Examination of Viscous Torque Relationships

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Viscous torque, the resistance that a fluid offers to rotational motion, was studied with air on a polished metallic sphere. This experiment examines whether the viscous torque, τ, is proportional to the angular velocity, ω, to the first power, or to some higher power, by observing the rate of decay in angular velocity of a rotating sphere, subject only to viscous torque. Plots of this decay rate on a semi-log scale produce linear results which show that $\tau = -k\omega^1$. Additional data taken with flags to increase torque suggests that for greater area, the viscous torque is actually proportional to the square of the angular velocity. This experiment also examines the relationship between the viscous coefficient, *k*, and the area, *A*, exposed to viscous torque. Data appear to suggest a linear relationship between *k* and *A*, but results are insufficient to be conclusive.

INTRODUCTION

The resistance that a fluid offers to movement is the fluid's viscosity. Viscosity measures "the internal friction that arises when there are velocity gradients within the system."[1] A common way to measure viscosity is to observe it when the viscous force is equal and opposite to the gravitational force on a falling sphere, as in a falling sphere viscometer.[1] This method relates viscosity to gravitation and demonstrates an exponential decay in velocity when the viscous force is proportional to the velocity. This proportionality is related to Stoke's law of the viscous force, *f*, on a sphere, which is $f = 6\pi\eta vr$, where η is the viscosity of the fluid, *r* is the radius of the sphere, and *v* is the constant velocity with which it is falling.[1] However, experimentation and observation can be made easier if the linear motion is translated into rotational motion. This can be done using a gyroscope consisting of a sphere rotating in air. After being accelerated and then allowed to coast, the relationship between angular velocity and viscous torque can be easily observed.

If the viscous force is proportional to the angular velocity to the first power, then the angular velocity will decay exponentially in time. The viscous coefficient, *k*, is said to relate viscous torque to angular velocity in the following manner, $\tau = -k\omega$. A new relationship can be studied by adding areas of resistance, or rigid flags, over which the viscous force is acting, to increase drag. The surface area added and the viscous coefficient can be observed and plotted against one another in an attempt to identify the relationship between them.

THEORY

The viscous force is proportional to the object's velocity, *v*, giving the relationship $F = -kv$, where *k* is the viscous coefficient. Relating this equation to rotational motion results in

$$\tau = -k\omega \qquad (1),$$

where τ is the torque, ω is the angular velocity. Equating this equation to Newton's 2nd law in terms of rotational motion ($\tau = I\alpha$, where *I* is the moment of inertia, and α is the angular acceleration) and then solving the differential equation for the angular velocity yields

$$\omega = \omega_o e^{-\lambda t} \qquad (2).$$

where $\lambda = k / I$, which clearly shows how the angular velocity decays exponentially under viscous torque, at a rate proportional to the viscous coefficient *k*. Hence, $-\lambda$ will be the slope of a semi-log plot of angular velocity as it decreases over time when subjected to a frictional viscous torque.

Included in the viscous coefficient, *k*, are dependencies on the radius, *r*, of the sphere, and the viscosity, η, of the fluid, as presented in the following equation[1]

$$k = 6\pi\eta r \qquad (3).$$

This equation shows that the viscous coefficient, and hence λ as well, is proportional to the viscosity of the fluid, and is also linearly related to the radius of the sphere for linear motion.

However, the above theory is assuming a linear relationship between the viscous force and the velocity. If, for the case of rotational motion, the viscous torque is proportional to any power of the velocity higher than the first power, the velocity's decay would not be exponential. As a representative example of all powers higher than 1, the square of angular velocity is examined as proportional to the viscous torque, known as the Newtonian proportionality, as shown in the following equation,

$$I\frac{d\omega}{dt} = -k\omega^2 \qquad (4).$$

Following the same procedure of solving this equation as was done to equation 11 yields

$$-\frac{1}{\omega} + \frac{1}{\omega_o} = -\frac{k}{I}t \qquad (5),$$

which when plotted as 1/ω versus t would give a straight line.

If the flags dominate in the second part of the experiment, the angular velocity is expected to decay at a rate greater than the linear Stoke's relationship depicted in equation 2. With the greater area exposed to the viscous torque, the angular velocity is expected to decay at a rate inversely proportional to time ($t \propto 1/\omega$) as depicted by the Newtonian relationship in equation 5. Hence the linear relationship is expected for a plot of *1 / ω* versus *t*.

This experiment seeks to determine which theory holds for the relationship between viscous torque and angular velocity for the sphere with no rigid flags, that is to what power of the angular velocity is the viscous torque proportional. Also, for cases with increased area due to added flags, this experiment seeks to determine if the relationship between viscous torque and angular velocity changes due to the flags, and if so, what is the new relationship. Also, the relationship between the viscous torque and the area of the flags is investigated.

