

Single-Photon Spin-Orbit Coupling for Cluster State Quantum Computation

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Outline

- **What is it?**

Overview of photon spin-orbit coupling (SOC)

- **How does it work?**

Physics of SOC

- **What is it good for?**

Applications of SOC for cluster state quantum computing

- **Conclusions**

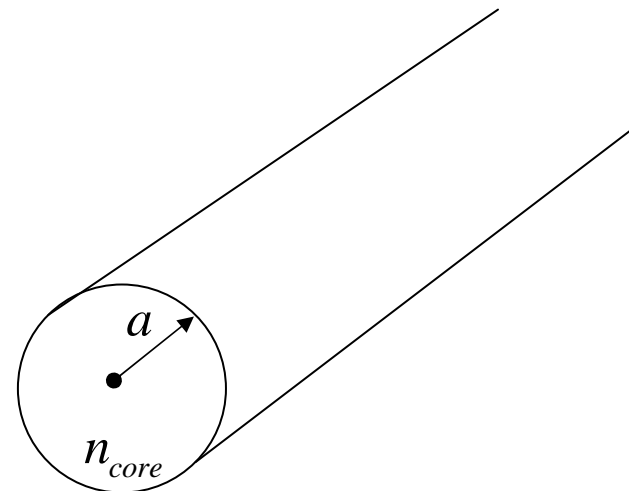
Progress toward observing and controlling the effect

What is it?

Overview of photon SOC

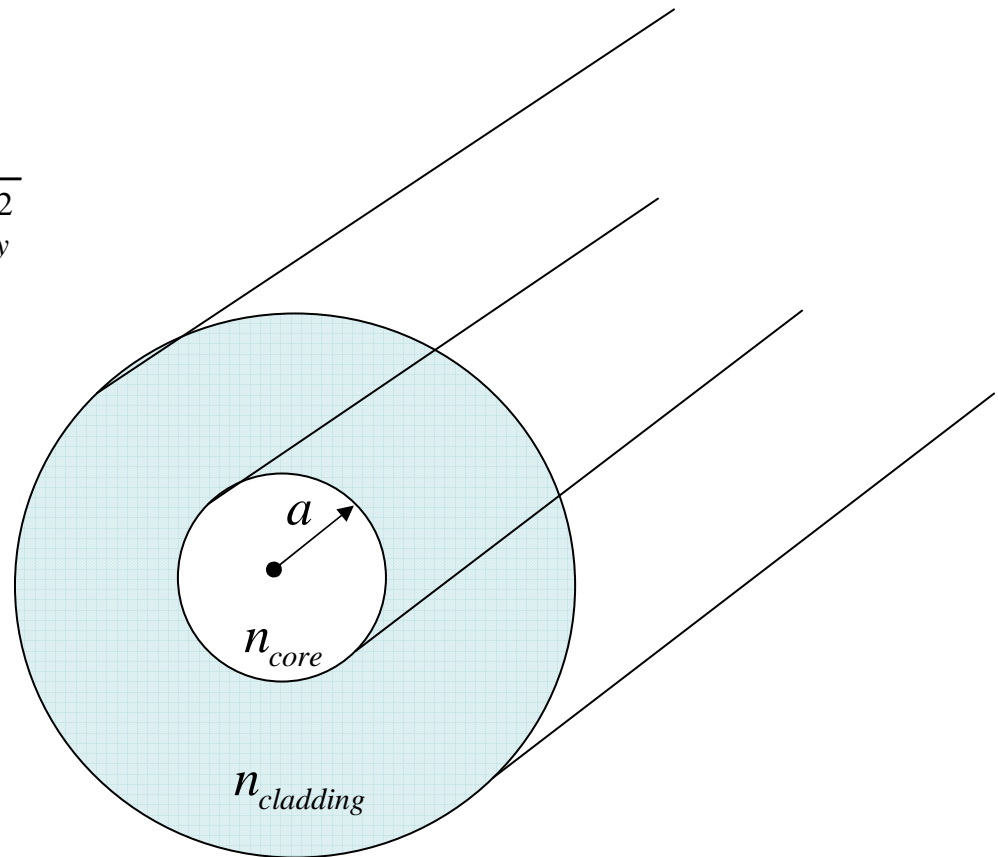
Overview of photon SOC

- Ingredients:
 - Inhomogeneous medium
 - Cylindrical symmetry
 - Quasi-paraxial photon:
 $\kappa \ll |\mathbf{k}|$, where $\kappa \equiv \sqrt{k_x^2 + k_y^2}$
 - Quasi-monochromatic
- Simplest example:
step-index optical fiber
 - $n_{core} > n_{cladding}$
 - Infinite cladding radius



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The spin-orbit coupling effect

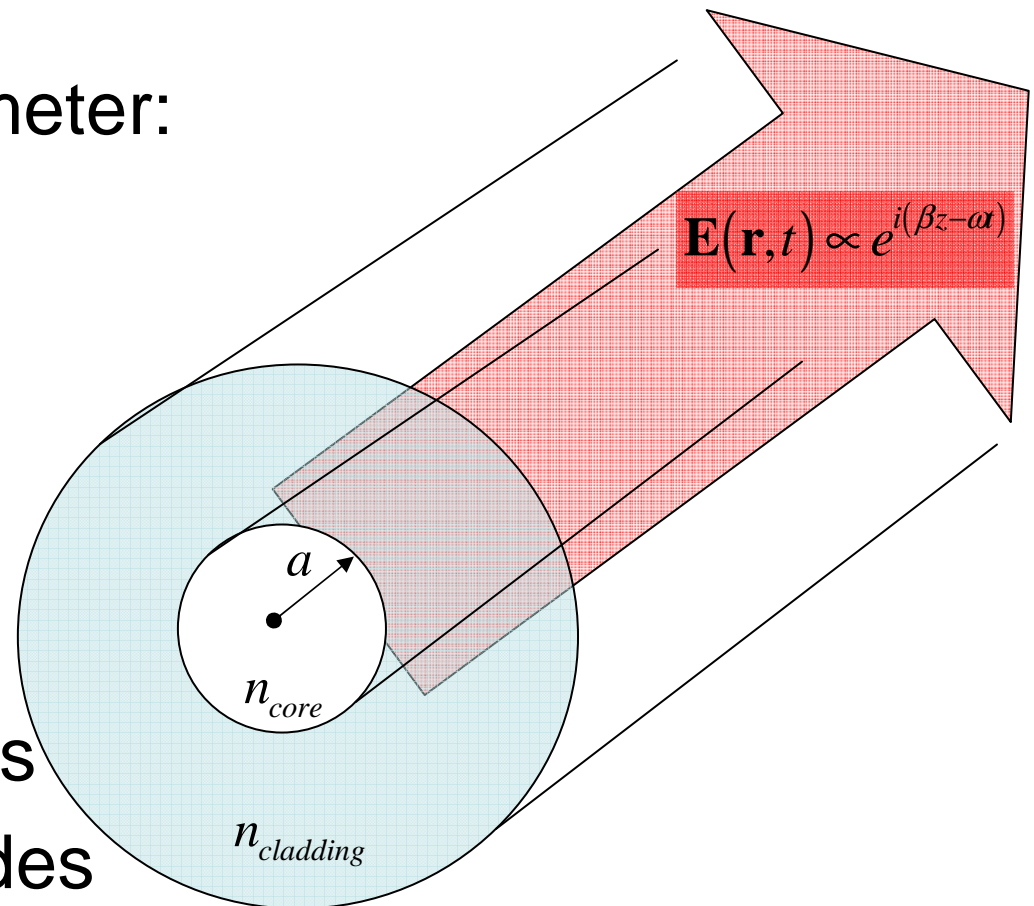
- Field in fiber proportional to $\mathbf{E}(\mathbf{r}, t) \propto e^{i(\beta z - \alpha t)}$

- Define the fiber V parameter:

$$V \equiv \frac{\omega a}{c} \sqrt{n_{core}^2 - n_{cladding}^2}$$

- Assume the far-from-cutoff limit: $V \gg 1$

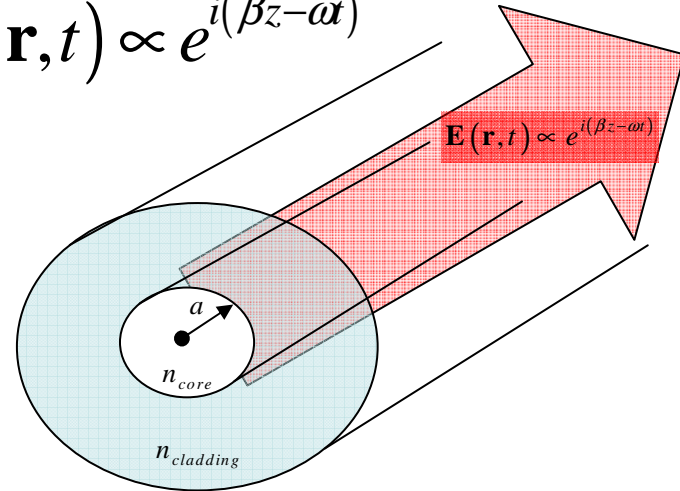
- Solve Maxwell equations to obtain fiber eigenmodes



The spin-orbit coupling effect

- Field in fiber proportional to $\mathbf{E}(\mathbf{r}, t) \propto e^{i(\beta z - \omega t)}$

- $$V \equiv \frac{\omega a}{c} \sqrt{n_{core}^2 - n_{cladding}^2}$$



- The effect:**

- Photon eigenmodes have well defined values of both spin σ and orbital angular momentum m_ℓ along z axis
- Modal propagation constants β split according to

