Spin and Orbital Rotation of Electrons and Photons via Spin-Orbit Interaction

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university of oregon

INTRODUCTION

- Photons and electrons have 4 degrees of freedom
- In cylindrical symmetry, these may be taken as:
  - Spectral (frequency): ω
  - Polarization (spin, or SAM): σ
  - Orbital angular momentum (OAM): m_ℓ (z component)
  - “Radial” quantum, number: n

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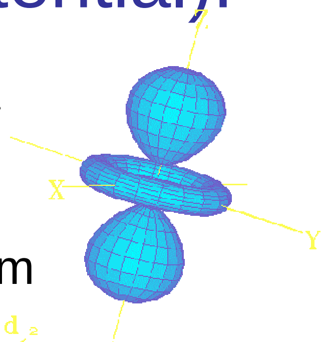
Hydrogen atom

(Spherical potential):

$$\left| n\,\ell\,m_\ell\,\sigma \right\rangle$$



$\hat{\mathbf{L}} \rightarrow \ell$ (Good quantum number)

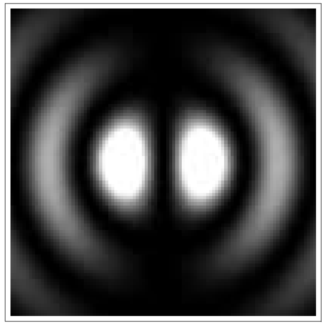
Cylindrical potential: $\Psi \propto e^{i(\beta z - \omega t)}$

$\ell \rightarrow \omega, \beta$

$$\left| \omega\, m_\ell\, \sigma\, n \right\rangle$$

$\updownarrow$

β



INTRODUCTION

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- In cylindrical symmetry, these may be taken as:
  - Spectral (frequency): ω
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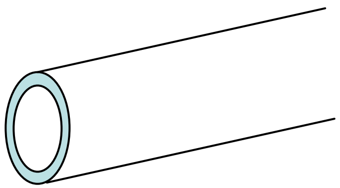
  - "Radial" quantum, number: n
- All previously studied (most recently photon OAM)
- This talk concerns:
  - The interaction between the SAM and OAM degrees of freedom of electrons and photons in cylindrical symmetry
  - The transfer of entanglement between the SAM and OAM degrees of freedom for two particle states

SOI

OUTLINE (elevator pitch)



1: Electron in an inhomogeneous potential– effective **B** field induces spin-orbit interaction

2: A photon in an inhomogeneous medium experiences an analogous spin-orbit interaction, even though it has no magnetic moment

3: **In a cylindrical waveguide, the spin-orbit dynamics are described by a single expression applying to both electrons and photons**

4: This leads to the prediction of several spin (SAM) and orbital (OAM) rotational effects:

a) spatial or time evolution of either particle's spin/polarization vector is controlled by the sign of its OAM

b) conversely, its spatial wavefunction evolution is controlled by its SAM

5: Two-particle states: Reversible transfer of entanglement between SAM and OAM

Question: Why such a close analogy?

quant-ph > arXiv:0905.3778

To what extent is there a photon-electron analogy? An Example:

1. Spin Hall Effect for Electrons: opposite spin accumulation on opposing lateral surfaces of a current-carrying sample. Its origin is spin-orbit interaction.

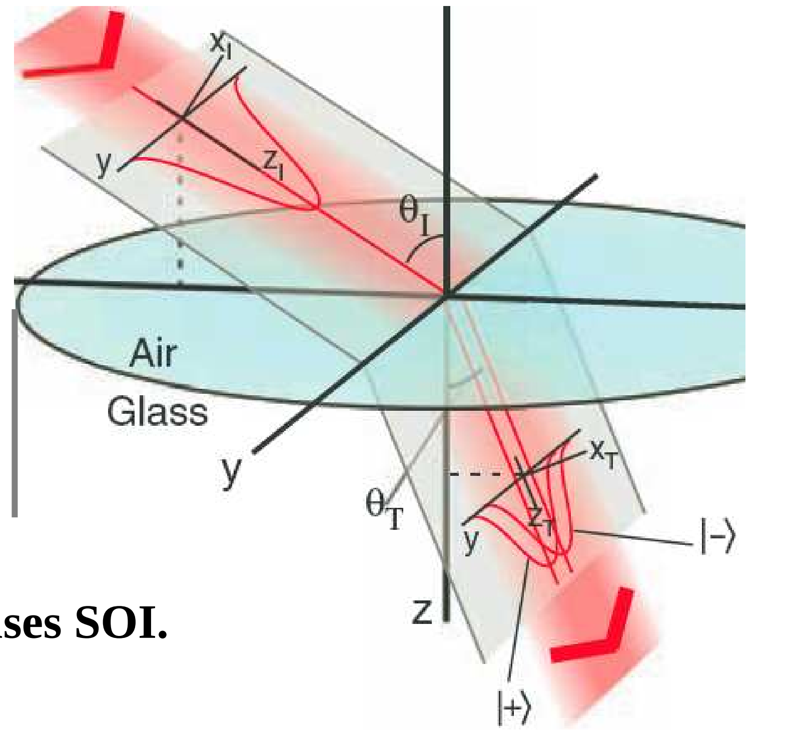
Dyakonov and Perel (1971) Sov. Phys. JETP Lett. 13, 467

Hirsch (1999) PRL 83, 1834

2. Spin Hall Effect for Light: spin-dependent displacement perpendicular to the refractive index gradient for photons passing through an air-glass interface.

M. Onoda, S. Murakami, N. Nagaosa, PRL 93, 083901 (2004)

Observed: Hosten, Kwiat Science 319 (2008)



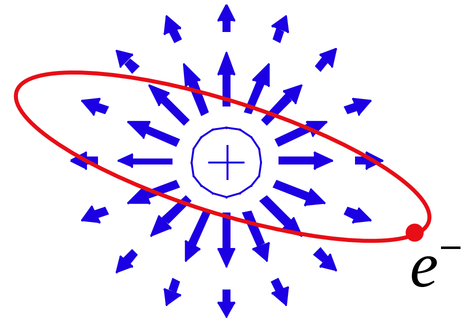
Inhomogeneity in refractive index causes SOI.

