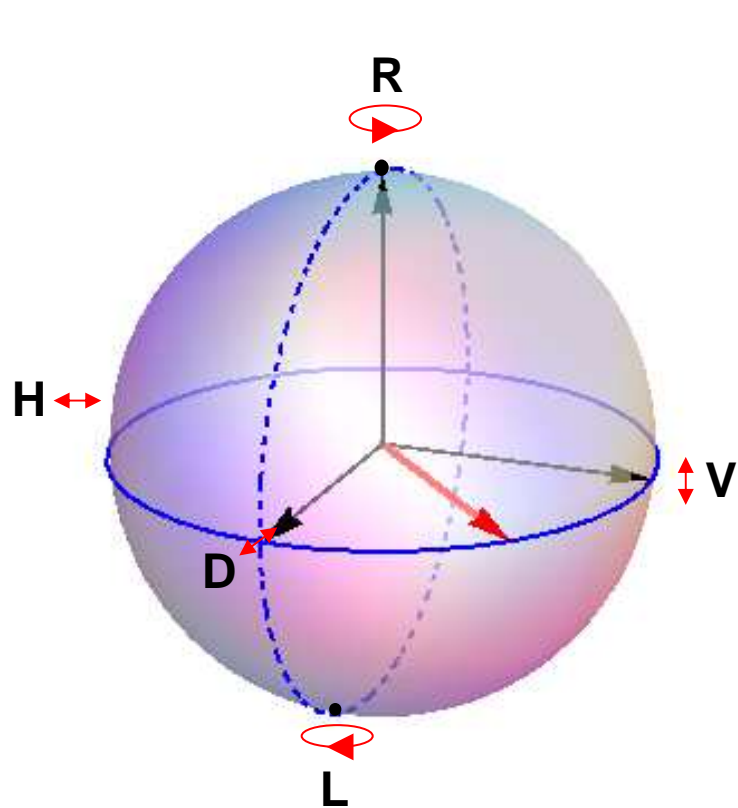
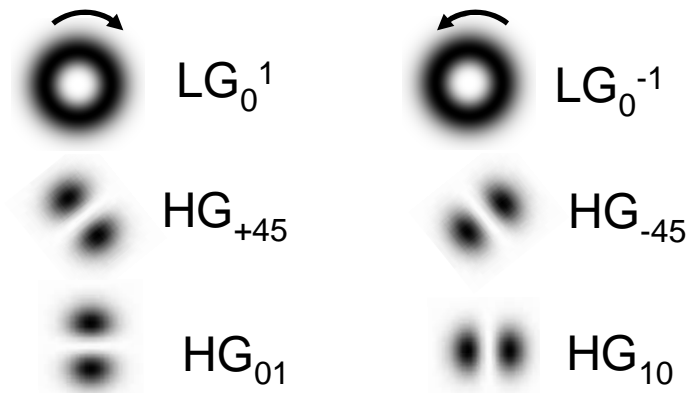
This talk concerns:

- The interaction between the SAM and OAM of electrons and photons in a cylindrical waveguide geometry
- The transfer of entanglement between the SAM and OAM degrees of freedom of a two-particle quantum state