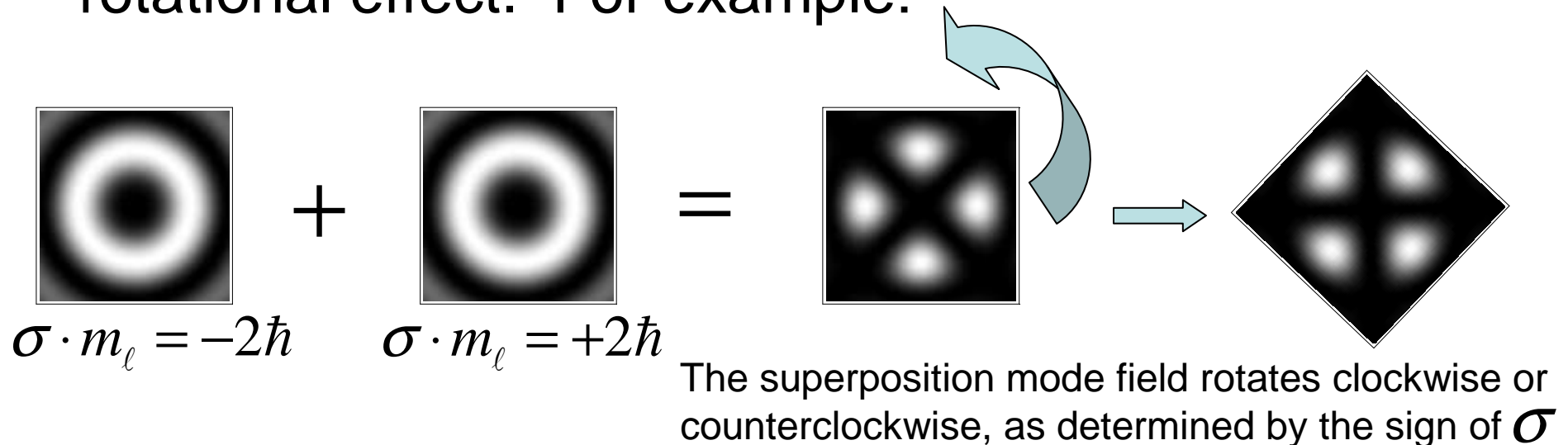
$$\beta \approx k \left\{ 1 - \frac{f_1}{V^2} + \frac{f_2}{V^3} - \frac{f_3}{V^4} (\sigma \cdot m_\ell) \right\}$$

when $n_{core}^2 \approx n_{cladding}^2$

(the “f” functions depend on simple fiber parameters)

Observable consequence of SOC

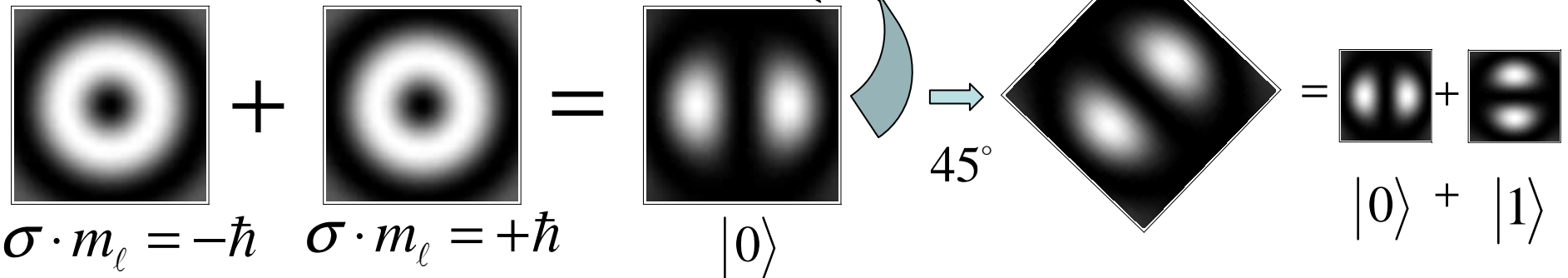
- If two fiber photon eigenmodes with different values of the quantity $\sigma \cdot m_\ell$ propagate in superposition, the relative phase between them will vary as the photon field propagates, due to their velocity mismatch.
- This phase shift gives rise to a σ -dependent rotational effect. For example:



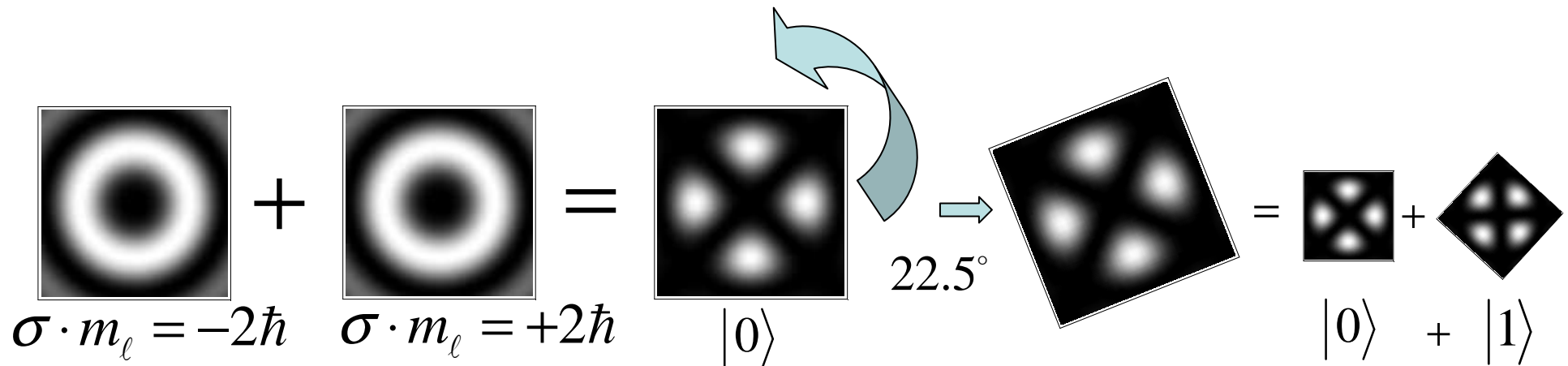
- **This is a spin-controlled spatial-mode Hadamard gate**

Spin-controlled Hadamard gate

$$\mathbf{E} \propto \cos[m_\ell \varphi] \longrightarrow \cos[m_\ell (\phi + \sigma \Delta \beta z)]$$



Flipping the photon spin (circular polarization) also flips the direction of rotation of the superposition spatial mode.



How does it work?

Physics of spin-orbit coupling

How does it work?

Physics of spin-orbit coupling

- It is the inhomogeneity of the medium gives rise to the effect— there is no paraxial free-space SOC
- B. Zeldovich (PRA '91) first mentioned photon SOC:
 - He treats a many-mode fiber with a parabolic index profile
 - Rotation of speckle pattern observed, but not of single modes
 - Kapany and Burke ('72) mention mode rotation, but not in the context of single photons or of spin-orbit coupling.
- We consider individual fiber modes propagating in a step-index few-mode fiber
 - The SOC effect is strongest for the step-index case
 - This is due to the $-\nabla \left[\mathbf{E} \cdot \nabla (\ln \varepsilon(\mathbf{r})) \right]$ term in the Helmholtz eqn

Photon angular momentum quantum numbers

- Define orbital angular momentum operators and quantum numbers:

$$\hat{\mathbf{L}} \equiv -i\hbar(\mathbf{r} \times \nabla) \quad \rightarrow \quad \hat{L}^2 |l\rangle \equiv (\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}) |l\rangle = -\hbar^2 l(l+1) |l\rangle$$

↓

$$\hat{L}_z = \hat{\mathbf{L}} \cdot \hat{\mathbf{z}} = -i\hbar \frac{\partial}{\partial \phi} \quad \rightarrow \quad \hat{L}_z |m_l\rangle = \hbar m_l |m_l\rangle$$

- Define “spin-1” operators and quantum numbers:

$$\hat{\mathbf{S}} \equiv -i\hbar \epsilon_{ijk} \hat{\mathbf{e}}_k \quad \rightarrow \quad \hat{S}_z = \hat{\mathbf{S}} \cdot \hat{\mathbf{e}}_3 = -i\hbar \epsilon_{ij3} = \begin{pmatrix} 0 & -i\hbar & 0 \\ +i\hbar & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \rightarrow \quad \hat{S}_z |\sigma\rangle = \hbar \sigma |\sigma\rangle$$

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$$\downarrow$$

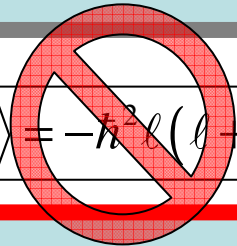
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The above are NOT generally good photon quantum numbers!

Photon angular momentum quantum numbers

$$\hat{L}^2 | \ell \rangle = -\hbar^2 \ell(\ell+1) | \ell \rangle$$


$$\hat{L}_z | m_\ell \rangle = \hbar m_\ell | m_\ell \rangle$$

$$\hat{S}_z | \sigma \rangle = \hbar \sigma | \sigma \rangle$$

The above are **NOT** generally good photon quantum numbers!

- However, in the aforementioned paraxial and far-from-cutoff limits, m_ℓ and σ **are** good quantum numbers for the photon fields in a step-index fiber:

$$\hat{L}_z \mathbf{E}(\mathbf{r}, t) \rightarrow -i\hbar \frac{\partial}{\partial \phi} \mathbf{E}(\mathbf{r}, t) = \hbar m_\ell \mathbf{E}(\mathbf{r}, t)$$

$$\hat{S}_z \mathbf{E}(\mathbf{r}, t) \rightarrow \begin{pmatrix} 0 & -i\hbar & 0 \\ +i\hbar & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{E}(\mathbf{r}, t) = \hbar \sigma \mathbf{E}(\mathbf{r}, t)$$

Fiber photon eigenmodes

- Fiber photon eigenmodes can be labeled as follows:

$$\mathbf{E} \propto J_{m_\ell}(kr) e^{im_\ell\phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

$$n = 1, 2, 3, \dots$$

$$\omega \in \mathbb{R}$$

$$m_\ell = 0, \pm\hbar, \pm 2\hbar, \dots$$

$$\sigma = \pm\hbar$$

$$\mathbf{e}_\sigma \equiv \hat{\mathbf{x}} + i\sigma\hat{\mathbf{y}}$$

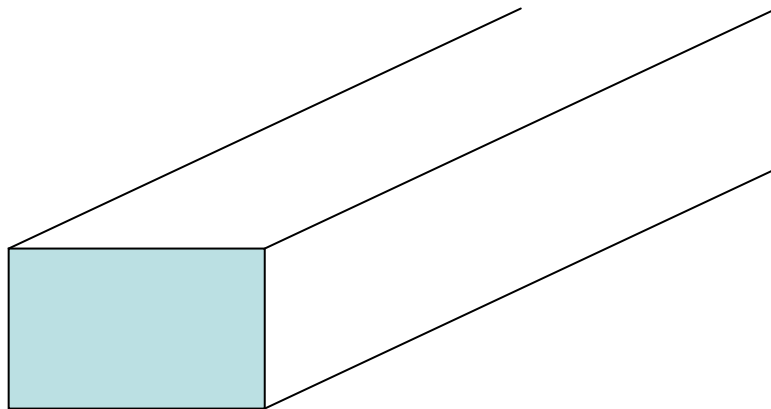
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$$n = 1, 2, 3, \dots \quad \omega \in \mathbb{R}$$

$$m_\ell = 0, \pm\hbar, \pm 2\hbar, \dots \quad \sigma = \pm\hbar$$



$$|k_x, k_y, k_z, \alpha\rangle$$

Fiber photon eigenmodes

- Fiber photon eigenmodes can be labeled as follows:

$$\mathbf{E} \propto J_{m_\ell}(\kappa r) e^{im_\ell \phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

$$n = 1, 2, 3, \dots \quad \omega \in \mathbb{R}$$

$$m_\ell = 0, \pm\hbar, \pm 2\hbar, \dots \quad \sigma = \pm\hbar$$

- Propagation constant $\beta \equiv \sqrt{k^2 - \kappa^2} \approx k \left(1 - \frac{1}{2} \left(\frac{\kappa}{k} \right)^2 \right)$
- Transverse wavenumber $\kappa(n, m_\ell, \omega, \sigma)$