PROCEDURE

A 10 cm, polished, solid metal sphere with a long pole attached to it was used along with a base mount built to hold the rotating sphere. The base mount is suited with an input valve through which compressed gas can be pumped and releases under the sphere to decrease the friction in rotation.

The angular velocity is determined by using a He-Ne laser reflecting off the shiny northern hemisphere where black stripes of electrical tape periodically interrupt the reflection, and from there reflect through a focusing convex lens onto a phototransistor. The Ne pressure that floats the sphere is set to a desired level of 10 psi so that the sphere is floating and is able to rotate with only viscous friction. The filter on the HP5385A frequency counter is turned on to ensure accuracy without external noise contaminating the data, and the gate time on the frequency counter is set at 1.0 second to help ensure the frequency counter is reading properly.

The LabView 3.1 program Frictional Torque is launched and the gate timer on the program is set to record at 10 second intervals. After spinning the pole protruding from the sphere's axis of rotation by hand up to a maximum frequency of approximately 7 Hz, read as 28 Hz on the frequency counter, and stabilizing the pole to eliminate any wobble, the Frictional Torque program was run to record data. The program was allowed to run for approximately 3000 seconds and stopped before any significant wobble could be observed that might skew the data. The data was saved as a file and opened in Igor for analysis.

To investigate the relationship between the viscous coefficient and area, styrofoam flags of increasing area are attached to the axis pole of the sphere. The goal is to obtain a plot of the viscous coefficient verses the area to observe how the viscous coefficient changes with respect to increasing area. After attaching each flag, the process above was repeated and data was taken, always making sure to stop the run before any significant wobble appeared which could skew data. Due to the inconvenience of working around the flags while spinning the pole, much lower maximum frequencies were reached. The size of the flags used were $(41.2\pm1.4)\text{cm}^2$, $(81.8\pm1.5)\text{cm}^2$, and $(121.4\pm2.3)\text{cm}^2$.

DATA and ANALYSIS

To determine the relationship between the viscous torque and the angular velocity data runs were taken with no flags put on,. The data was plotted and analyzed using Igor Pro. A sample of the first data run, as recorded by the computer program, can be seen in Table 1.

Table 1. Sample of data from Run 1 (no flags), with time in units of seconds, frequency in units of Hz, and the error for every time measurement being ±0.01%, and the error for every frequency measurement being 1 part over (40*the frequency value).

time_s	frequency
47.94	6.03765
58.42	6.00346
68.94	5.9634
79.364	5.9239
89.833	5.88655
100.273	5.84698

The graph of frequency versus time of the Run 1's data is shown in Figure 1.

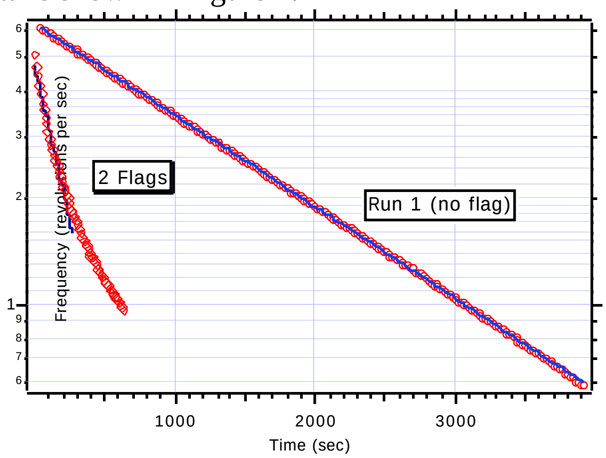

Figure 1. Graph of Frequency versus Time for Run 1 on a semi-log axis, with exponential line fit. The exponential line fit fits the data so well that it runs right through the middle of most of the data points and is hard to see. Over-laid on this graph is a graph of Frequency versus Time for the run with two flags, Flag 2, on a semi-log axis, with exponential line fit for initial decay down to an agular frequency ~1.7 s^{-1}. The exponential line fits the initial decay data well, yet the data shows some systematic deviation from the line fit at the later time and lower frequency.

When plotted on a semi-log axis, the data for the run without flags showed extreme linearity. This means that the data is well explained by an exponential with constant decay rate. An exponential line fit was performed to test the linearity, and to find a value for the slope of the line which, as was shown in Figure 1, is the viscous coefficient, λ. Igor drew the fit line according to the following equation

$$\omega = k_1 + k_2 e^{-k_3 t} \qquad (6),$$

which, if the variable k_1 is forced to be zero on Igor, fits the form from equation 2, with ω_o corresponding to k_2, and, as previously stated, k_3 being the slope of the line corresponding to λ, the viscous coefficient. The values for run 1 were $k_2 = 6.214 \pm 0.001$, $k_3 = (5.882 \pm 0.004)*10^{-4}$.