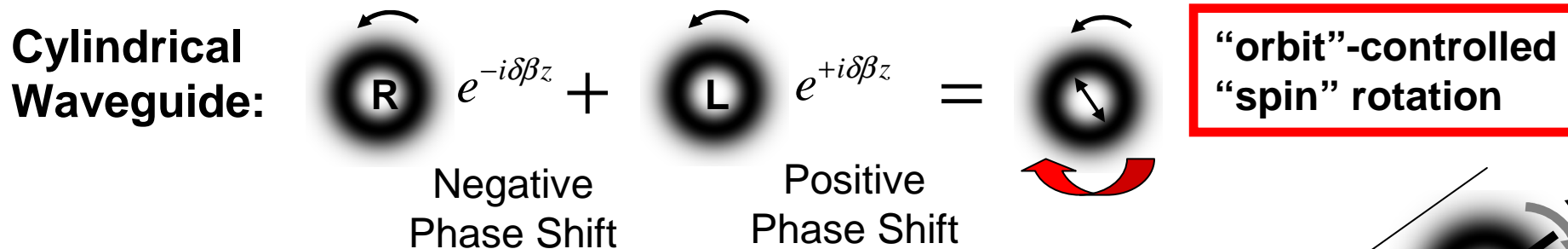
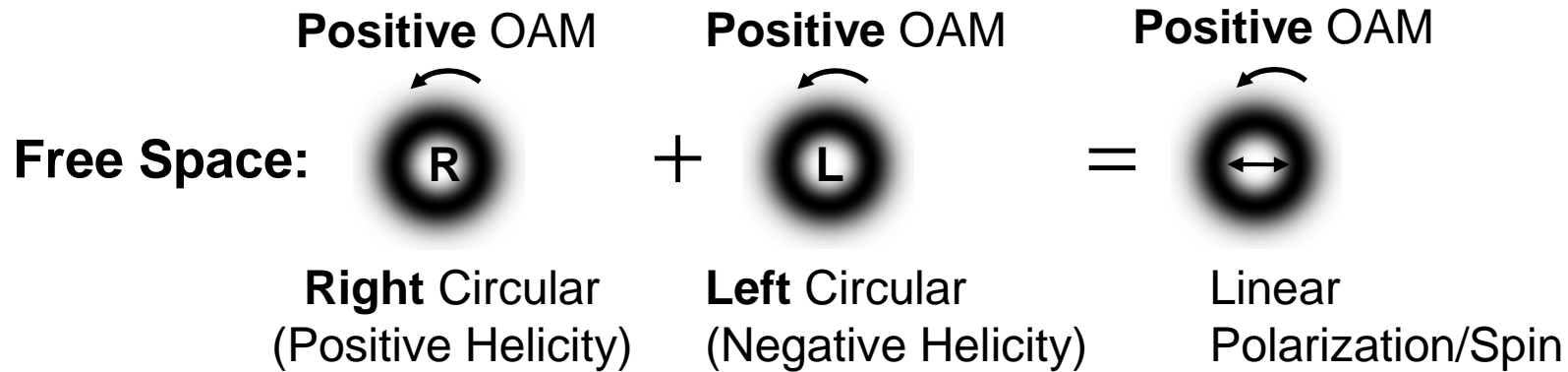
Spin (SAM) “Bloch” Sphere