Fiber photon eigenmodes

- The roles of m_ℓ and σ are now explicit:

$$\mathbf{E} \propto J_{m_\ell}(kr) e^{im_\ell\phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

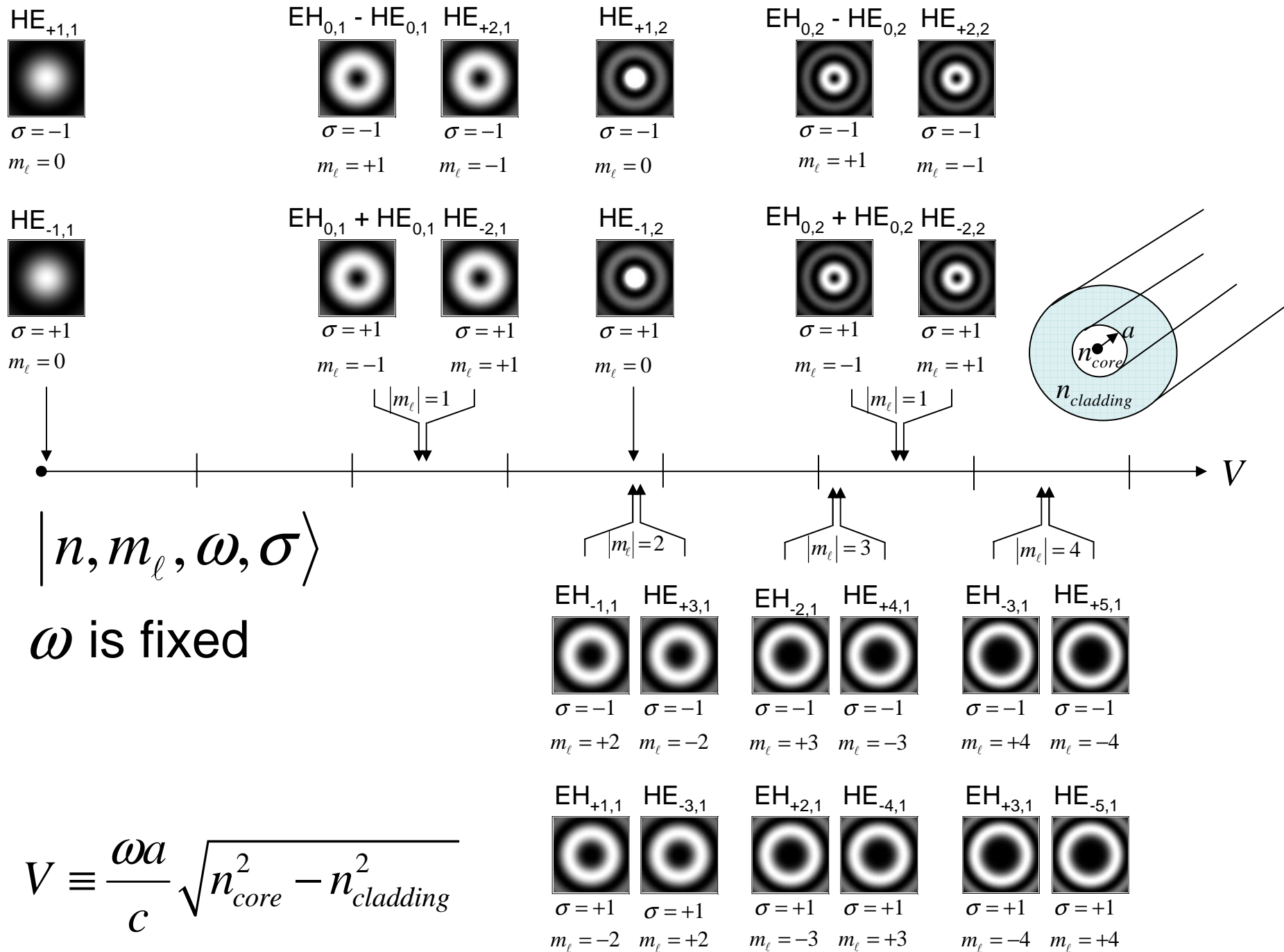
$$-i\hbar \frac{\partial}{\partial \phi} \mathbf{E}(\mathbf{r}, t) = \hbar m_\ell \mathbf{E}(\mathbf{r}, t)$$

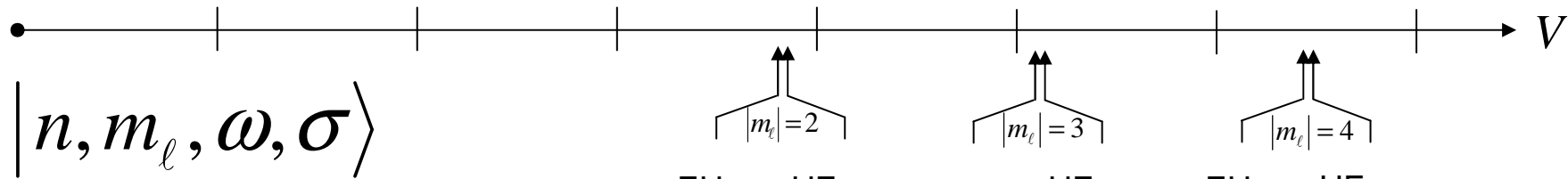
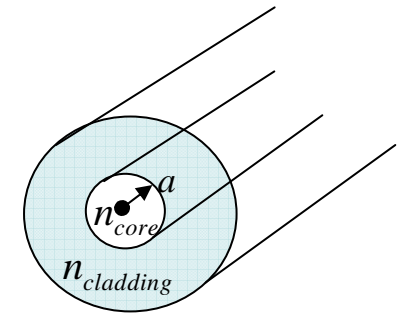
$$\begin{pmatrix} 0 & -i\hbar & 0 \\ +i\hbar & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} E(\mathbf{r}, t) \begin{pmatrix} 1 \\ i\sigma \\ 0 \end{pmatrix} = \hbar \sigma E(\mathbf{r}, t) \begin{pmatrix} 1 \\ i\sigma \\ 0 \end{pmatrix}$$

$$\beta \approx k \left\{ 1 - \frac{f_1}{V^2} + \frac{f_2}{V^3} - \frac{f_3}{V^4} (\sigma \cdot m_\ell) \right\}$$

Eigenmode pictures

$$\mathbf{E} \propto J_{m_\ell}(Kr) e^{im_\ell\phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

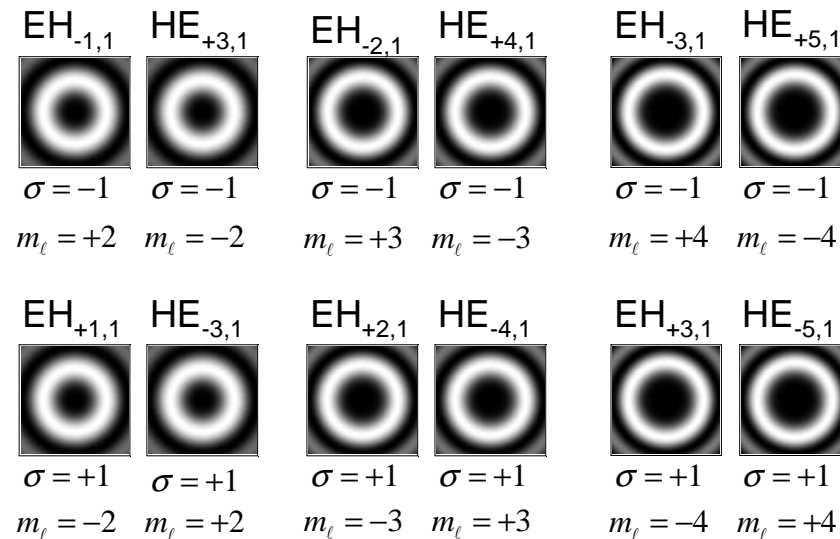




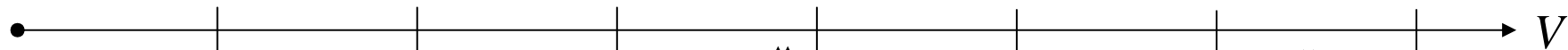
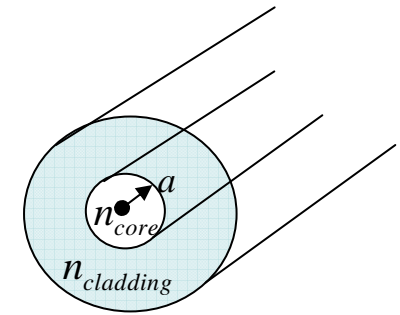
ω is fixed
 $n = 1$