Spin-Orbit Interaction (SOI) in <u>Spherical</u> Potentials:

• ELECTRON IN AN INHOMOGENOUS SPHERICAL ELECTRIC POTENTIAL (ATOM)

$$H' = -\frac{e^2}{2m^2c^2}\frac{1}{r}\frac{\partial V}{\partial r}\mathbf{S}\cdot\mathbf{L}$$

$$\mathbf{S} = SAM \qquad \mathbf{r}\times\mathbf{p} = \mathbf{L} = OAM$$



(atomic fine structure)

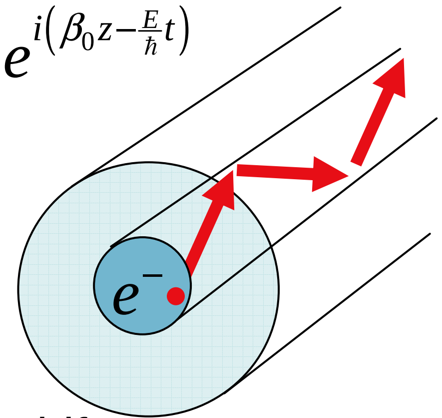
Spin-Orbit Interaction (SOI) in <u>Cylindrical</u> Potentials?

C Leary, D Reeb, M Raymer, NJP, 10, 103022 (2008).

$$H' = -\frac{e^2}{2m^2c^2}\frac{1}{\rho}\frac{\partial V}{\partial \rho}S_z L_z$$

$$\Psi \propto e^{i\left(\beta_0 z - \frac{E}{\hbar}t\right)}$$

$$S_z = \mathbf{S}\cdot\hat{\mathbf{z}} \qquad L_z = \mathbf{L}\cdot\hat{\mathbf{z}}$$



Traveling wave
Schrodinger equations:

$$\hat{H}\Psi = E\Psi$$

Energy shift

$$\hat{H}\Psi = \beta^2\Psi$$

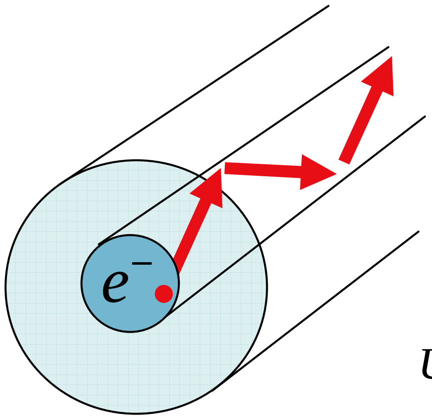
Propagation constant shift
(phase velocity)

Perhaps the cylindrical electron case is not so surprising…

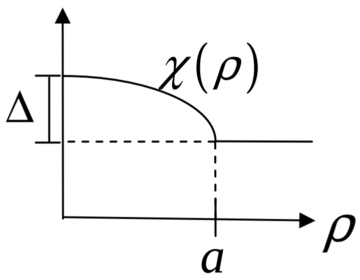
$$\hat{H}\Psi = \beta^2\Psi \qquad \hat{H} = \hat{H}_0 + \hat{H}'$$

$$\Psi_0 \propto e^{i\left(\beta_0 z - \frac{E}{\hbar}t\right)}$$

Solve Schrodinger (Dirac) equation:



$V(\rho) \propto -U(\rho)$



$$U(\rho) = \left(U(0) - \Delta\chi(\rho)\right)$$

$$\hat{H}'_{e^-} = -\frac{\Delta}{4\beta_0}\frac{1}{\rho}\frac{\partial\chi}{\partial\rho}S_z L_z$$

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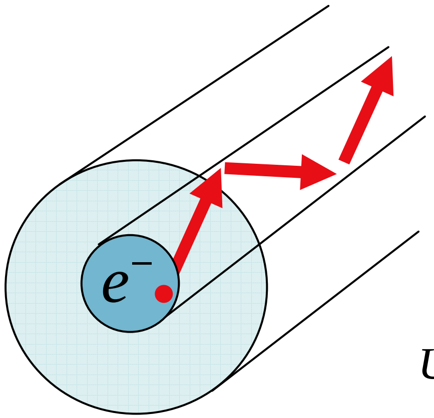

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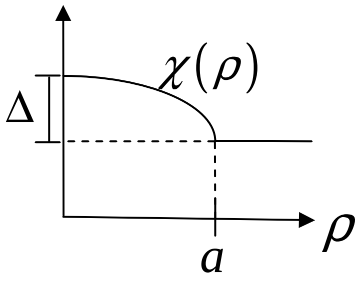

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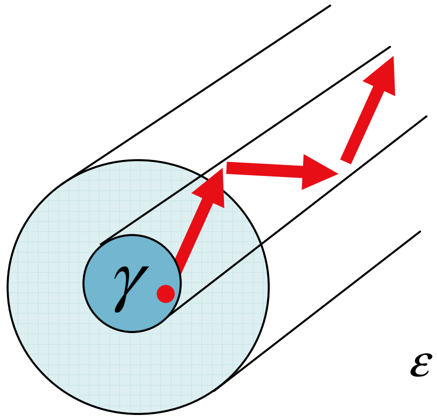
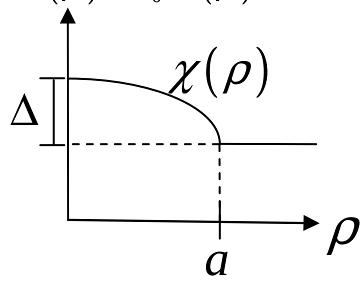
$$\Psi_0 \propto e^{i(\beta_0 z - \omega t)}$$

Solve Helmholtz (Maxwell) equations:



$\varepsilon(\rho) = \varepsilon_0 n^2(\rho)$



$$\varepsilon(\rho) = \varepsilon(0)\left(1 - \Delta\chi(\rho)\right)$$

$$\hat{H}'_{\gamma} = -\frac{\Delta}{4\beta_0}\frac{1}{\rho}\frac{\partial\chi}{\partial\rho}S_z L_z$$

but it is remarkable that the photon SOI is completely analogous!

