Measurement and Control of Photon Spatial Wave Functions

Cody Leary
Oberlin College, April 2012
Agenda for today

• Measuring the spatial wave function of a single photon

• Manipulating the wave function of a single photon
  • Imparting a quantum of orbital angular momentum to a photon

• Interaction between wave functions of two “colliding” photons
What is a photon?

- A photon is an oscillating electric field (wave) that propagates through space and time
- A photon makes a photon detector go “click!”
- A photon has 4 degrees of freedom (DOFs):
  1. It oscillates with a certain frequency (energy)
  2. It oscillates in a certain plane (polarization)
  3. Its intensity (& E-field) in the transverse x direction has a certain spatial shape
  4. Its intensity (& E-field) in the transverse y direction has a certain spatial shape

This talk concerns:
- The measurement and control of transverse spatial DOFs of photons
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

- The electric field of one “lobe” is out of phase with the other
- Both modes have either even or odd parity in each dimension:
  - The vertical mode is odd under reflection in x
  - The horizontal mode is even under reflection in x
- Phase structure lies at the heart of photon quantum mechanics
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$

- Transverse spatial modes are analogous to spherical harmonics in a hydrogen atom
- Transverse spatial modes are thus the photon's wavefunction
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$

- An equal superposition of the vertical and horizontal coffee bean modes results in a new mode: a “diagonal coffee bean”.

$$E_{TOT}(x, y) = E_{Vert}(x, y) + E_{Hor}(x, y)$$

What I show What I mean
Transverse spatial modes

1.) Vertical “Coffee Bean”

2.) Horizontal “Coffee Bean”

Intensity = $|E|^2$

- An equal superposition of the vertical and horizontal coffee bean modes results in a new mode: a “diagonal coffee bean”.

- How to measure a photon's mode? (vertical, horizontal, diagonal)
How **not** to measure the spatial mode of a single photon

- A photon counting detector alone cannot measure the transverse spatial mode of a single photon.
- Suppose a “vertical” photon impinges on such a detector:

  ![Diagram](image.png)

  At this point, the photon could be in either mode:

- It would take many identical photons to build up the full spatial intensity pattern.
How to measure the spatial mode of a single photon

- We need a “magic box” that:
  - Accepts an unknown state as input
  - Routes photons to one output
  - Routes photons to a separate output
  - Detector click yields the desired measurement
  - Let's call it a “Sorter”
1-D parity sorting interferometer

- Parity sorter is based on the superposition principle of the phase of an electric field
1-D parity sorting interferometer
1-D parity sorting interferometer
1-D parity sorting interferometer
1-D parity sorting interferometer

\[ \text{Laser} \rightarrow + \rightarrow y \rightarrow x \]

\[ \text{BS1} \]

\[ \text{BS2} \]

\[ + + + = 0 \]

\[ A \]

\[ B \]
1-D parity sorting interferometer

“Odd” modes exit port A
1-D parity sorting interferometer

“Even” modes exit port B
A diagonal coffee bean input mode is split into its corresponding vertical and horizontal "components".
### Experimental results

**Experiment**

<table>
<thead>
<tr>
<th>Input Mode</th>
<th>Input Intensity Profile</th>
<th>Phase Shift ($\phi$)</th>
<th>Port A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HG_{45^\circ}$</td>
<td>![Intensity Profile]</td>
<td>0</td>
<td>![Port A Image]</td>
</tr>
</tbody>
</table>

**Experimental Setup**

![Experimental Diagram]
Experimental results

**Experiment**

<table>
<thead>
<tr>
<th>Input Mode</th>
<th>Input Intensity Profile</th>
<th>Phase Shift ((\phi))</th>
<th>Port A</th>
</tr>
</thead>
<tbody>
<tr>
<td>HG_{45^\circ}</td>
<td>[Image]</td>
<td>0</td>
<td>[Image]</td>
</tr>
<tr>
<td>HG_{45^\circ}</td>
<td>[Image]</td>
<td>(\frac{\pi}{2})</td>
<td>[Image]</td>
</tr>
<tr>
<td>HG_{45^\circ}</td>
<td>[Image]</td>
<td>(\pi)</td>
<td>[Image]</td>
</tr>
<tr>
<td>HG_{45^\circ}</td>
<td>[Image]</td>
<td>(\frac{3\pi}{2})</td>
<td>[Image]</td>
</tr>
</tbody>
</table>

(a)

**Data: Zach Bond**

Experimental Setup
With glass slide inserted

Output mode is controlled by tilting the glass slide
New kid on the block: the “donut” mode