$$|m_\ell| > 1$$

$$V \equiv \frac{\omega a}{c} \sqrt{n_{core}^2 - n_{cladding}^2}$$



- Eigenmodes have cylindrical symmetry



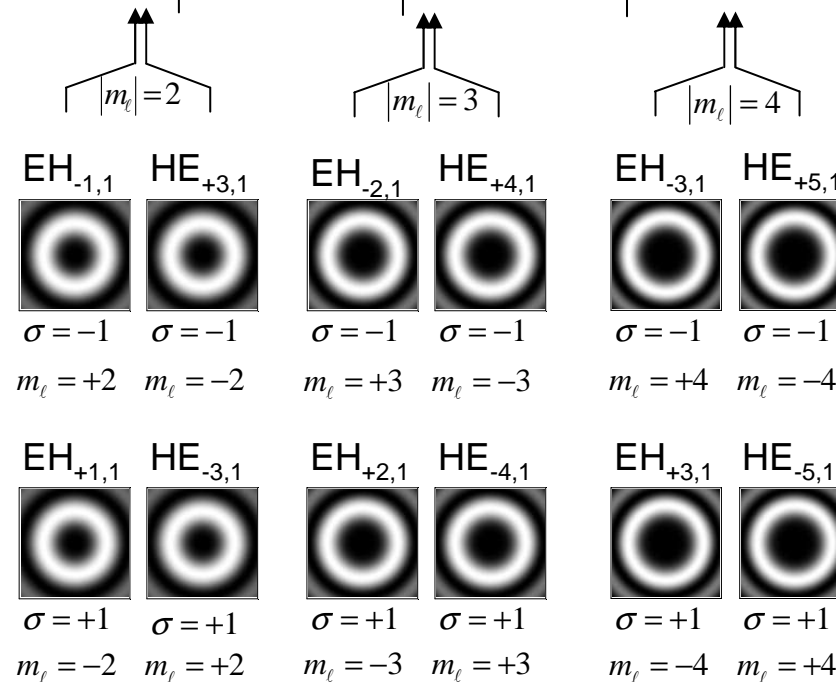
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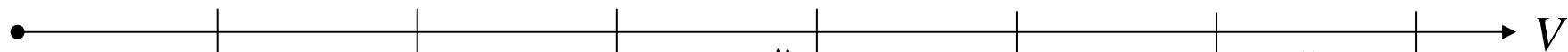
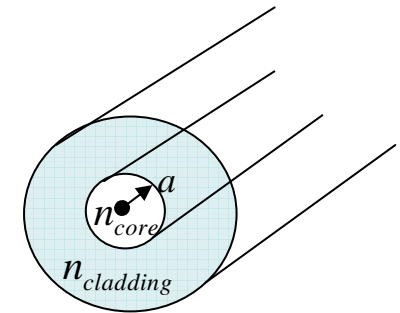
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- Eigenmodes have cylindrical symmetry
- Grouped into families of $|m_\ell|$



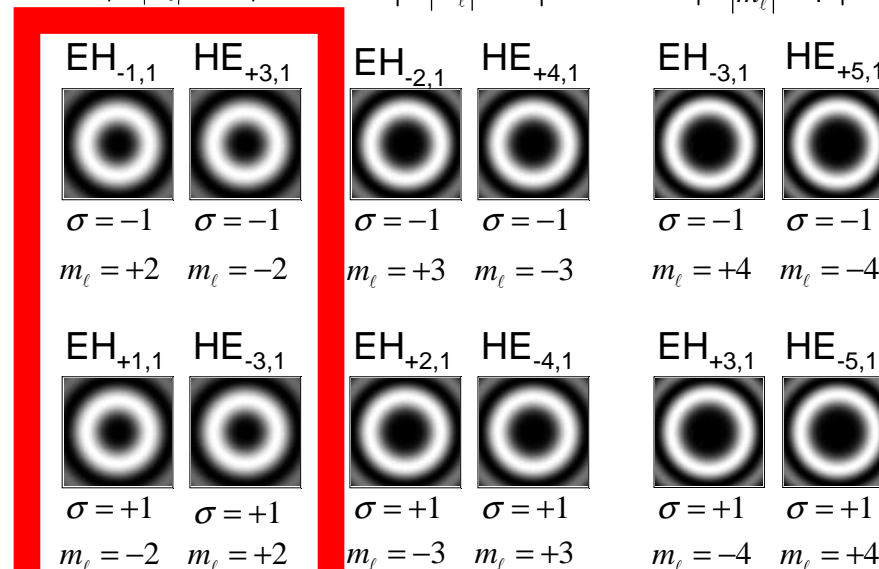
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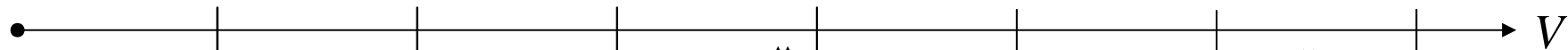
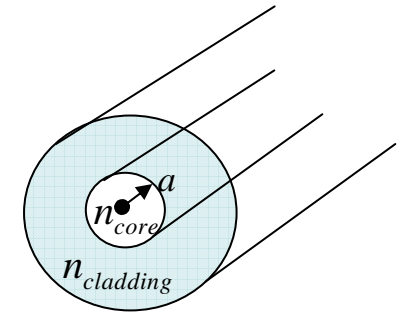
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- Eigenmodes have cylindrical symmetry
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- Paired according to σ



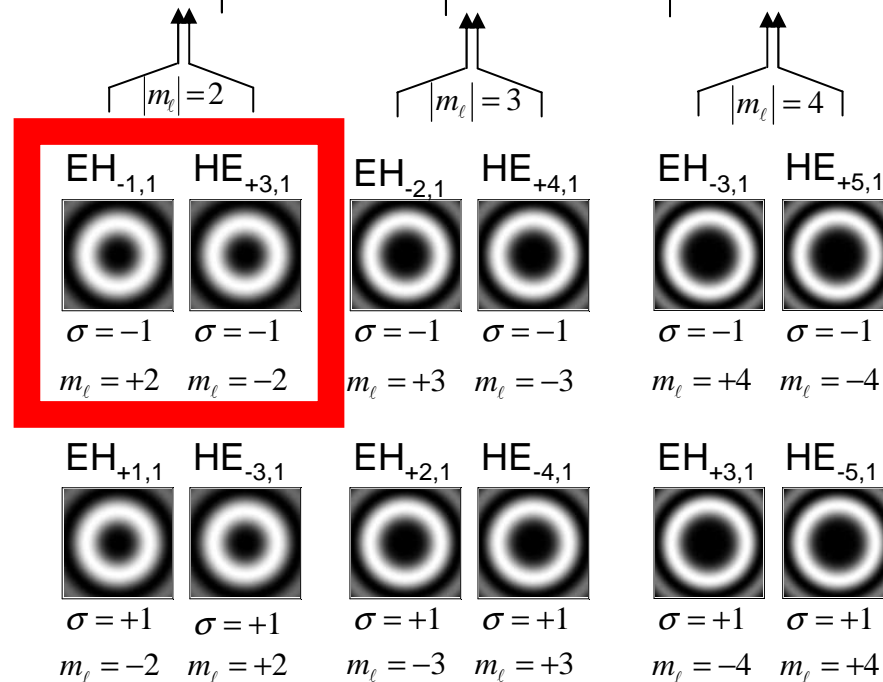
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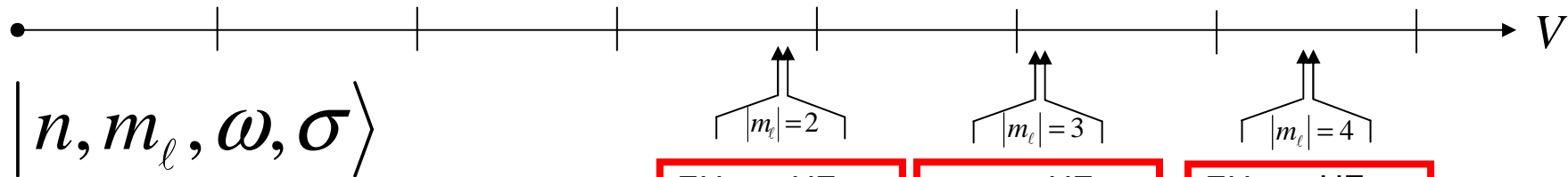
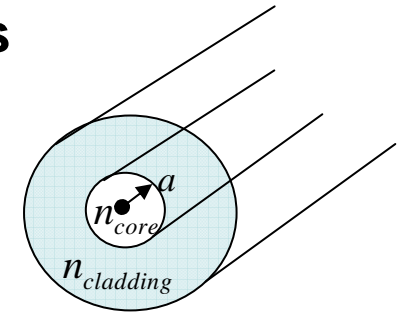
$$n = 1$$

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- Eigenmodes have cylindrical symmetry
- Grouped into families of $|m_\ell|$
- Paired according to σ
- Each mode has well defined spin and orbital angular momentum
- The modes of each family have **identical field distributions**
- Parallel vs. anti-parallel momenta: $\sigma \cdot m_\ell = \pm 1$
- Propagation constant $\sigma \cdot m_\ell$ dependent; mode pairs have a small degeneracy lifting in propagation constant