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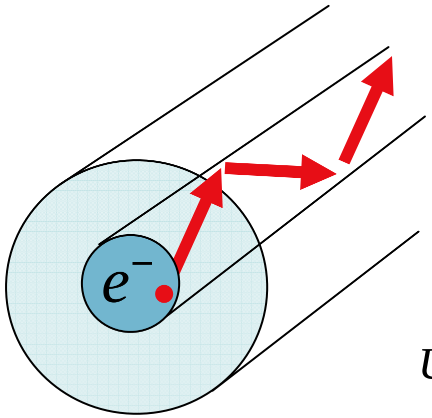

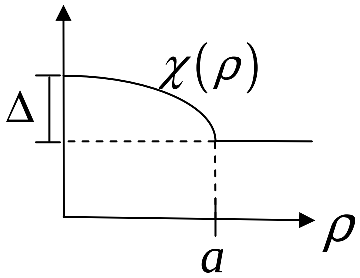

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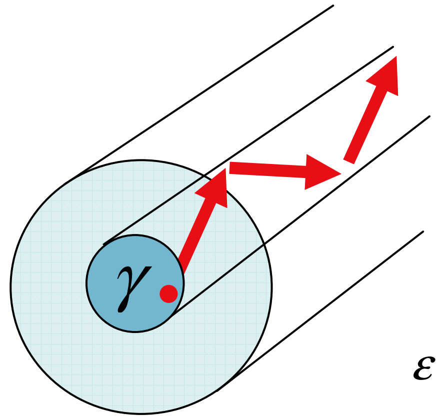

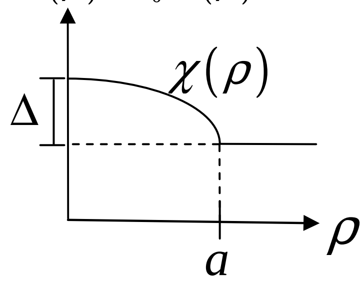

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but it is remarkable that the photon SOI is completely analogous!

Main Point: the $S_z L_z$ part of the Hamiltonian means that β will undergo a small positive/negative shift depending upon whether S_z and L_z are oriented parallel/antiparallel to each other

Wavefunction: $\Psi_{\sigma m_\ell} = \psi(\rho)\, e^{-i m_\ell \phi} e^{i(\beta_0 z - \omega t)} \hat{\mathbf{e}}_\sigma$

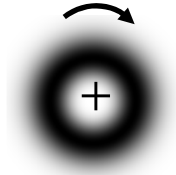
$e^{-i m_\ell \phi}$: OAM $\hat{\mathbf{e}}_\sigma$: SAM

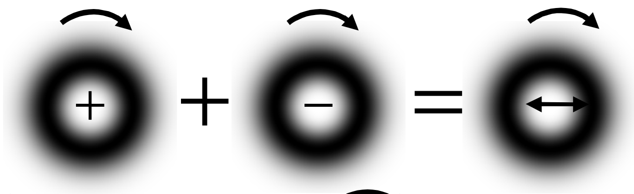
This slide common to both particle types

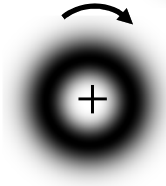
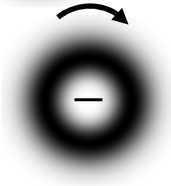
$m_\ell = 0, \pm 1, \pm 2, \ldots =$ OAM quantum number $\sigma = \pm 1 =$ SAM quantum number

Hamiltonian: $$H'_{SOI} = -\frac{\Delta}{4\beta_0}\frac{1}{\rho}\frac{\partial \chi}{\partial \rho} S_z L_z \propto S_z L_z$$

Perturbation theory: $\delta\beta = \left\langle \Psi_{\sigma m_\ell} \middle| H'_{SOI} \middle| \Psi_{\sigma m_\ell} \right\rangle \propto \sigma m_\ell \Rightarrow \Psi_{\sigma m_\ell} \rightarrow \Psi_{\sigma m_\ell} e^{i\delta\beta z}$

"Notation": $\Psi_{+1,+2} = \psi(\rho)\, e^{+2i\phi} e^{i(\beta_0 z - \omega t)} \hat{\mathbf{e}}_{+1} \Leftrightarrow$  etc. **(no more math!)**

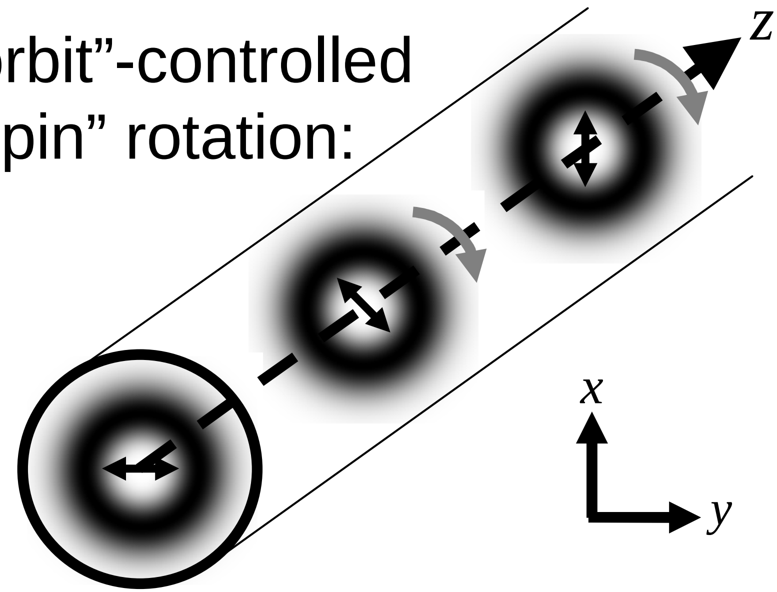
rotation effect: 

