Measurement and Control of a Photon's Spatial Wavefunction

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The College of Wooster, January 2011

Institute of Experimental Physics
Optics Division
University of Warsaw
Poland
# A brief history of light and matter

<table>
<thead>
<tr>
<th>Electrons (Matter)</th>
<th>Photons (Light)</th>
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Quantum Electrodynamics
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Relativistic Wave Optics  
Particle creation!

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Particle creation!

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Motivation: What I do and why I do it

- I develop new ways of measuring and controlling the wavefunction (or intensity distribution) of single photons

- This is worth doing:
  - It allows for direct experimental probing of the fundamentals of quantum theory (wavefunctions tell you everything!)
  - A photon's wavefunction carries quantum information (quantum bits) which have practical applications

- This is fun to do:
  - All my experiments may be done on as “tabletop physics”
  - Large group collaboration is not required
  - Undergraduates can (and have!) make crucial contributions
What is a photon?

- A photon is an oscillating electric field (wave) that propagates through space and time
- A photon makes a photon detector go “click!”
- A photon has 4 degrees of freedom (DOFs):
  1. It oscillates with a certain frequency (energy)
  2. It oscillates in a certain plane (polarization)
  3. Its intensity (& E-field) in the transverse x direction has a certain spatial shape
  4. Its intensity (& E-field) in the transverse y direction has a certain spatial shape

This talk concerns:
- The measurement and control of transverse spatial DOFs of photons
Transverse spatial modes

1.) Vertical “Coffee Bean”

- The electric field of one “lobe” is out of phase with the other
- Both modes have either even or odd parity in each dimension:
  - The vertical mode is odd under reflection in x
  - The horizontal mode is even under reflection in x
- Phase structure lies at the heart of photon quantum mechanics

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Transverse spatial modes are analogous to spherical harmonics in a hydrogen atom

Transverse spatial modes are thus the photon's wavefunction
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$

• An equal superposition of the vertical and horizontal coffee bean modes results in a new mode: a “diagonal coffee bean”.

$E_{TOT}(x, y) = E_{Vert}(x, y) + E_{Hor}(x, y)$

What I show

What I mean
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$

- An equal superposition of the vertical and horizontal coffee bean modes results in a new mode: a “diagonal coffee bean”.

- How to measure a photon's mode? (vertical, horizontal, diagonal)
How not to measure the spatial mode of a single photon

- A photon counting detector alone cannot measure the transverse spatial mode of a single photon.
- Suppose a “vertical” photon impinges on such a detector:

  ![Diagram](image)

  At this point, the photon could be in either mode:

- It would take many identical photons to build up the full spatial intensity pattern.
How to measure the spatial mode of a single photon

• We need a “magic box” that:
• Accepts an unknown state as input
• Routes photons to one output
• Routes photons to a separate output
• Detector click yields the desired measurement
• Let's call it a “Sorter”
1-D parity sorting interferometer

- Parity sorter is based on the superposition principle of the phase of an electric field
1-D parity sorting interferometer
1-D parity sorting interferometer
1-D parity sorting interferometer

Laser

BS1

BS2

A

B

\[ + + + = 0 \]
1-D parity sorting interferometer
"Odd" modes exit port A.

1-D parity sorting interferometer
1-D parity sorting interferometer

“Even” modes exit port B
A diagonal coffee bean input mode is split into its corresponding vertical and horizontal “components”.

1-D parity sorting interferometer
Experimental results

Experiment

<table>
<thead>
<tr>
<th>Mode</th>
<th>Input</th>
<th>Intensity</th>
<th>Phase</th>
<th>Port A</th>
</tr>
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<tbody>
<tr>
<td>HG45°</td>
<td></td>
<td></td>
<td>0</td>
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Experimental Setup

- Laser
- BS1
- BS2
- Camera

A diagram showing the experimental setup with a laser, beam splitters (BS1 and BS2), and a camera.
Experimental results

Output mode is controlled by tilting the glass slide
New kid on the block: the “donut” mode

Well-defined orbital angular momentum (2-D parity)

- Delaying the phase between the below superposed modes results in evolution of the “diagonal coffee bean” mode to a “donut” mode

\[
\begin{align*}
\text{No phase delay} & : \cos(\omega t) + \cos(\omega t) \\
\text{90° phase delay} & : \cos(\omega t) + i \sin(\omega t)
\end{align*}
\]
**Experimental results**

Conclusion: the 1-D parity interferometer imparts one quantum of orbital angular momentum to a single photon!
The 1-D parity interferometer is the simplest of several devices I designed and built along with undergraduates. Working under my direction, an undergraduate summer student (Zach Bond) built and tested this sorter over 10 weeks, taking the data just shown and presenting it at a national undergraduate symposium. Another undergraduate (Ashleigh Baumgardner) is a coauthor on a paper we published in the international journal *Optics Express*. Next up: proposed senior I.S. experiment. Conclude with a broader view of my research.
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must “CHOOSE” whether to be in a “vertical” or “horizontal” mode at the output.
- Only one detector can click.
- This device measures parity of single photons!

“Global” measurement with “local” detectors!
- This experiment has never been done.
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Broader view of my research

- Measuring a photon's wavefunction (1-D parity sorter)
- Imparting orbital angular momentum to a single photon
- Measuring a photon's wavefunction (2-D parity sorter)
- Controlling a photon's wavefunction with its own polarization state in optical fibers
- Direct analogies between the dynamics of photons and electrons in cylindrical geometries
- Hydrogen atom for photons?
2-D parity sorting interferometer

(a) \[ R \xrightarrow{\hat{\Pi}_x} R \xrightarrow{\hat{\Pi}_y} R \]

(b) \[ R \xrightarrow{\hat{\Pi}_{xy} = \hat{R}_{180^\circ}} R \]

Diagram showing the 2-D parity sorting interferometer with labels and arrows indicating the sequence of operations.
Experimental results: 2-D parity sorting

<table>
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<tr>
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<th>Port A</th>
<th>Port B</th>
<th>Interference Intensity Profile</th>
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Controlling a photon's wavefunction with its polarization state

“orbit”-controlled “spin” rotation

“spin”-controlled “orbit” rotation

• The effects occur analogously for electrons and photons!
• Independent of mass, charge, magnetic moment, etc.

Conclusions

• Just like electrons, photons have wave functions (transverse spatial modes)
• I have learned to measure and control them directly:
  – 1-D Parity
  – 2-D Parity
• And indirectly: spin-controlled wavefunction rotation
  – Used to transfer quantum info between DOFs
• Together, the above methods allow (in principle) for universal quantum information processing
• Electrons and photons evolve identically in cylinders
• What about spherical symmetry? (Hydrogen atom!)
• Trap photons in glass spheres- store quantum info!
Spin-orbit interaction experiment

SM Fiber Laser → Glass plates → 3 Mode Fiber → Fiber Crusher → 2-D Sorter

Even Port → Odd Port

Odd Port → To 1-D Sorter

To Polarization Analyzer

Test Fiber
Quantum experiment: two photons

- Put two photons in different input ports, and you get out **EITHER** two “vertical” photons **OR** two “horizontal” ones, but **NEVER** one of each!

- This experiment has never been done
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- "Two photon interference" probabilities cancel out!
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