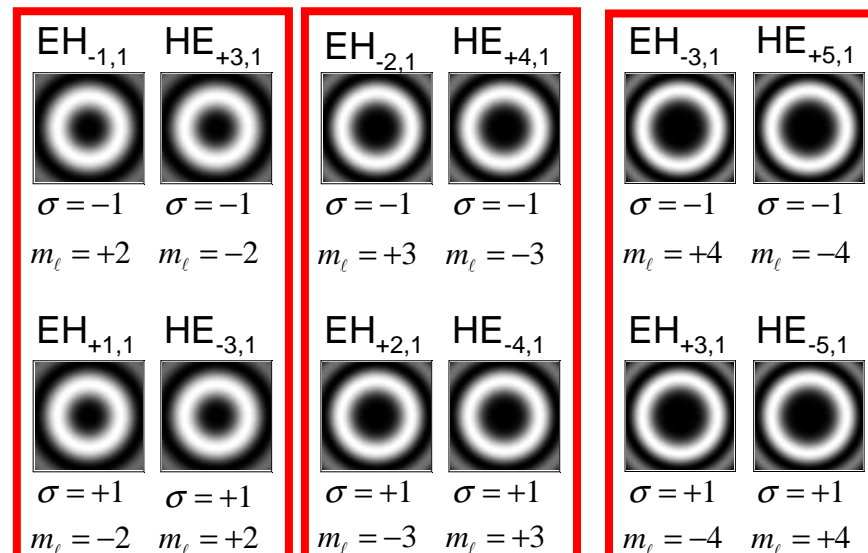


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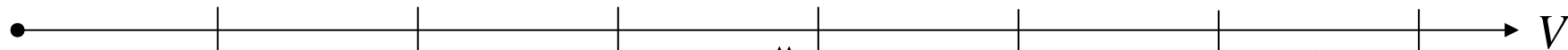
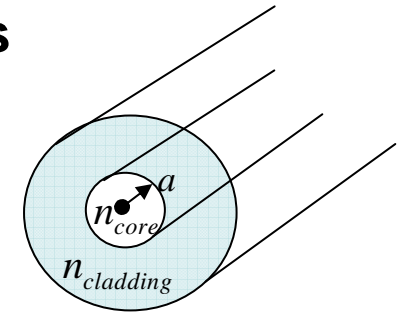
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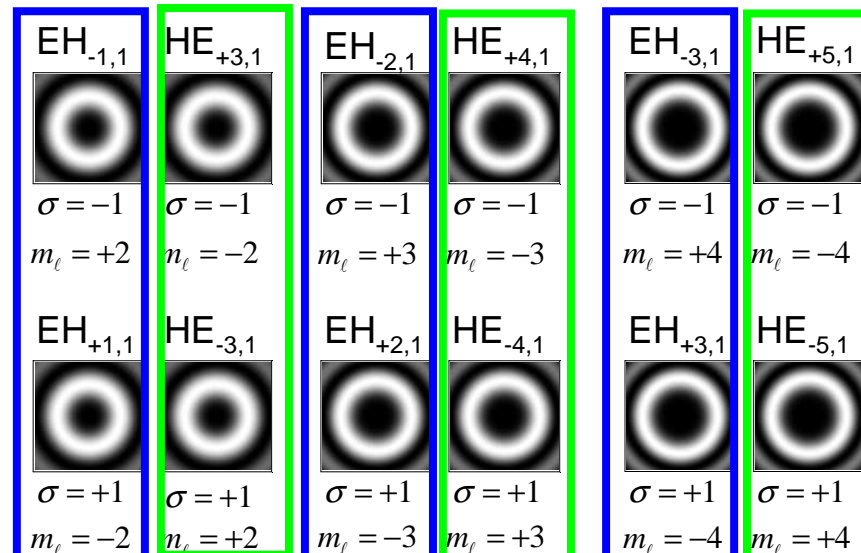
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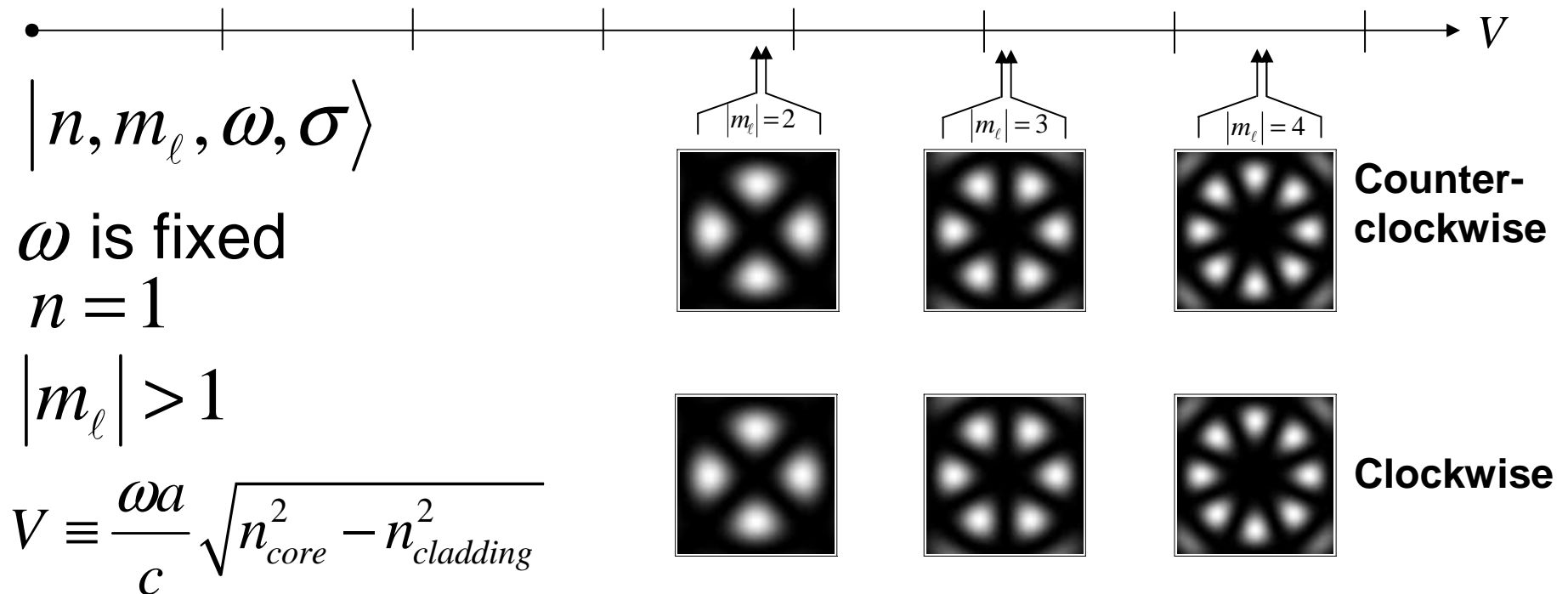
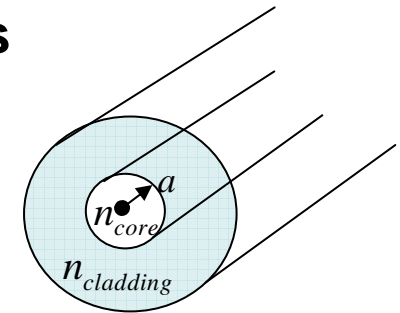
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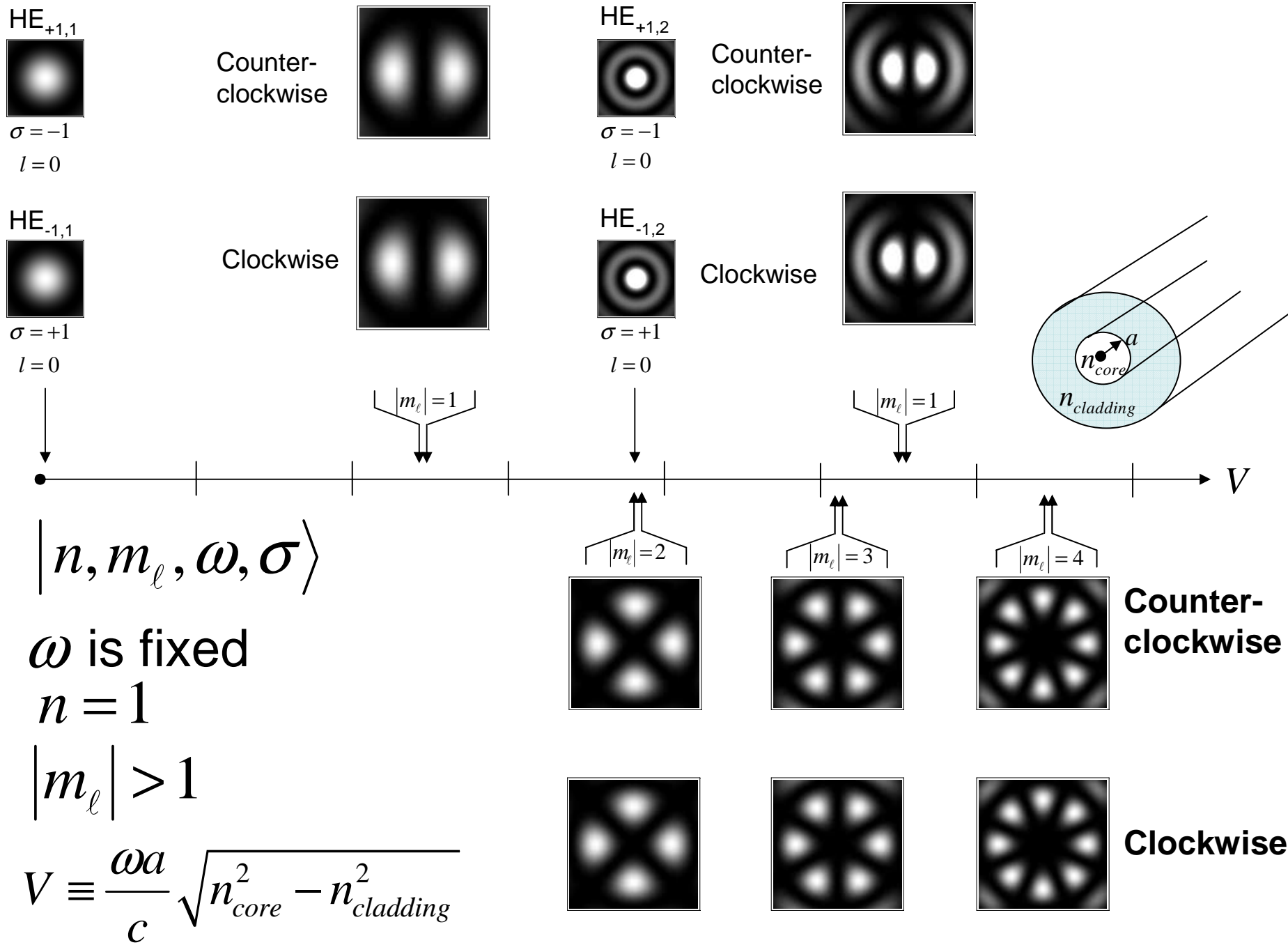
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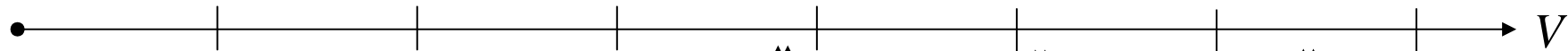
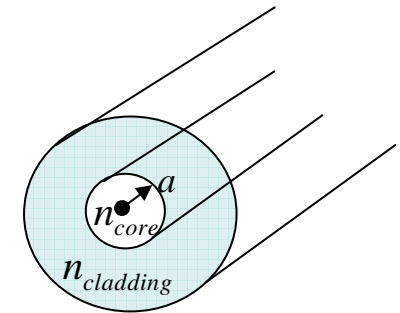
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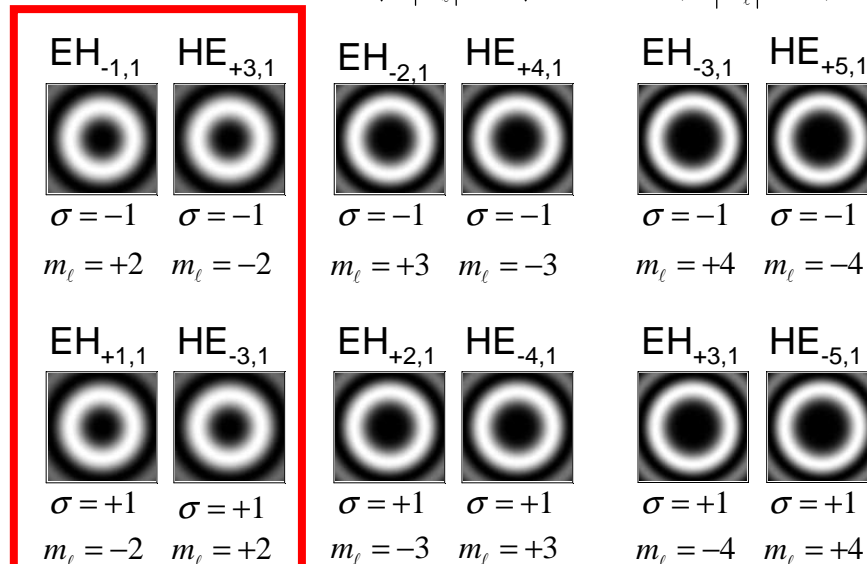
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- The modes of each family have **identical field distributions**
- Parallel vs. anti-parallel momenta: $\sigma \cdot m_\ell = \pm 1$
- Propagation constant $\sigma \cdot m_\ell$ dependent; mode pairs have a small degeneracy lifting in propagation constant
- **Superposition modes exhibit spin-controlled rotation**







We focus next on
the subset of
modes where $|m_\ell| = 2$

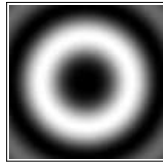
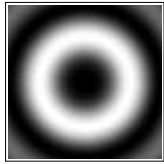


How does it work?