$\longrightarrow$  $e^{+i|\delta\beta| z}$ $+$  $e^{-i|\delta\beta| z}$

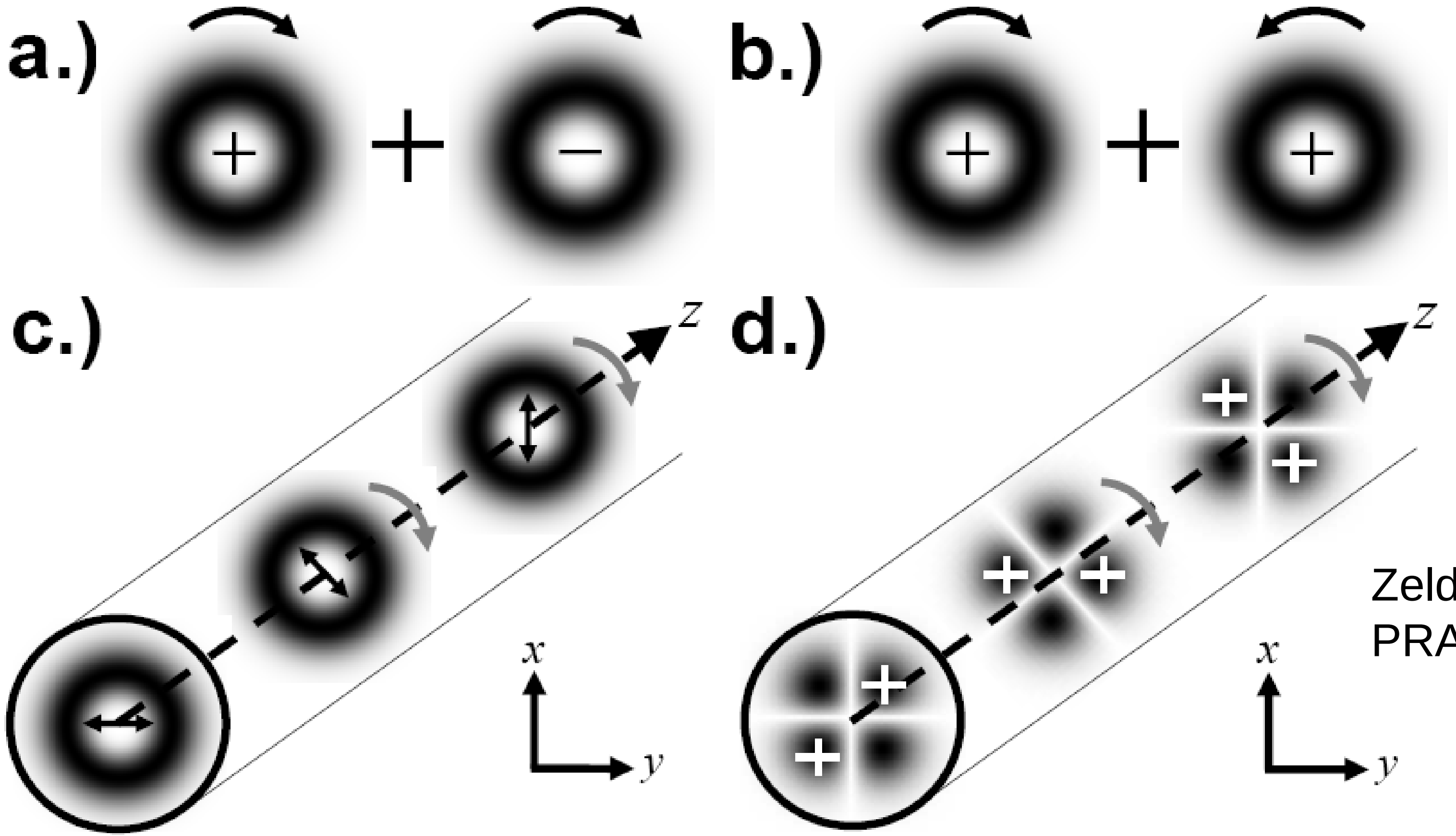
$$\propto \cos\left(|\delta\beta| z\right)\hat{\mathbf{x}} + \mu \sin\left(|\delta\beta| z\right)\hat{\mathbf{y}}$$

Spin/polarization rotation direction controlled by sign of $\mu = \pm 1 \equiv \frac{m_\ell}{|m_\ell|}$

"orbit"-controlled "spin" rotation:



Complementary spin-orbit effects



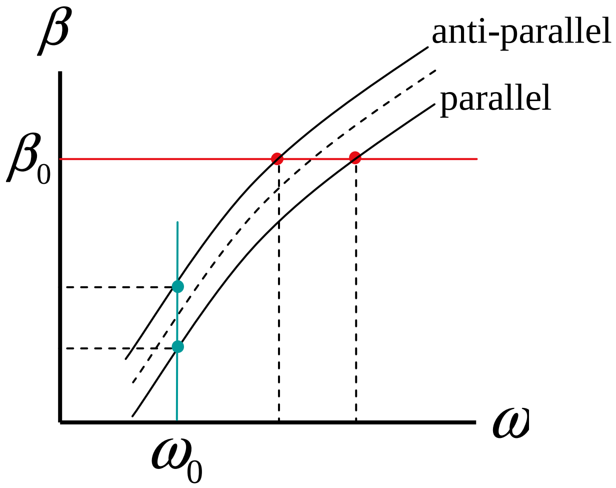
Zeldovich, PRA ‘91

“orbit”-controlled “spin” rotation (OAM eigenstate)

“spin”-controlled “orbit” rotation (SAM eigenstate)

Both of these effects may occur either in space or time

Spatial vs. Temporal Rotation



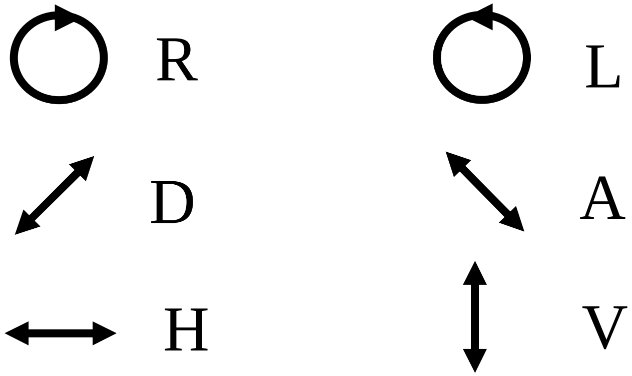
- The SOI may be thought of a a splitting of the dispersion curve for “parallel” vs. “anti-parallel” states.