Well-defined orbital angular momentum

- Delaying the phase between the below superposed modes results in evolution of the “diagonal coffee bean” mode to a “donut” mode

\[
\begin{align*}
\cos(\omega t) & \quad + \quad \cos(\omega t) \\
\downarrow & \\
\text{No phase delay} & \\
\end{align*}
\]

\[
\begin{align*}
\cos(\omega t) & \quad + \quad \cos(\omega t) \\
\uparrow & \\
i = e^{i\frac{\pi}{2}} = 90^\circ \text{ phase delay} & \\
\downarrow & \\
\cos(\omega t) & \quad + \quad \sin(\omega t)
\end{align*}
\]
Experimental results

Conclusion: the 1-D parity interferometer imparts one quantum of orbital angular momentum to a single photon!
Bloch Sphere

Arbitrary two-mode superposition represented by sphere

Each point on Bloch sphere surface represents distinct state

\[ E(x, y) = \cos \left( \frac{\theta}{2} \right) + e^{i\phi} \sin \left( \frac{\theta}{2} \right) \]
Parity interferometer “drawback”

Interferometer has TWO exit ports
We can control the mode, but can’t predict exit port for 1 photon
Fiber stress breaks symmetry, imparts relative phase

Data: Margaret Raabe

Solution: optical fiber crushing
Fiber crushing simulation
Orbital angular momentum may be deterministically imparted to a single photon.
Two-photon interference at a beam splitter.

Discovered by Hong, Ou, and Mandel in 1985.
It is commonly thought that photons MUST be indistinguishable.
Two photon “collision”

Use fiber crushing to control input photons

Output:
- **EITHER** two “right handed” photons out A
- **OR** two “left handed” photons out B
- **NEVER** one of each
- **ALWAYS** exit in pairs
Two photon “collision”

Use fiber crushing to control input photons

Output:
- EITHER two “right handed” photons out A
- OR two “left handed” photons out B
- NEVER one of each
- ALWAYS exit in pairs
Two photon "collision"

Use fibers to control input photons

Single Photon input

Output:
- EITHER two "right handed" photons out A
- OR two "left handed" photons out B
- NEVER one of each
- ALWAYS exit in pairs

- First demonstration of distinguishable photons interacting in quantum gate

- "Two photon interference" probabilities cancel out!
Conclusions: Measurement and Control of Photon Wave Functions

- **Measurement:**
  - 1-D parity interferometer “sorts” photons with even and odd x-parity into different ports.

- **Control:**
  - Demonstrated phase control between even and odd modes.
  - Imparted a quantum of orbital angular momentum to photons in two ways
    - interferometer
    - optical fiber.

- “Collisions”:
  - Overlapping photons with nontrivial spatial structure within a 1-D parity interferometer causes their wave functions to interact in surprising ways-- interfering photons can be distinguishable!
Movie: states that exhibit two-photon interference
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must "CHOOSE" whether to be in a "vertical" or "horizontal" mode at the output

- "Global" measurement with "local" detectors

- Only one detector can click

- This device measures parity of single photons
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must "CHOOSE" whether to be in a "vertical" or "horizontal" mode at the output.

- "Global" measurement with "local" detectors!
- This experiment has never been done.

- Only one detector can click.
- This device measures parity of single photons!
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must "CHOOSE" whether to be in a "vertical" or "horizontal" mode at the output.

- "Global" measurement with "local" detectors!
- This experiment has never been done.

- Only one detector can click.
- This device measures parity of single photons!
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must "CHOOSE" whether to be in a "vertical" or "horizontal" mode at the output.
- "Global" measurement with "local" detectors!
- This experiment has never been done.
- Only one detector can click.
- This device measures parity of single photons!
Quantum experiment: single photon

- Put one diagonal photon in the input port, and it must “CHOOSE” whether to be in a “vertical” or “horizontal” mode at the output.

- “Global” measurement with “local” detectors!
- This experiment has never been done.

- Only one detector can click.
- This device measures parity of single photons!
2-D parity sorting interferometer

(a) $y \xrightarrow{R_x} \hat{\Pi}_x \xrightarrow{R_y} \hat{\Pi}_y \xrightarrow{R_x} x$

(b) $y \xrightarrow{R} \hat{\Pi}_{xy} = \hat{R}_{180^\circ} \xrightarrow{R_x} x$
### Experimental results: 2-D parity sorting

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>HG&lt;sub&gt;00&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;00&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;01&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;01&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;10&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;10&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;45°&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;45°&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;11&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;11&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;15&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;15&lt;/sub&gt;</td>
</tr>
<tr>
<td>HG&lt;sub&gt;32&lt;/sub&gt;</td>
<td>HG&lt;sub&gt;32&lt;/sub&gt;</td>
</tr>
<tr>
<td>3 Mode Fiber</td>
<td>3 Mode Fiber</td>
</tr>
</tbody>
</table>

The effects occur analogously for electrons and photons!