Propagation depends on absolute m_j

$$\mathbf{E} \propto J_{m_\ell}(\kappa r) e^{im_\ell\phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

$$|1, +2, \omega, -1\rangle \quad |1, -2, \omega, -1\rangle$$



$$\sigma = -1$$

$$m_\ell = +2$$

$$\sigma = -1$$

$$m_\ell = -2$$

$$\sigma \cdot m_\ell = -2 \quad \sigma \cdot m_\ell = +2$$

(anti-parallel) (parallel)

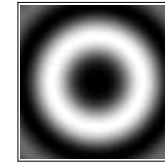
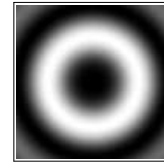
$$|m_j| = 1$$

$$|m_j| = 3$$

(faster)

(slower)

$$|1, -2, \omega, +1\rangle \quad |1, +2, \omega, +1\rangle$$



$$\sigma = +1$$

$$m_\ell = -2$$

$$\sigma = +1$$

$$m_\ell = +2$$

$$\sigma \cdot m_\ell = -2 \quad \sigma \cdot m_\ell = +2$$

(anti-parallel) (parallel)

$$|m_j| = 1$$

$$|m_j| = 3$$

(faster)

(slower)

$$\hat{J}_z \equiv \hat{L}_z + \hat{S}_z \quad m_j \equiv m_\ell + \sigma \quad \hat{J}_z |n, m_\ell, \omega, \sigma\rangle = \hbar m_j |n, m_\ell, \omega, \sigma\rangle$$

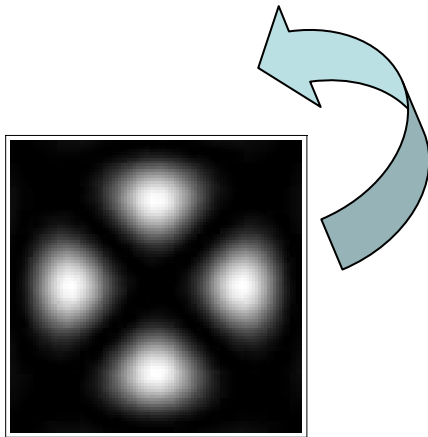
How does it work?

Propagation depends on absolute m_j

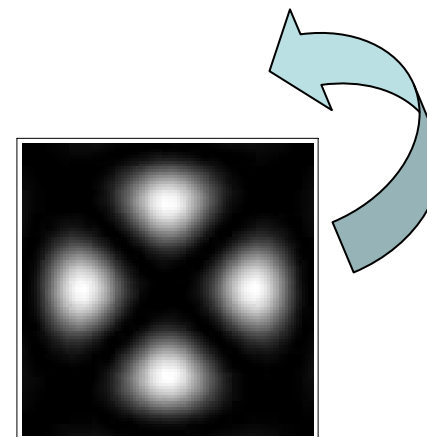
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$$|1, +2, \omega, -1\rangle + |1, -2, \omega, -1\rangle$$

$$|1, -2, \omega, +1\rangle + |1, +2, \omega, +1\rangle$$



**Counter-
clockwise**



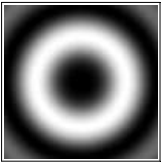
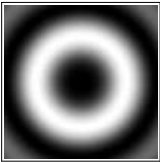
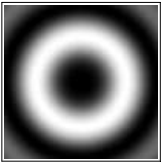
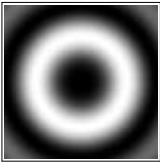
Clockwise

$$\hat{J}_z \equiv \hat{L}_z + \hat{S}_z \quad m_j \equiv m_\ell + \sigma \quad \hat{J}_z |n, m_\ell, \omega, \sigma\rangle = \hbar m_j |n, m_\ell, \omega, \sigma\rangle$$

How does it work?

Propagation depends on absolute m_j

$$\mathbf{E} \propto J_{m_\ell}(\kappa r) e^{im_\ell\phi} e^{i(\beta z - \omega t)} \mathbf{e}_\sigma \leftrightarrow |n, m_\ell, \omega, \sigma\rangle$$

$ 1, +2, \omega, -1\rangle$		$ 1, -2, \omega, -1\rangle$?				$ 1, -2, \omega, +1\rangle$		$ 1, +2, \omega, +1\rangle$	
											
$\sigma = -1$	$\sigma = -1$	$\sigma = +1$	$\sigma = +1$								
$m_\ell = +2$	$m_\ell = -2$	$m_\ell = -2$	$m_\ell = +2$								
$\sigma \cdot m_\ell = -2$	$\sigma \cdot m_\ell = +2$	$\sigma \cdot m_\ell = -2$	$\sigma \cdot m_\ell = +2$								
(anti-parallel)	(parallel)	(anti-parallel)	(parallel)								
$ m_j = 1$	$ m_j = 3$	$ m_j = 1$	$ m_j = 3$								
(faster)	(slower)	(faster)	(slower)								

$$\hat{J}_z \equiv \hat{L}_z + \hat{S}_z \quad m_j \equiv m_\ell + \sigma \quad \hat{J}_z |n, m_\ell, \omega, \sigma\rangle = \hbar m_j |n, m_\ell, \omega, \sigma\rangle$$

What is it good for?

Applications of SOC: cluster states

What is it good for?

Applications of SOC: Hermite-Gauss (HG) spatial-mode-entangled cluster states

$$|0\rangle = \text{[Image of two Gaussian spots in a square frame]}$$

We denote the Bell state

$$|00\rangle + |11\rangle \text{ by } |\bullet\bullet \bullet\bullet\rangle + |\bullet\bullet \bullet\bullet\rangle$$

$$|1\rangle = \text{[Image of two Gaussian spots in a square frame, vertically offset]}$$

And the GHZ state

$$|00\rangle + |11\rangle \text{ by } |\bullet\bullet\bullet\bullet\bullet\bullet\rangle + |\bullet\bullet\bullet\bullet\bullet\bullet\rangle$$

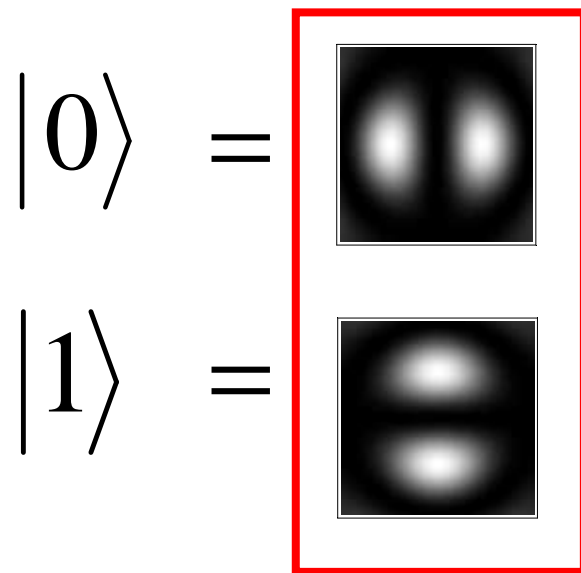
$$|0\rangle = \text{[Image of four Gaussian spots in a square frame, arranged in a cross pattern]}$$

The following is applicable to either of the choices to the left: we focus on first order modes

$$|1\rangle = \text{[Image of four Gaussian spots in a diamond frame, arranged in a cross pattern]}$$

What is it good for?

Applications of SOC: Hermite-Gauss (HG) spatial-mode-entangled cluster states

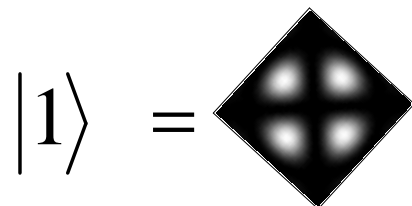


We denote the Bell state

$$|00\rangle + |11\rangle \text{ by } |\bullet\bullet \bullet\bullet\rangle + |\bullet\bullet \bullet\bullet\rangle$$

And the GHZ state

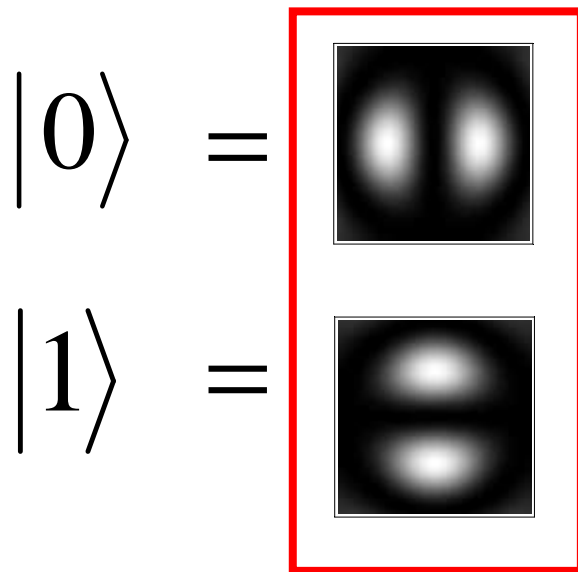
$$|00\rangle + |11\rangle \text{ by } |\bullet\bullet\bullet\bullet\bullet\bullet\rangle + |\bullet\bullet\bullet\bullet\bullet\bullet\rangle$$



The following is applicable to either of the choices to the left: we focus on first order modes

What is it good for?