- For a given frequency, there are two β values, so that the SOI splits the propagation constant (spatial rotation)

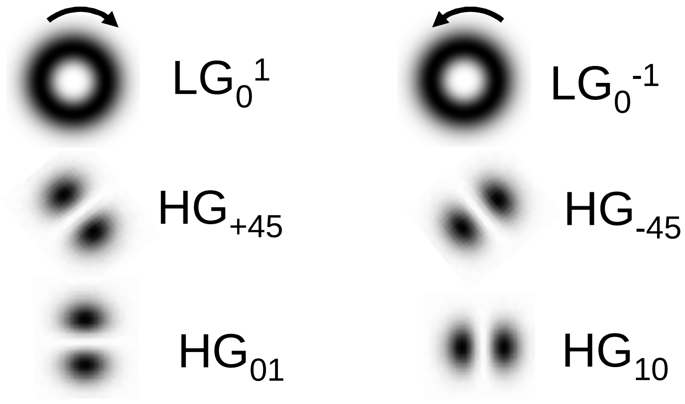
- If the both the frequencies and propagation constants of the parallel and anti-parallel states are originally slightly different, than the SOI acts to *restore* the propagation constants to degeneracy (temporal rotation)

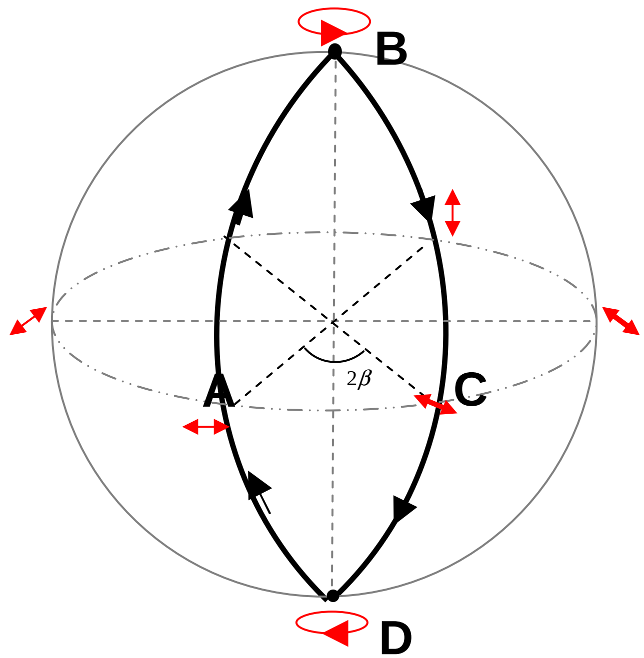
SAM to OAM entanglement transfer

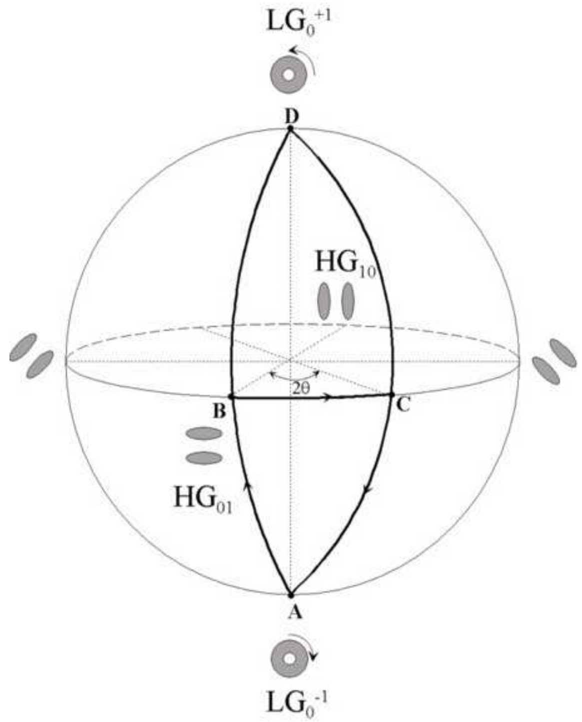
Polarization (SAM)



Transverse Beam Shape (OAM)

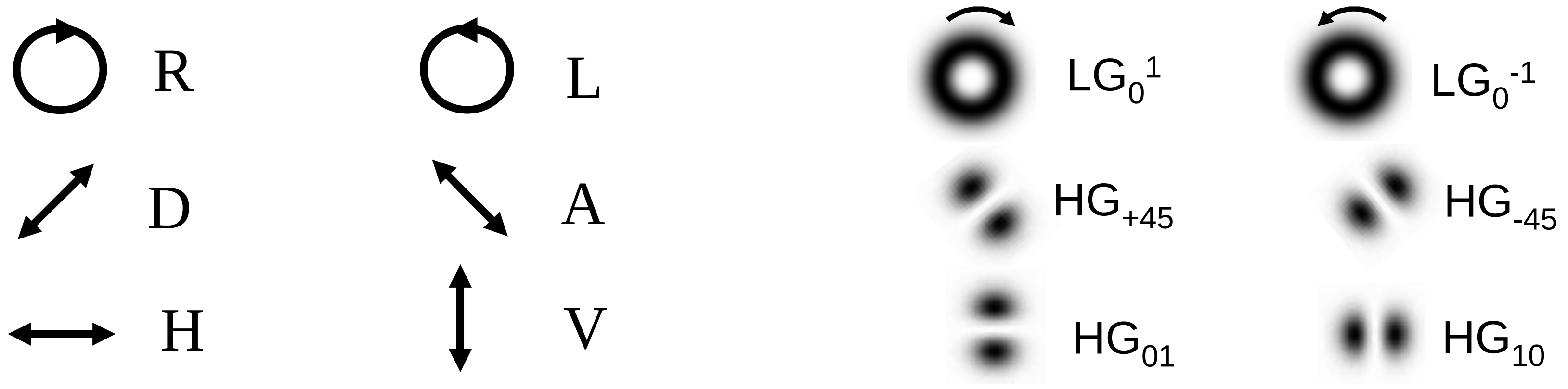






SAM to OAM entanglement transfer

Polarization (SAM) | Transverse Beam Shape (OAM)



