

# Determining Surface Tension by Light Scattering

David Walkenhorst

*Physics Department, The College of Wooster, Wooster, Ohio 44691*

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This experiment examines the relationship of the wavevector to the frequency of oscillation when looking at surface tension waves. A value is found for the surface tension of water by using light scattering techniques. Theory states that the wavevector,  $q$ , goes as the two-thirds power of the frequency of oscillation. As the frequency is changed, the distance between bright spots of the diffraction pattern also changes. By using the known frequencies and the separation distances for the diffraction pattern a value can be found for the surface tension of water. Experimentally, the surface tension of water as determined by the light scattering method is  $71 \pm 31 \text{ mN/m}$ , compared to the surface tension found in the conventional way with a value of  $77 \pm 1.0 \text{ mN/m}$ .

## INTRODUCTION

In this experiment, light scattering is used as a way of measuring the surface tension of water. Waves at the surface of water are a simple illustration of the concept of waves, and wave dynamics can be dominated by either gravity or by surface tension. The wavelength of the surface tension waves can be measured by light scattering. The water-air interface appears as a diffraction grating for the laser beam. The diffraction pattern on the wall is centered around the reflected beam. Because the waves produced by the steel needle are cylindrical, the diffraction pattern is more complicated than it would be if plane waves were being produced. If that were the case, one would observe only two spots that were symmetrical with respect to the reflected spot.<sup>1</sup>

The ordinary static method of measuring surface tension involves the measurement of the force to be exerted on a piece of metal as it is pulled from a liquid. A shortcoming of this static method is that it cannot be used to continuously measure the surface tension. It is also difficult to use in some environments, for example, close to the critical point. This method was used in this experiment as a supplement to the light scattering method, so that there could be some comparison between the two methods.<sup>1</sup>

## THEORY

This experiment looks at the relationship between the frequency,  $\omega$ , and the wavevector,  $q$ , when water is made to oscillate with surface tension waves. Using a theory developed by Klemens<sup>2</sup>, a dispersion relation is found based on the Lagrange equation. The potential energy given from the surface deformation arises from gravity and surface tension. The gravity and surface tension pieces are added and the result is integrated over a finite length. The kinetic energy is found by integrating over the entire velocity field. The flow is assumed to be irrotational, so that the velocity  $v$  can be derived from a velocity potential  $\phi$ . The velocity potential  $\phi$  and the wave displacement  $\eta$  are related at the surface, since both are a measure of the vertical component of the velocity at the surface.<sup>2</sup> The total kinetic energy describes a harmonic oscillator from which the simplified dispersion relation<sup>1</sup> for surface tension waves is

$$\omega^2 = \frac{A}{\rho} q^3 \quad (1)$$

The wavevector,  $q$ , is given by the equation<sup>1</sup>:

$$q = \frac{2\pi}{\lambda} \sin \frac{r}{2} \sin \theta - \frac{r}{2} + \sin \theta + \frac{r}{2} \quad (2)$$

Because  $\sin \theta \approx \theta$  at small angles, this can be simplified to:

$$q = \frac{4}{\lambda} \sin \frac{d}{2l} \quad (3).$$

Note that  $r = \frac{d}{l}$ .

## SETUP/PROCEDURE

The main part of the experiment resulted in a log-log plot of wavevector versus frequency, and the slope and y-intercept of this curve were found. The frequency is adjusted on the Pasco PI-9587A digital function generator, and the wavevector is calculated using equation (2). The wavevector is dependent on the angle that the laser beam is scattered. This angle can be found if the distance between the bright spots is known. To measure the distance between bright spots, pieces of paper were taped to the wall, and pencil marks were used to indicate the bright spots. The mean of the distances was calculated along with the standard deviation. The logarithm of the wavevector is then plotted against the logarithm of the frequency. The slope of the curve should verify the power law predicted by the dispersion relation. The intercept of the curve gives the surface tension.