Applications of SOC: Hermite-Gauss (HG) spatial-mode-entangled cluster states



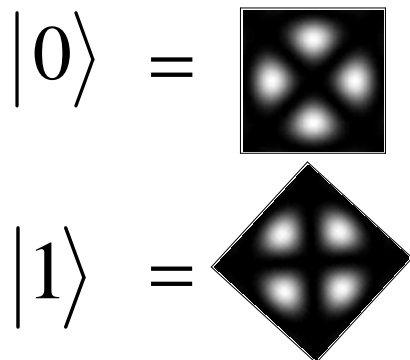
Our states have two key properties:

- 1.) Hadamard gate equivalent to a rotation
- 2.) They have opposing 1-D parities

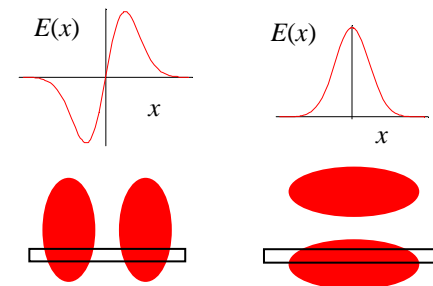
$$\frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{HG10} \\ \text{HG01} \end{array} + \begin{array}{c} \text{HG01} \\ \text{HG10} \end{array} \right) = \begin{array}{c} \text{HG}'10 \\ \text{HG}'01 \end{array}$$

$$\frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{HG10} \\ \text{HG01} \end{array} - \begin{array}{c} \text{HG01} \\ \text{HG10} \end{array} \right) = \begin{array}{c} \text{HG}'10 \\ \text{HG}'01 \end{array}$$

Sums/differences make rotations



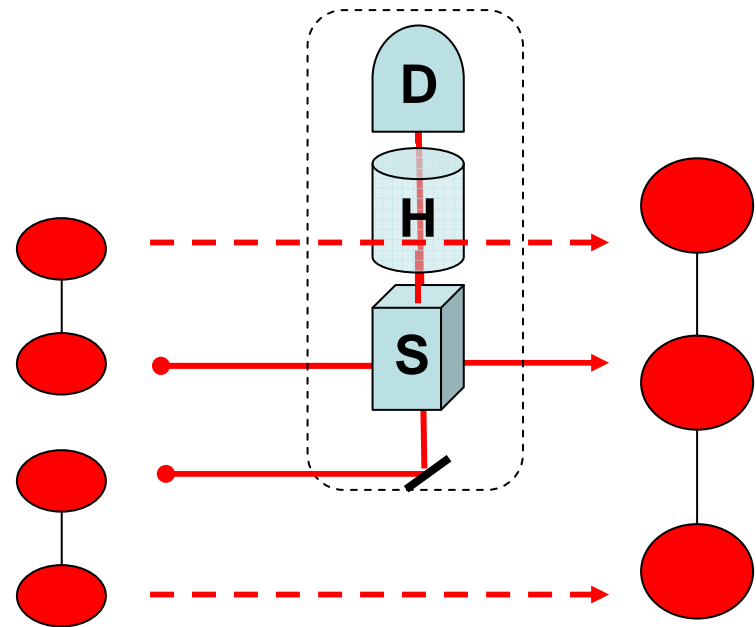
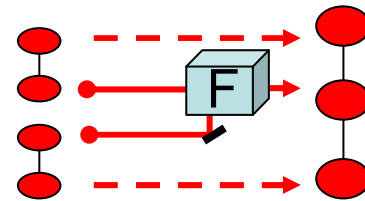
Odd/even upon y axis reflection



What is it good for?

The simplest example: type-I fusion (Brown et.al., 2005)

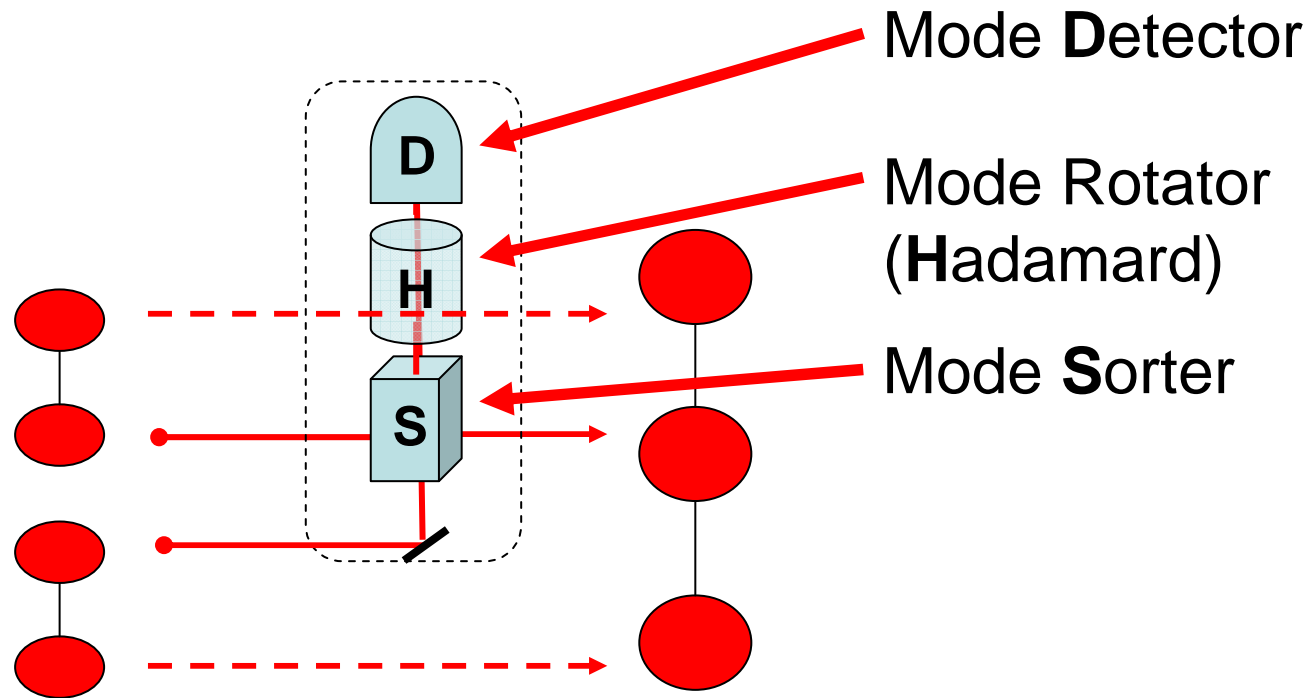
- Initial resource: two Bell states
- Input one photon from each Bell state
- The remaining two photons propagate freely
- One of the input photons is measured, the other exits the gate as output
- The result is an entangled three-qubit cluster state: $|\bullet\bullet\bullet\rangle + |\bullet\bullet\bullet\rangle \rightarrow |\bullet\bullet\bullet\bullet\bullet\rangle \pm |\bullet\bullet\bullet\rangle$



$$|\bullet\bullet\bullet\rangle + |\bullet\bullet\bullet\rangle \rightarrow |\bullet\bullet\bullet\bullet\bullet\rangle \pm |\bullet\bullet\bullet\rangle$$

What is it good for?

Fusion gate elements



What is it good for?

fusion gate elements

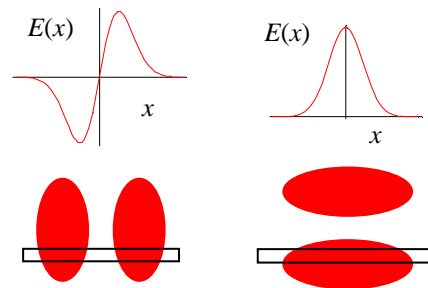
Use transverse spatial modes as qubits

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \text{HG10} & + & \text{HG01} \end{pmatrix} = \text{HG}'10$$

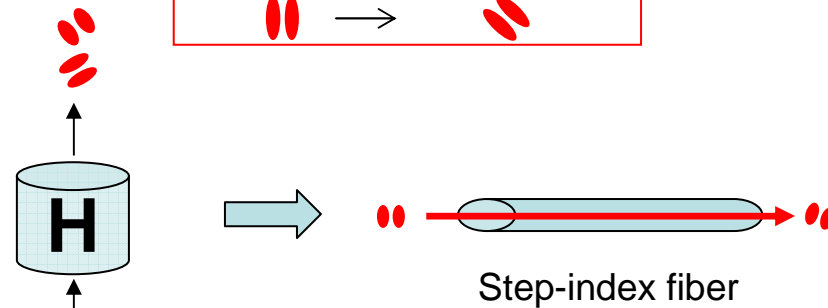
$$\frac{1}{\sqrt{2}} \begin{pmatrix} \text{HG10} & - & \text{HG01} \end{pmatrix} = \text{HG}'01$$

Sums/differences make rotations

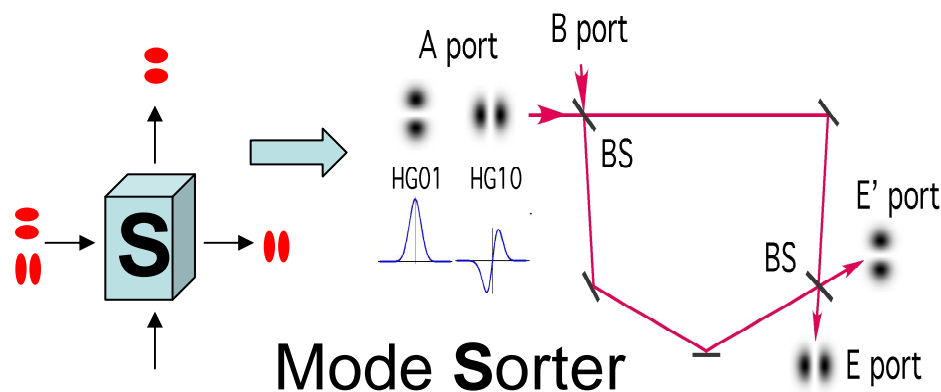
Odd/even upon y axis reflection



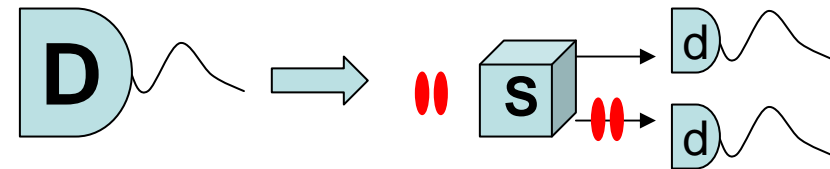
$$|0\rangle \rightarrow |0\rangle + |1\rangle$$