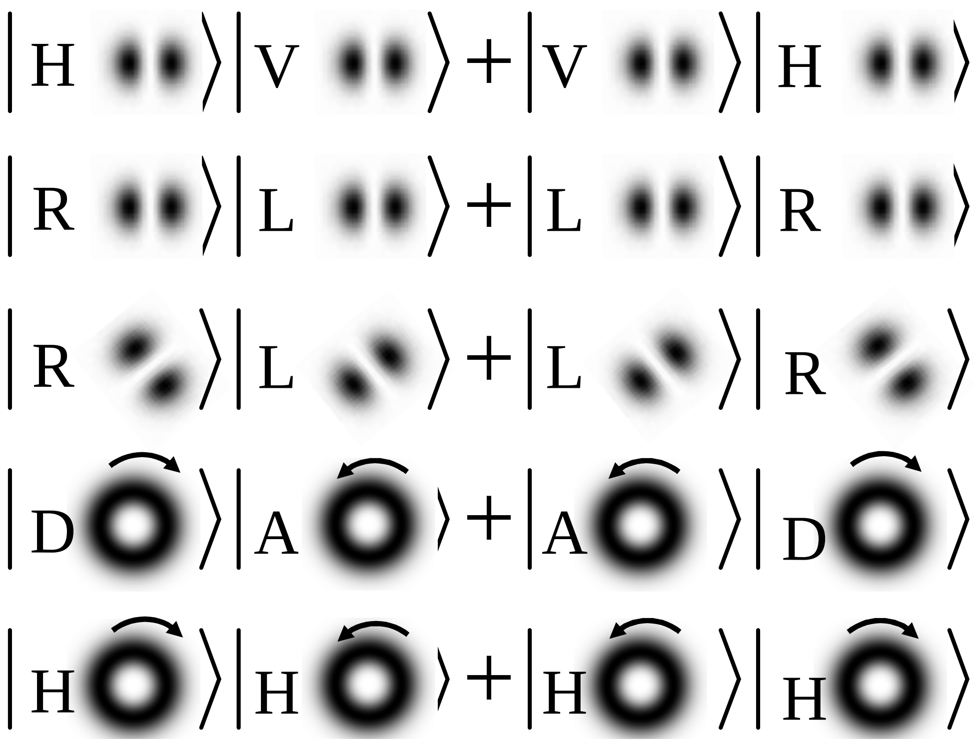
Begin in a purely polarization Bell state:

Quarter wave plates:

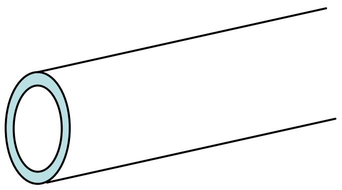
“Orbital” rotation SOI gate:

Quarter wave plates
and mode converters:

“Spin” rotation SOI gate:
(end in purely OAM–entangeld state)



OUTLINE REVISITED



1: Electron in an inhomogeneous potential– effective **B** field induces spin-orbit interaction

2: A photon in an inhomogeneous medium experiences an analogous spin-orbit interaction, even though it has no magnetic moment

3: In a cylindrical waveguide, the spin-orbit dynamics are described by a single expression applying to both electrons and photons

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5: Two-particle states: Reversible transfer of entanglement between SAM and OAM

Question: Why such a close analogy?

quant-ph > arXiv:0905.3778

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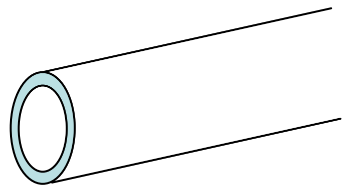

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Question: Why such a close analogy?

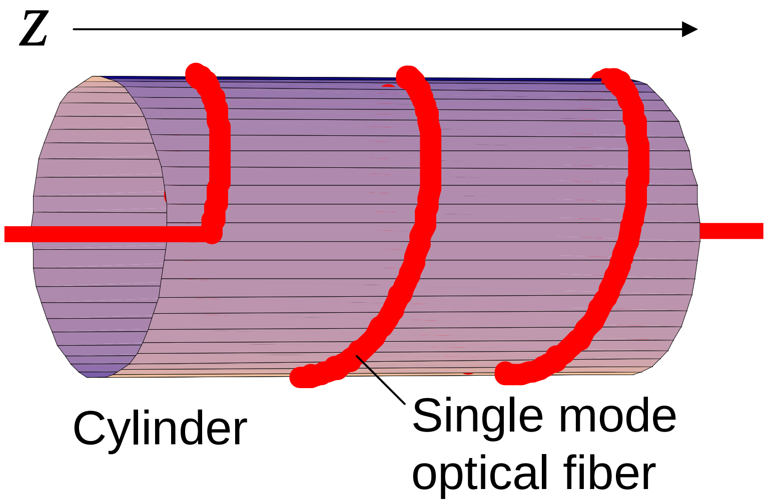
Answer: The spin-orbit interaction is a purely geometric effect (Berry phase)