The light scattering value for the surface tension is verified using the conventional method for measuring surface tension. The surface tension is found by dividing the force exerted by the water by the length of the paper clip that is in the water. The paper clip is immersed in the distilled water. A Mettler digital balance measures the force that the water exerts on the paper clip as the beaker of water is lowered away from the paper clip. The length of the paper clip that touches the water is measured.

## DATA/ANALYSIS

The values were recorded from the function generator, and the distance between the bright spots from the paper on the wall, the mean and standard deviation were calculated for the distance. The angle of incidence was calculated by measuring the distances  $h$  and  $l$  as shown in Figure 1. Solving for  $\theta$  the angle of incidence and assuming that  $\tan \theta \approx \theta$  for small angles,

$$\theta = \frac{h}{l} = \frac{.47\text{m}}{3.25\text{m}} = 0.15 \pm 0.02 \text{ radians} \quad (14).$$

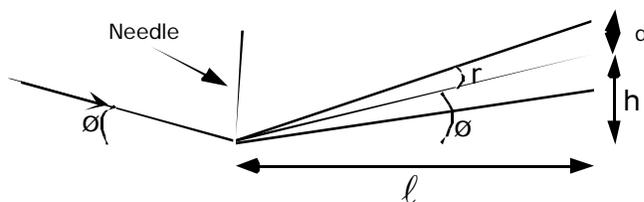


FIG 1. This figure illustrates the angle of incidence,  $\theta$ , and the distances  $h$  and  $l$  which are used to find  $\theta$ . The angle  $r$  and the distance  $d$  are related to the wavevector  $q$ .

The uncertainty in this measurement comes from the way that the distances  $h$  and  $l$  were measured. A tape measure was used and the uncertainty in  $h$  and  $l$  is 0.01m for each, giving an uncertainty of 0.02 radians. To find the angle  $r$ ,  $r = \frac{h}{l} = \frac{d}{l}$ .

The uncertainty in the distance  $d$  is taken to be 2mm because of the difficulties in measuring the bright spots. The uncertainty in the distance  $l$  is 1 cm because of the large distance that was measured using a tape measure. The wavelength of the laser is  $632.8 \pm 0.1$  nm. The uncertainty is given in Table 2. The error in frequency of the function generator is taken to be 0.5 Hz.

**Table 1.** This table shows the distance in cm, in Hz, wavevector  $q$  and error  $dq$  in  $\text{m}^{-1}$ ,  $r$  and  $dr$  in radians.

Point	d	w	'_q'	dq	'_r'	dr
0	0.53	1.01e+03	2.34e+03	1.02e+03	0.0016	6.15e-06
1	0.65	1.3e+03	2.87e+03	1.25e+03	0.0020	6.15e-06
2	0.65	1.33e+03	2.87e+03	1.25e+03	0.0020	6.15e-06
3	0.73	1.51e+03	3.22e+03	1.41e+03	0.0022	6.15e-06
4	0.88	2.02e+03	3.89e+03	1.7e+03	0.0027	6.15e-06
5	1.05	2.49e+03	4.64e+03	2.02e+03	0.0032	6.15e-06
6	1.00	2.56e+03	4.42e+03	1.93e+03	0.0031	6.15e-06
7	1.10	2.89e+03	4.86e+03	2.12e+03	0.0034	6.15e-06
8	1.22	3.23e+03	5.39e+03	2.35e+03	0.0038	6.15e-06

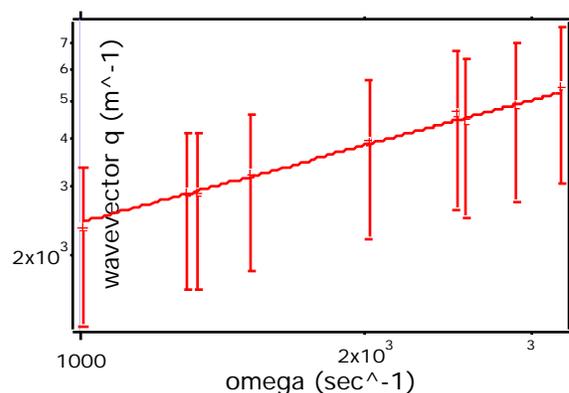


FIG 2. A log-log plot of the wavevector  $q$  vs. the frequency  $\omega$  has a line of slope of  $0.69 \pm 0.02$ .