Orbital (OAM) “Bloch” Sphere

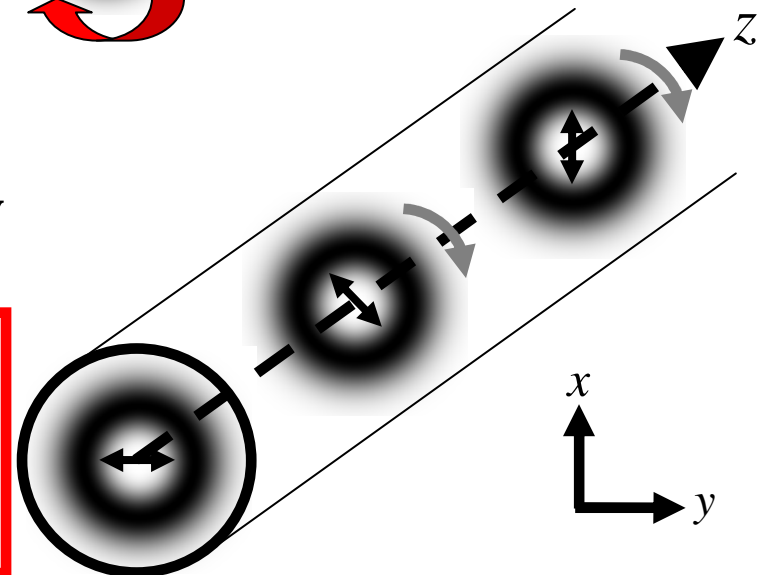


Spin-Orbit Interaction Effect

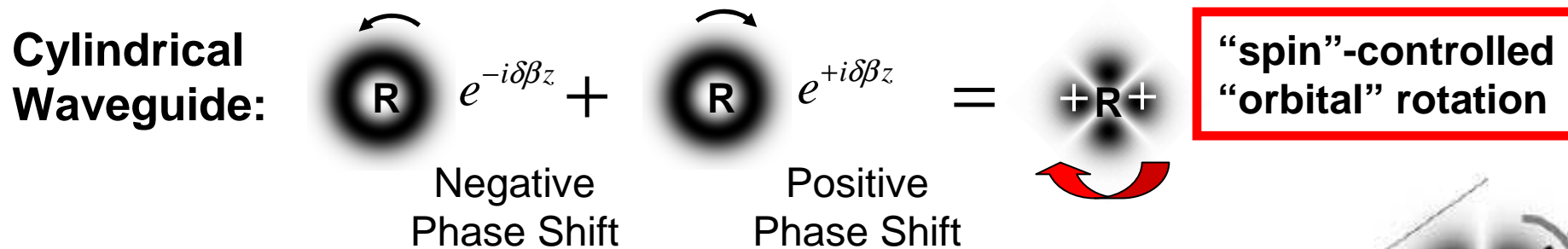
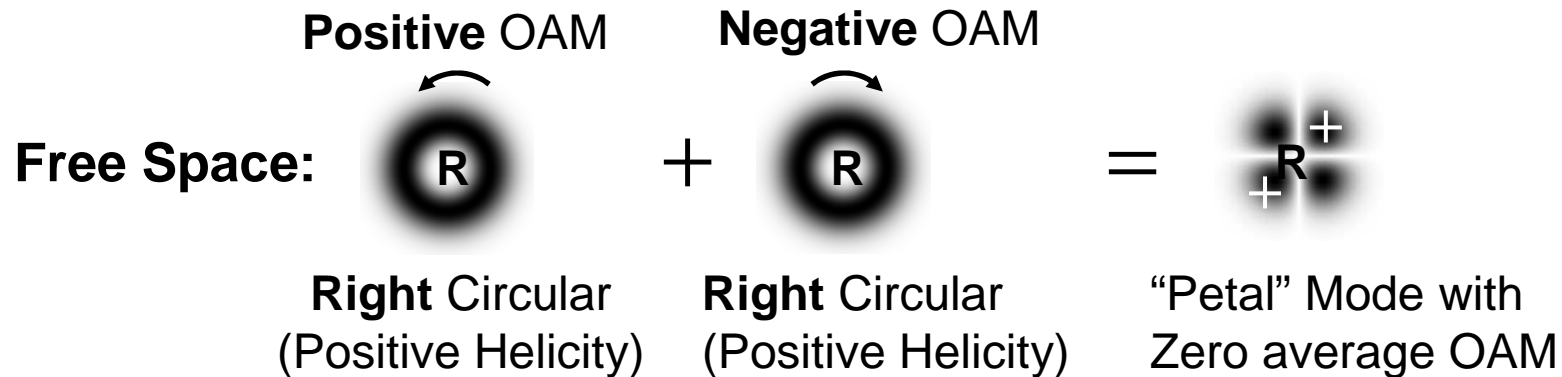


Polarization/Spin: $\propto \cos(\delta\beta z) \hat{x} \pm \sin(\delta\beta z) \hat{y}$

States with parallel SAM and OAM experience a negative phase shift, while anti-parallel states undergo a positive shift. This results in rotation of the **polarization/spin** with propagation in z .