Data was taken with each size of flag attached to the pole. The graph of the 2 Flag frequency versus time can be found over-laid on the graph of Run 1 in Figure 1. This portion of the experiment only investigates the relationship between area and the viscous coefficient, so for each data run with a different flag area, λ was determined by finding k_3 in the same manner as it was found for the earlier runs. However, the exponential fit was performed, with k_0 forced to zero, only on the initial decay, due to the data not being very linear when looked at as a whole. It is believed that perhaps the initial decay for the different runs will provide a consistent approach.

The line fit for Flag 2 produced a value of $k_3 = \lambda = (4.0 \pm 0.1)*10^{-3}\ s^{-1}$, for the initial decay down to an angular frequency ~1.70 s^{-1}. It is noted that the data line is not at all straight, and also that there is a small amount systematic deviation of the data from the exponential line fit at later times and lower frequencies. The data is possibly starting to show behavior consistent with theories other than that of angular velocity to the first power.

As a further test of other theoretical relationships, the data for 2 Flag was taken and a plot was made of 1 / ω versus Time. A linear line fit was also then performed on the graph to observe linearity predicted by equation 5.

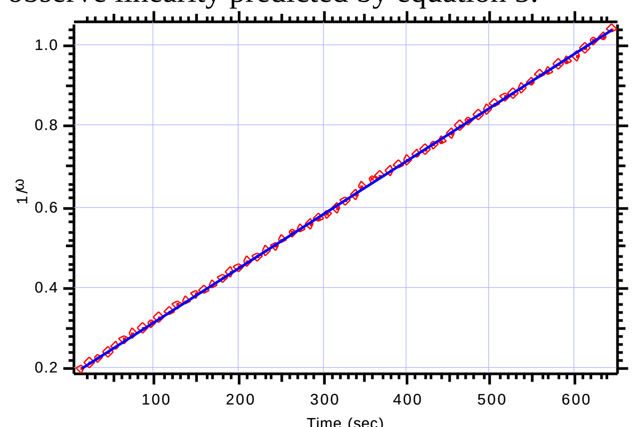

Figure 2. Graph of 1 / ω versus Time for the run with two flags , 2 Flag , with a linear line fit. The data is well explained by the linear fit, which implies the Newtonian form of the viscous torque is applicable.

The data produced a graph that illustrates very linear behavior between 1 / ω and time. This supports the Newtonian theory depicted in equations 4 and 5, where the viscous torque is proportional to the square of the angular velocity. The linear fit produced a slope of $(1.320\pm0.002)*10^{-3}\ s^{-2}$.

CONCLUSIONS AND DISCUSSION

One can observe, record, and infer the relationship between frictional viscous torque on and angular velocity of a smooth, rotating sphere from a plot of angular velocity, ω, versus time, *t*, and find it to be consistent with the theory that the frictional viscous torque is proportional to the angular velocity to the first power. One can also observe, record, and plot the relationship between the viscous coefficient , λ , and the area of flags attached and find the relationship to be linear as suspected from gathered data, although not enough data was gathered in this experiment to make a conclusive statement on the relationship.

The frictional viscous torque was found to be proportional to the angular velocity to the first power for the sphere rotating with no flags. The viscous coefficient for angular velocity with no flags was found to be $(6.0\pm0.2)*10^{-4}\ s^{-1}$. The viscous coefficients for the 1 Flag of area $(41.2\pm1.4)\ cm^2$, the 2 Flag of area $(81.8\pm1.5)\ cm^2$, and the 3 Flag of area $(121.4\pm2.3)\ cm^2$ were found to be $\lambda = (1.77\pm0.02)*10^{-3}\ s^{-1}$, $(4.0\pm0.1)10^{-3}\ s^{-1}$, and $(4.5\pm0.2)*10^{-3}\ s^{-1}$, respectively. The graphs of these data did not appear very linear on a semi-log plot for the data as a whole. Yet when only the initial decay was examined with a linear fit for each of the 3 flag runs, that region could be described by a line corresponding to an exponential decay of the angular velocity, but the data is much more consistent with a viscous torque model that varies as the square of ω.

For further examination, the data for 2 Flag was taken and a plot was made of 1 / ω versus Time to test the Newtonian theory that holds viscous torque to be proportional to the square of the angular velocity. A linear line fit was also then performed on the graph, producing a slope of $(1.320\pm0.002)*10^{-3}$. The data fit the line fit extremely well, indicating that with additional area added to increase drag, the viscous torque is in fact proportional to the square of the angular velocity, and hence the connection to area is an open question.

[1]McGraw-Hill Encyclopedia of Science & Technology. 7th ed. Vol. 19. (McGraw-Hill, New York, 1987). pp. 247-248.

[2]Reynolds, James and Stanton Hillier. "A Demonstration of the Proportional Relationship Between Frictional Torque and Angular Velocity," The Ealing Review. Nov./Dec., 1977. pp. 2-3.