The fitting function used by Igor was of the form:

$$fit = K_1 x^{K_2} \quad (16),$$

where  $K_2$  is the slope of a straight line on a log-log plot. This shows that  $q$  is proportional to  $\omega^{2/3}$ . The slope in Figure 2 is what is consistent with the dispersion relation power of 2/3. By holding the power constant, a more accurate value can be found for the surface tension, which is the  $y$  intercept in this case. The fit was done in Igor and was of the form given in equation 16. With  $K_2$  held constant at 0.66, Igor found  $K_1$  to be  $24.2 \pm 0.2 \text{ m}^{-1}(\text{sec}/\text{rad})^{.66}$ .

The simplified dispersion relation is used to solve for surface tension:

$$A = \frac{F}{K_1^3} = 71 \frac{mN}{m} \quad (17).$$

The surface tension was found directly by measuring the force that was exerted on a piece of metal. The length  $\ell$  of the paper clip that touched the water was  $\ell = 3.5 \pm 0.4 \text{ cm}$ . This error came from the curvature of the paper clip. The mass of the string and paper clip together was  $0.555 \pm 5 \times 10^{-4} \text{ g}$ . The force that was recorded for the surface tension corresponded to a mass of  $0.273 \pm 5 \times 10^{-4} \text{ g}$ . The uncertainty in this value comes from the precision of the digital balance. The surface tension

$$A = \frac{F}{l} = \frac{.0027 N}{.035 m} = (77 \pm 1.0) \frac{mN}{m} \quad (18).$$

The surface tension as measured by the conventional method is  $77 \pm 1.0 \text{ mN/m}$ . The accepted value<sup>5</sup> is  $72 \text{ mN/m}$ .

## CONCLUSION

The value of the power that relates the wavevector  $q$  to the frequency  $\omega$  is experimentally determined to be  $0.69 \pm 0.02$ . This value is approximately 3% higher than the theoretically determined value of 0.666. However, there are other variables that

need to be mentioned. One possible reason for the slightly different value is that the positions of the bright spots were not clear enough to make a solid measurement. The frequency could be controlled with a good degree of precision. The fit was done a second time while the slope of the fit was held constant at 0.666. This gave a much better value for the  $y$ -intercept, which is used to find the value for the surface tension.

In the second part of the experiment, the source of uncertainty was from the equipment that was being used (namely the digital balance) and from estimating the length of water over which the paper clip adhered to the water.

This experiment demonstrates a new method for finding the surface tension, provided that great care is taken when making measurements and calculations. There is a large opportunity for error when measuring the separation between the bright spots on the diffraction pattern. A two-thirds power law became evident when the powerlaw fit function was used in Igor. If this two-thirds power law is assumed, the value for the surface tension agrees very well with the accepted value.

There are many different ways that this lab could be extended. A particularly interesting way could be to put some kind of surfactant on the surface of the water. Then the effects of the surfactant on the wave's motion could be studied. This experiment could be repeated with different temperatures of water, mostly warmer than room temperature.

<sup>1</sup>Garbay, F., Weisbuch, G. "Light scattering by surface tension waves" *American Journal of Physics* **47**, 355-356 (1979).

<sup>2</sup>Klemens, P.G. "Dispersion relations for waves on liquid surfaces" *American Journal of Physics* **52**, 451-452 (1984).

<sup>3</sup>Taylor, John R. *An Introduction to Error Analysis* (University Science Books, Mill Valley, California, 1982). pp. 119-120.