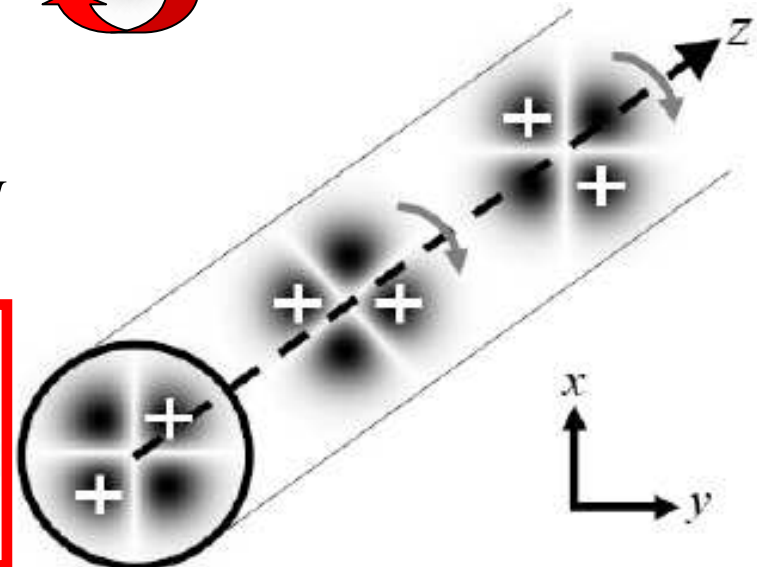


Spin-Orbit Interaction Effect

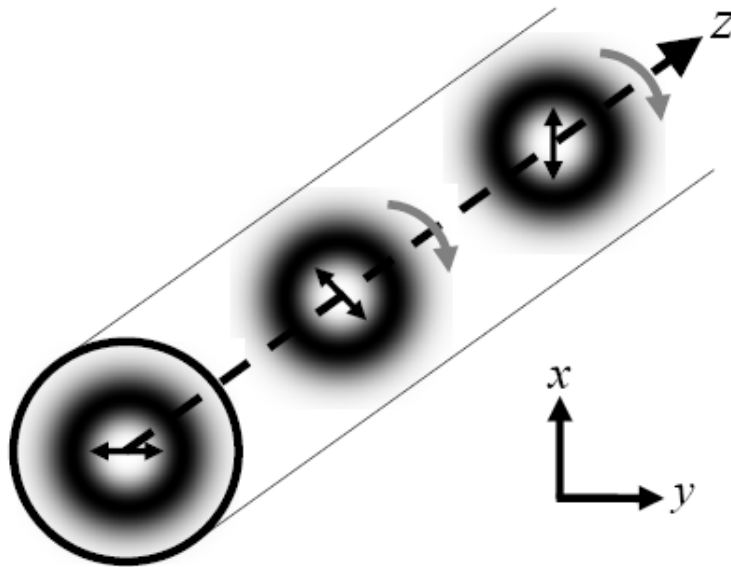


Wave Function: $\propto \cos(m\phi \pm \delta\beta z)$; $m = OAM$

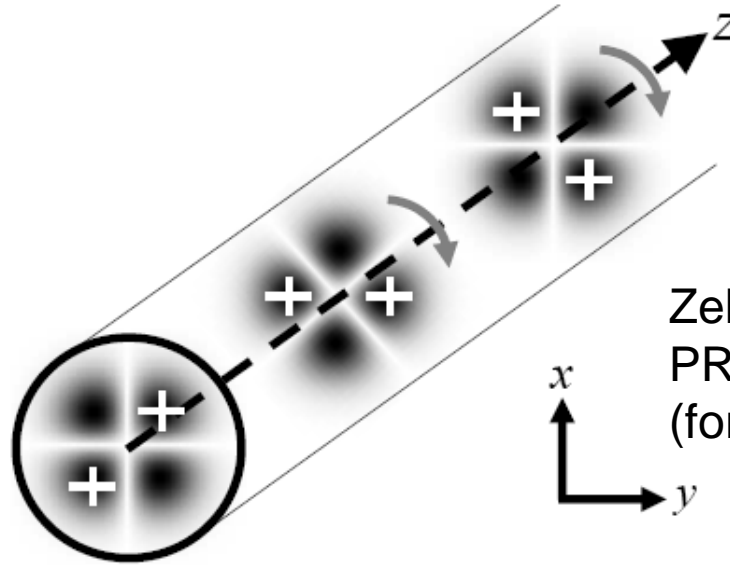
States with parallel SAM and OAM experience a negative phase shift, while anti-parallel states undergo a positive shift. This results in rotation of the **transverse mode** with propagation in z .