- Independent of mass, charge, magnetic moment, etc.

# A brief history of light and matter

<table>
<thead>
<tr>
<th>Electrons (Matter)</th>
<th>Photons (Light)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Mechanics</td>
<td>Classical (Ray) Optics</td>
</tr>
<tr>
<td>- Galileo, Brahe, Kepler, Newton</td>
<td>- Hero, Ptolemy, Sahl, al-Haytham, Kepler, Newton</td>
</tr>
<tr>
<td>Electrons are waves!</td>
<td>Light is a wave!</td>
</tr>
<tr>
<td>Quantum (Wave) Mechanics</td>
<td>Wave Optics</td>
</tr>
<tr>
<td>-- Bohr, De Broglie, Heisenberg, Schrodinger</td>
<td>-- Hooke, Huygens, Young, Fresnel</td>
</tr>
<tr>
<td>Relativity!</td>
<td>Relativity!</td>
</tr>
<tr>
<td>Relativistic Quantum (Wave) Mechanics</td>
<td>Relativistic Wave Optics</td>
</tr>
<tr>
<td>- Dirac</td>
<td>- Maxwell, Heaviside, Gibbs, Hertz</td>
</tr>
<tr>
<td>Particle creation!</td>
<td>Particle creation!</td>
</tr>
<tr>
<td>Quantum Field Theory</td>
<td>Quantum Optics</td>
</tr>
<tr>
<td>- Feynman, Tomonaga, Schwinger, Dyson</td>
<td>- Dirac</td>
</tr>
</tbody>
</table>

Quantum Electrodynamics
# A brief history of light and matter

<table>
<thead>
<tr>
<th>Electrons (Matter)</th>
<th>Photons (Light)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Mechanics</td>
<td>Classical (Ray) Optics</td>
</tr>
<tr>
<td>- Galileo, Brahe, Kepler, Newton</td>
<td>- Hero, Ptolemy, Sahl, al-Haytham, Kepler, Newton</td>
</tr>
<tr>
<td>Electrons are waves!</td>
<td>Light is a wave!</td>
</tr>
<tr>
<td>Quantum (Wave) Mechanics</td>
<td>Wave Optics</td>
</tr>
<tr>
<td>-- Bohr, De Broglie, Heisenberg, Schrodinger</td>
<td>- Hooke, Huygens, Young, Fresnel</td>
</tr>
<tr>
<td>Relativity!</td>
<td>Relativity!</td>
</tr>
<tr>
<td>Relativistic Quantum (Wave) Mechanics</td>
<td>Relativistic Wave Optics</td>
</tr>
<tr>
<td>- Dirac</td>
<td>- Maxwell, Heaviside, Gibbs, Hertz</td>
</tr>
<tr>
<td>Particle creation!</td>
<td>Particle creation!</td>
</tr>
<tr>
<td>Quantum Field Theory</td>
<td>Quantum Optics</td>
</tr>
<tr>
<td>- Feynman, Tomonaga, Schwinger, Dyson</td>
<td>- Dirac</td>
</tr>
</tbody>
</table>

- **MY RESEARCH**
# A brief history of light and matter

<table>
<thead>
<tr>
<th>Electrons (Matter)</th>
<th>Photons (Light)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Mechanics</td>
<td>Classical (Ray) Optics</td>
</tr>
<tr>
<td>-Galilei, Brahe, Kepler, Newton</td>
<td>-Hero, Ptolemy, Sahl, al-Haytham, Kepler, Newton</td>
</tr>
<tr>
<td></td>
<td>Electrons are waves!</td>
</tr>
<tr>
<td></td>
<td>Light is a wave!</td>
</tr>
<tr>
<td>Quantum (Wave) Mechanics</td>
<td>Wave Optics</td>
</tr>
<tr>
<td>-- Bohr, De Broglie, Heisenberg, Schrodinger</td>
<td>-Hooke, Huygens, Young, Fresnel</td>
</tr>
<tr>
<td></td>
<td>Relativity!</td>
</tr>
<tr>
<td>Relativistic Quantum (Wave)</td>
<td>Relativistic Wave Optics</td>
</tr>
<tr>
<td>Mechanics</td>
<td>-Maxwell, Heaviside, Gibbs, Hertz</td>
</tr>
<tr>
<td>-Dirac</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Relativity!</td>
</tr>
<tr>
<td></td>
<td>Particle creation!</td>
</tr>
<tr>
<td>Quantum Field Theory</td>
<td>Quantum Optics</td>
</tr>
<tr>
<td>-Feynman, Tomonaga, Schwinger, Dyson</td>
<td>-Dirac</td>
</tr>
</tbody>
</table>