Mode Rotator
(Hadamard)



Mode **S**orter

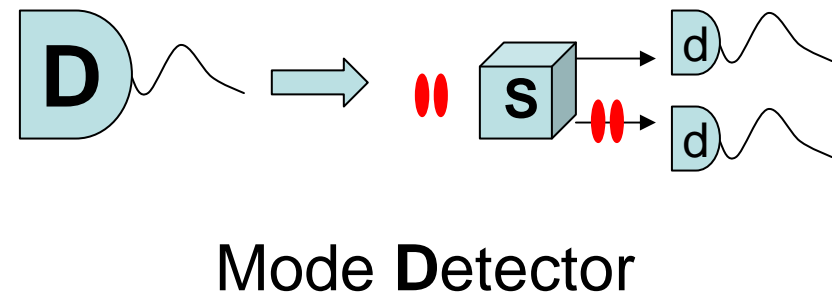
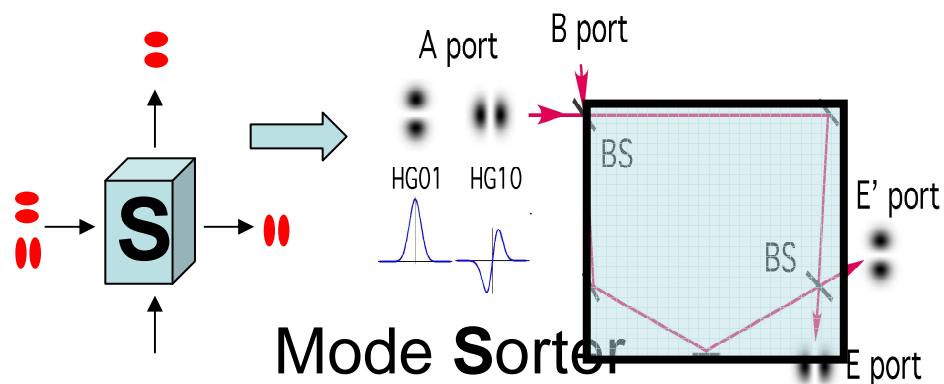
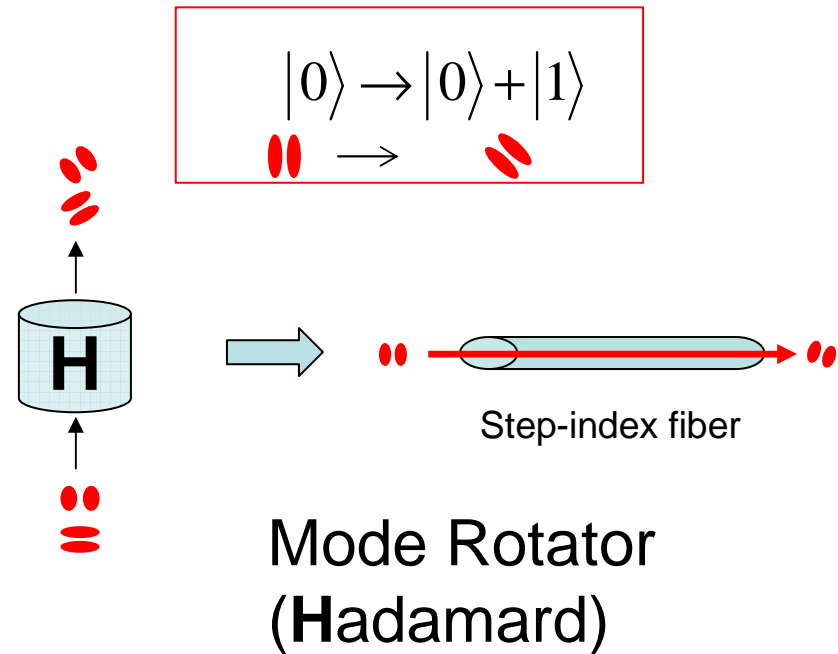


Mode **D**etector

What is it good for?

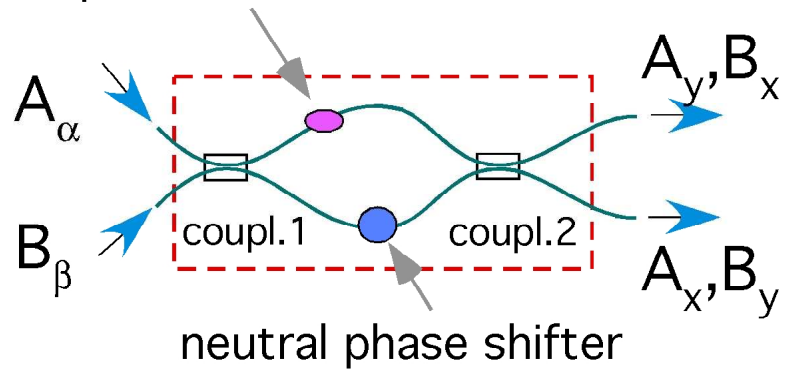
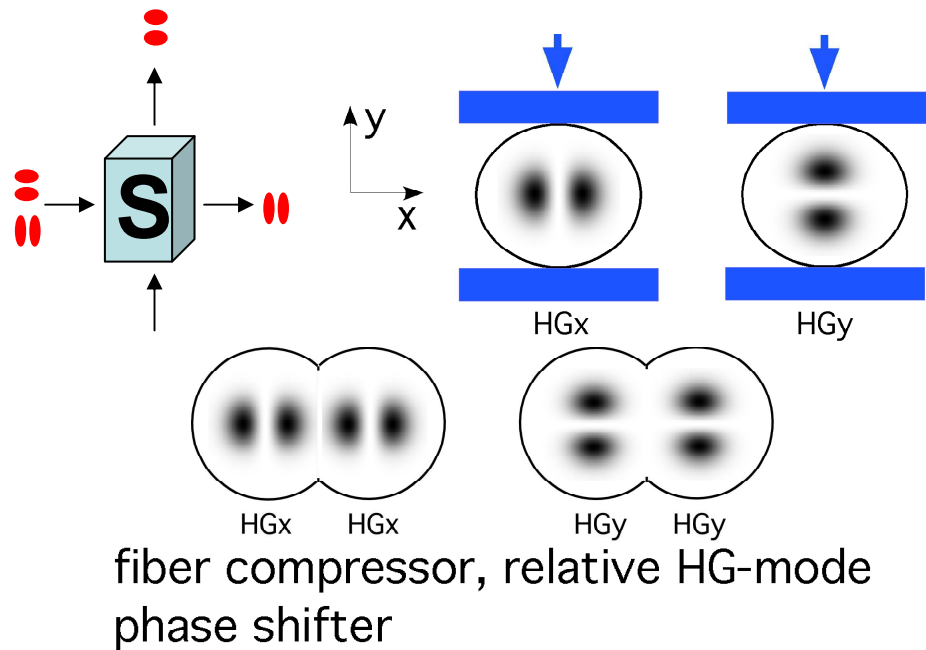
fusion gate elements

- The Mach-Zehnder style interferometer below is not stable, but we can overcome this by constructing a monolithic prism-like device
- The reflections and interference occur at internal interfaces in the “prism”

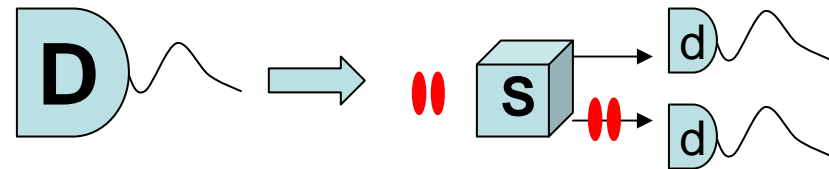
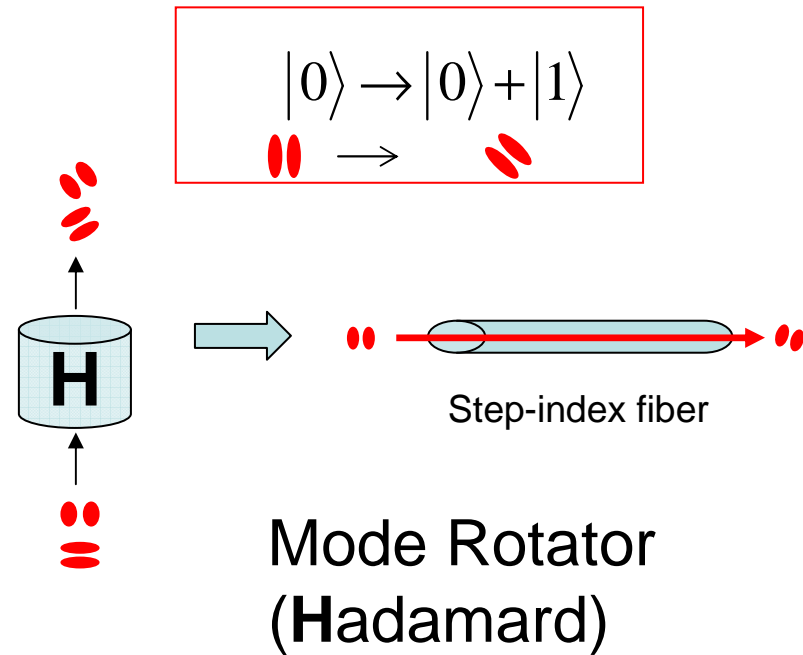


What is it good for?

All fiber-based fusion gate elements



Mode Sorter



Mode Detector

Outlook

Single-photon spin-orbit coupling

- No clear advantages to using photonic spatial modes over the standard polarization encoding for cluster states
- But there are physical differences!
 - Polarization “parity” is **local**, spatial “parity” is **global**
 - Spatial modes must be sorted via interferometry (not a PBS)
- Could this make spatial modes more robust in some situations?
- Interferometry could be in principle provide a less lossy way to sort qubits than polarizing beam splitters
- Fibers restrict the number of degrees of freedom for losses
- All-fiber devices are practically convenient to work with
- Encode **both** the photon’s spin and spatial degrees of freedom?

Conclusions

Single-photon spin-orbit coupling

- **What is it?**

β splitting due to degeneracy lifting in $\sigma \cdot m_\ell$: a larger $|m_j| = |m_\ell + \sigma|$ gives rise to a slower propagation speed

- **How does it work?**

$\sigma \cdot m_\ell$ -split modes have a varying relative phase

- **What is it good for?**

SOC produces a mode rotation effect that can be used to implement a spin-controlled Hadamard gate for multiple HG-entangled spatial modes.

- **How will we observe it?** Experiments are underway...