Complementary spin-orbit effects

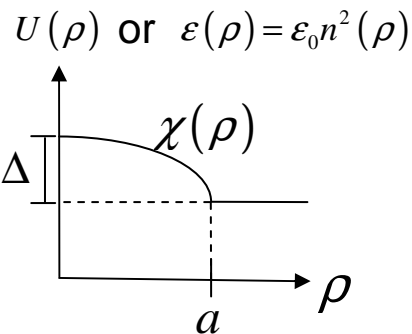


“orbit”-controlled “spin” rotation



“spin”-controlled “orbit” rotation

Zel'dovich,
PRA '91
(for photons)



Electrons, photons have same Hamiltonian!

Dirac and Maxwell equation both give:

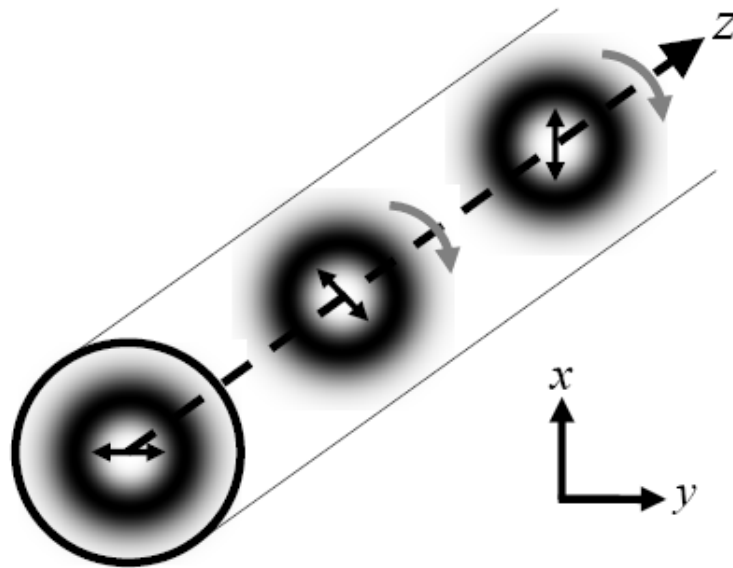
$$\hat{H}' = -\frac{\Delta}{4\beta_0} \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} S_z L_z$$

Electrons: $U(\rho) = (U(0) - \Delta\chi(\rho))$

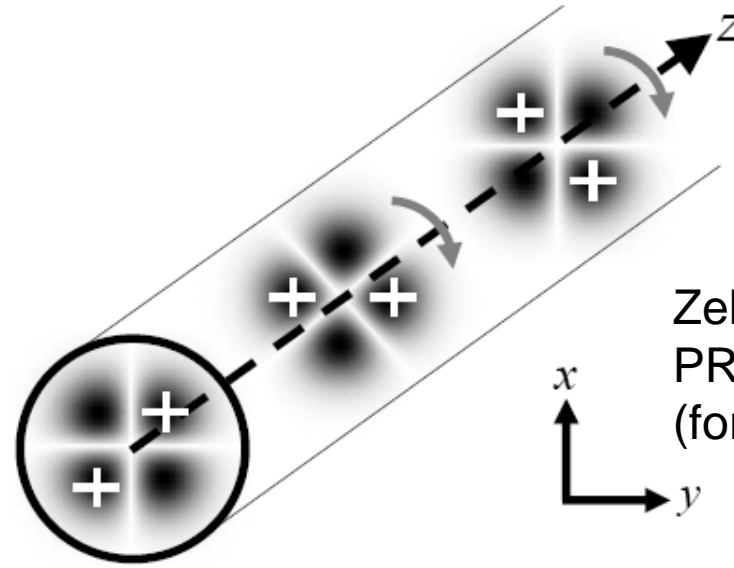
Photons: $\varepsilon(\rho) = \varepsilon(0)(1 - \Delta\chi(\rho))$

Rotation rates for both particles analogous

Complementary spin-orbit effects



“orbit”-controlled “spin” rotation



“spin”-controlled “orbit” rotation

Zel'dovich,
PRA '91
(for photons)