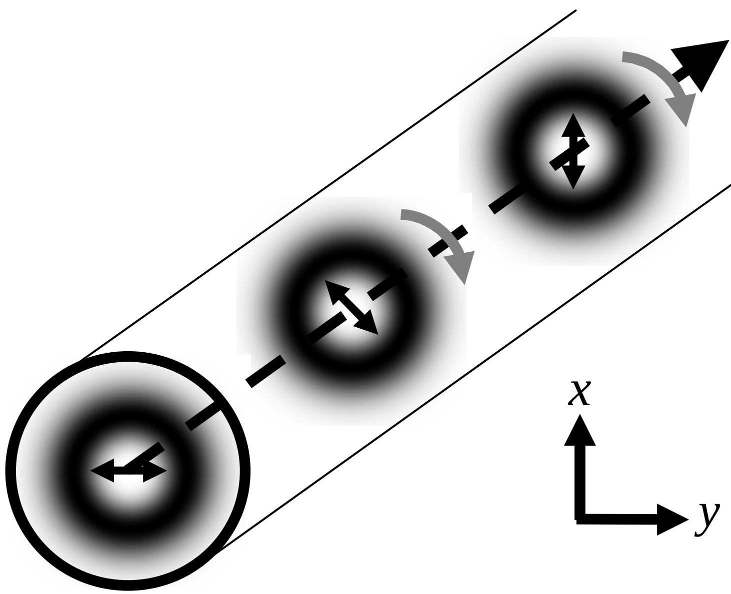
quant-ph > arXiv:0905.3778

Seem familiar?

The polarization rotation effect is reminiscent of an experiment of Chaio, in which the geometric (Berry) phase was first observed as polarization rotation in a coiled fiber.

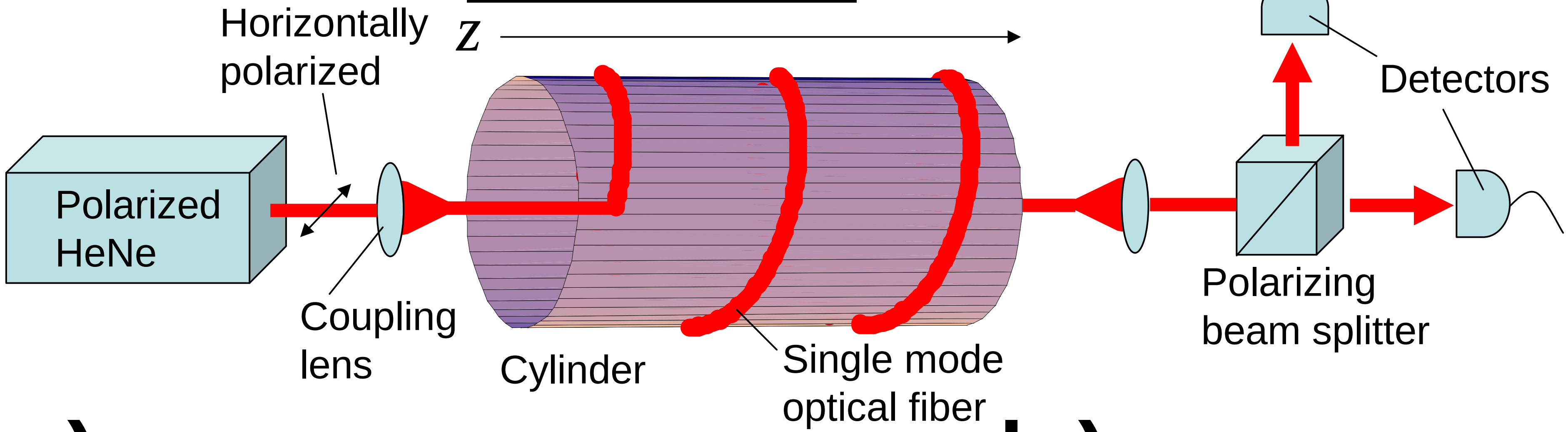
Chaio Experiment

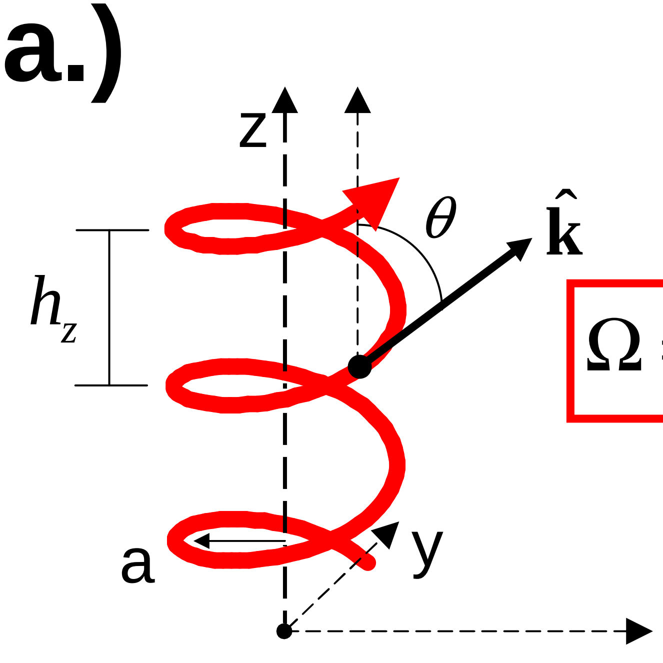




However, the Chaio effect involves only the fundamental Gaussian mode, which has zero OAM, and predicts an accumulated geometric phase of zero for a straight fiber! Nevertheless, considering this effect provides a way in which to understand the straight fiber rotation phenomena for modes with OAM.

Chaio Experiment

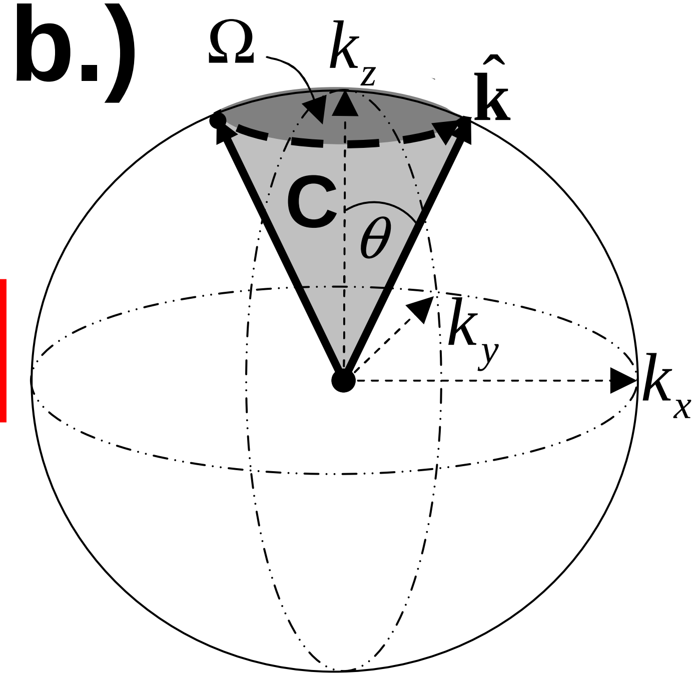




$$\boxed{\Omega = 2\pi N\left(1-\cos\theta\right)}$$

Where N = # of coils

$$\boxed{E \rightarrow E e^{-i\sigma \mu_{cl} \Omega}}$$



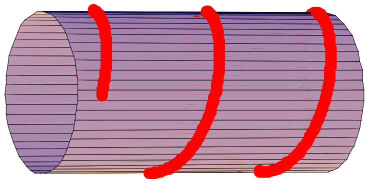
The geometric phase equals the surface area enclosed by the curve C, which is traced out by the momentum vector **k**. This phase causes a “beating” effect: polarization rotation

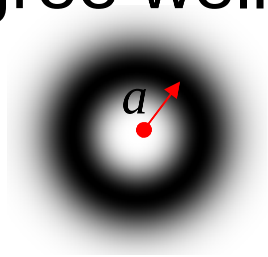
Geometric phase approach

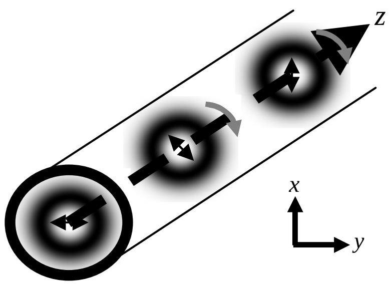
- "Classical" geometric phase: $E \rightarrow Ee^{-i\sigma\mu_{cl}\Omega(\mathbf{k})} \equiv Ee^{-i\gamma}$
  ("Classical" particle with spin; Bialynicki-Birula)
- For quantum states, we quantize: $\gamma \rightarrow \hat{\gamma} \equiv -\sigma\mu \frac{\Delta}{2\beta_0 a^2} \nabla_T$
- The geometric phase $\gamma = \delta\beta z$ is then given by the expectation of Ω in the unperturbed states:

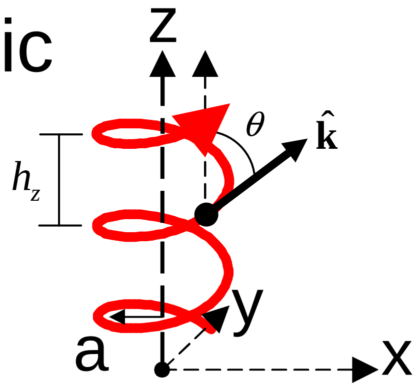
$$\gamma = \delta\beta z = \left\langle \Psi_{\sigma m_\ell} \middle| \hat{\gamma} \middle| \Psi_{\sigma m_\ell} \right\rangle \propto \sigma m_\ell \Rightarrow \Psi_{\sigma m_\ell} \rightarrow \Psi_{\sigma m_\ell} e^{i\gamma} = \Psi_{\sigma m_\ell} e^{i\delta\beta z}$$

- If *a* is set equal to the radius of the wavefunction "peak", then the perturbative and geometric approaches agree well.







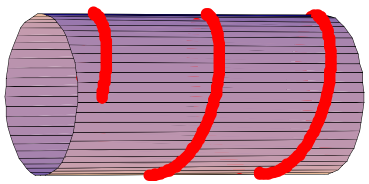


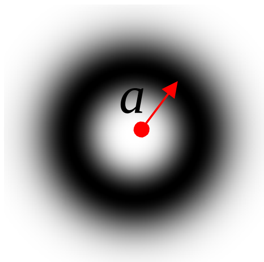
Geometric phase approach

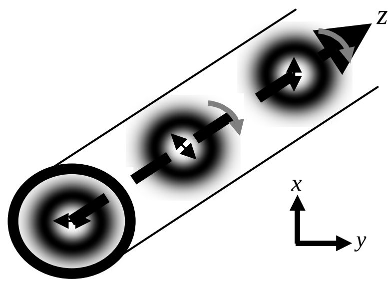
- Specifically, one gets a particularly simple form for the accumulated phase $e^{i\delta\beta z}$ in the special case of a step-index/potential of large radius *a*, for OAM states which are "near cutoff"– that is, their wavefunction "peak" is near the fiber radius:

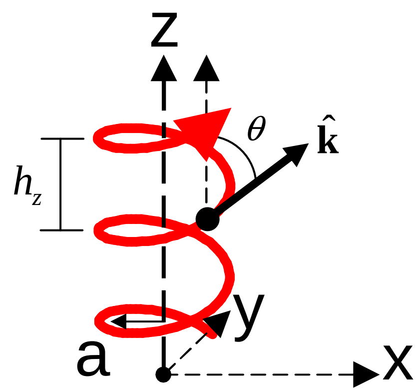
$$\boxed{\gamma = \delta\beta z = \sigma m_\ell \frac{\Delta}{2\beta_0 a^2} z}$$

- In this case the two approaches agree exactly.









Conclusions

- The spin-orbit interaction dynamics of the electron and photon are identical to first order in perturbation theory
- They have a common geometric origin
- The role of the electron's potential energy is played by the permittivity in the photon case
- Any particle with spin (i.e. neutrons) will also undergo SOI
- The SOI allows for the construction of both spin and orbital "gates", which may enable reversible transfer of entanglement between SAM and OAM
- Experiments are underway to observe these effects