- Both of these effects may occur either in space or time
- The effects occur for both electrons and photons
- Independent of mass, charge, magnetic moment, etc.
- This spin-orbit interaction is a universal geometric phase (see PRA 80, 061804 (2009), NJP 10 103022 (2008))

SAM to OAM Entanglement Transfer

Begin in a purely SAM-entangled Bell state:

$$|H \bullet\bullet\rangle |V \bullet\bullet\rangle + |V \bullet\bullet\rangle |H \bullet\bullet\rangle$$

Apply quarter wave plates:

$$|R \bullet\bullet\rangle |L \bullet\bullet\rangle + |L \bullet\bullet\rangle |R \bullet\bullet\rangle$$

Apply "OAM" rotation SOI gate:

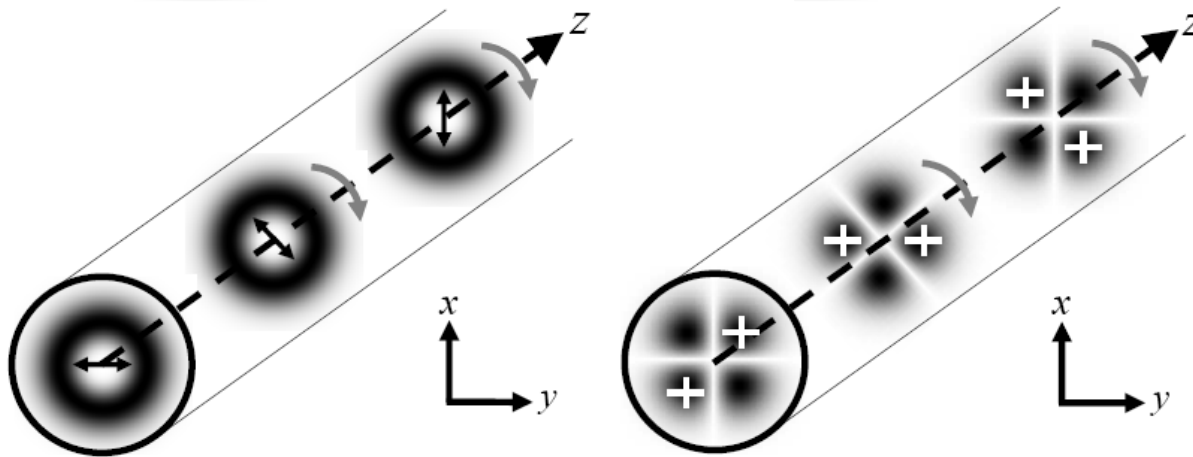
$$|R \text{ } \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \rangle |L \text{ } \begin{array}{c} \diagdown \\ \bullet \\ \diagup \end{array} \rangle + |L \text{ } \begin{array}{c} \diagdown \\ \bullet \\ \diagup \end{array} \rangle |R \text{ } \begin{array}{c} \diagup \\ \bullet \\ \diagdown \end{array} \rangle$$

Apply quarter wave plates and mode converters:

$$|D \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowleft \end{array} \rangle |A \text{ } \begin{array}{c} \curvearrowleft \\ \bullet \\ \curvearrowright \end{array} \rangle + |A \text{ } \begin{array}{c} \curvearrowleft \\ \bullet \\ \curvearrowright \end{array} \rangle |D \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowleft \end{array} \rangle$$

Apply "SAM" rotation SOI gate:
(End in purely OAM-entangled state)

$$|H \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowright \end{array} \rangle |H \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowright \end{array} \rangle + |H \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowright \end{array} \rangle |H \text{ } \begin{array}{c} \curvearrowright \\ \bullet \\ \curvearrowright \end{array} \rangle$$



Thank You



R



L



D



A



H



V



LG_0^1



LG_0^{-1}



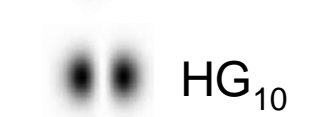
HG_{+45}



HG_{-45}



HG_{01}



HG_{10}