

A Study of the Phenomenon of Spontaneous Parametric Down-Conversion

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The phenomenon of type-I spontaneous parametric down-conversion (SPDC) was studied and an experimental setup was designed in an attempt to use SPDC to create entangled photons to be detected by two single photon detectors, one for each entangled photon beam. The average dark count rates of the two single photon detectors were experimentally measured to be 257 counts/s and 189 counts/s having a percent difference of 4 % and 18 % from the dark count rates determined by the manufacturers of the detectors. The accidental coincidence rate between the two detectors was 1.6 counts/s when illuminated with a 120 V non-dimming green light. The coincidence rate between the two detectors when entangled photons were produced was experimentally measured to be zero, indicating that the down-converted light was not registered by the detectors. In addition, it was computationally found for our experimental setup, that angles of 27° , 28.9° and 30° between the direction of propagation of the incident pump beam and the optical axis of the non-linear crystal used in SPDC, result in 5° , 3° and 0° half-angle splits of the down-converted photons respectively.

I. INTRODUCTION

Quantum optics is a field of research that uses semi-classical and quantum mechanical physics to investigate phenomena that involve light and its interaction with matter at submicroscopic levels. Quantum theory considers light to be an electro-magnetic wave as well as a stream of particles, that travel at the speed of light, called photons [1]. One of the many results of research in the field of quantum optics is the demonstration of quantum entanglement.

Just like human twins who evoke a sense of mystery and amazement by reporting empathetic experiences across great distances, quantum entangled photons that are born in pairs have an analogous quantum correlated behavior. Photons created two at a time with entangled quantum states highlight many of the most fundamental and unsettling aspects of quantum mechanics, such as non-locality [2]. Two particle entanglement also has several applications in metrology because two photon states

do not require a previously calibrated standard in order to perform absolute optical measurements [2].

An example of a quantum phenomenon that produces entangled photons is spontaneous parametric down-conversion (SPDC). SPDC was described as early as 1970 by D. C. Burnham and D.L. Weinberg [3]. Experiments using this technique were first prevalent in 1980 and with an advancement in equipment and technology several experiments have been conducted using this phenomenon.

SPDC is a phenomenon where a non-linear and birefringent crystal is used to split photons into pairs of photons that in accordance with the laws of conservation of energy and momentum have combined energies and momenta equal to that of the original photon [3]. The twin photons that are produced by the crystal have correlated polarizations. In this phenomenon the light that is incident on the crystal has a shorter wavelength and a higher frequency than each of the twin photons that are produced by the crystal.

There are two types of spontaneous parametric down-conversion, type-I and type-II [4]. In type-I SPDC the twin photons produced by the crystal have the same polarization as the photons incident on the crystal. In type-II SPDC the twin photons produced do not have correlated polarizations. Figure 1 illustrates the phenomenon of type-I spontaneous parametric down-conversion.

In SPDC, the entangled photons produced by the crystal forms a cone and when the trajectory of the photons are mapped in space over time, a photon ring is formed as seen in Fig. 1. Previously, the photon ring has been mapped using CCD cameras. More recently, the photon ring was mapped by a two-dimensional scan in the space where the ring was formed using a single photon counting module.

Down-conversion has several applications in physics. The entangled photons can be used to determine the absolute quantum efficiency of a photon detector without the use of a previously calibrated standard. In addition, the entangled photons can be used for quantum imaging purposes where the photons make a picture even though

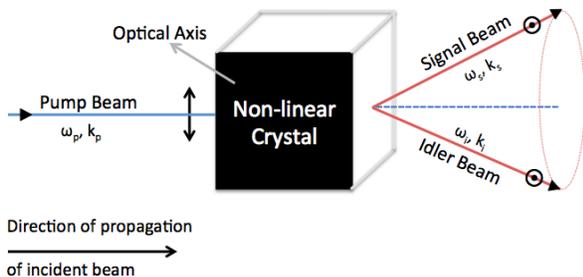


FIG. 1: A diagram illustrating the phenomenon of type-I spontaneous parametric down-conversion where a non-linear crystal is used to split an incoming pump beam into two quantum entangled photon beams namely the signal and the idler. Both the angular frequency ω , hence the energy, and the wavenumber k , hence the momentum, are conserved in this system.

they do not directly interact with the object being photographed [5, 6]. Quantum entanglement also has applications in both quantum cryptography and quantum computing.

In the past, researchers designed the electronics behind the experimental apparatus used in our setup in order to connect the single photon counting modules that are used to detect the down-converted photons to a LabVIEW-computer interface in order to read the signals received by the single photon detectors.

In this experiment an experimental setup was designed in an attempt to use the entangled photons produced by type-I SPDC to develop a technique for the absolute calibration of a single photon detector and determine its quantum efficiency by studying the coincidence between the entangled photons. In addition, characteristics such as the dark count rate of the single photon detectors, as well as accidental coincidences between the entangled photons are examined. The different types of SPDC for different angles between the optical axis of the non-linear crystal and the direction of propagation of the incident pump beam was also studied.

II. THEORY

In SPDC, as illustrated in Fig. 1, a uniaxial, birefringent and non-linear crystal is used to split a photon pump beam having angular frequency ω_p , wavenumber k_p and wavelength λ_p into a pair of entangled twin photon beams, namely the signal beam and the idler beam. The signal beam has an angular frequency ω_s , wavenumber k_s and wavelength λ_s . Similarly, the idler beam is designated by angular frequency ω_i , wavenumber k_i and wavelength λ_i .

The core theory behind SPDC is based on the Maxwell equations and the concept of non-linear polarization and is described in detail in ref. [7].

A. Theory behind SPDC

Non-linear optics is the branch of optics that describes the behavior of light in a non-linear medium in which the dielectric polarization \vec{P} of the medium responds non-linearly to the electric field \vec{E} of the light [1]. A parametric non-linearity is an interaction in which the quantum state of the non-linear material is not changed by the interaction with the optical field resulting in an instantaneous process.

For a given non-linear medium, the dielectric polarization density $\vec{P}(t)$, which is the dipole moment per unit volume, at a time t can be expressed in terms of $\vec{E}(t)$ and is given by

$$\vec{P}(t) = \epsilon_0(\chi^1 \vec{E}(t) + \chi^2 \vec{E}(t)^2 + \chi^3 \vec{E}(t)^3 + \dots + \chi^n \vec{E}(t)^n) \quad (1)$$

where ϵ_0 is the permittivity of free space and the coefficients χ^n are the n -th order susceptibilities of the non-linear medium. For SPDC, we only include terms up to $n = 2$.

The energy E_p of the pump beam before the light is incident on the crystal is given by $E_p = \hbar\omega_p$. The energies E_s and E_i of the twin photons produced by the crystal are given by $E_s = \hbar\omega_s$ and $E_i = \hbar\omega_i$, where \hbar is the reduced Planck's constant.

The total energy of the system before and after down-conversion must be equal. Therefore, we equate E_p with the sum of E_s and E_i to get

$$\omega_p = \omega_s + \omega_i, \quad (2)$$

which illustrates energy conservation.

Momentum is conserved both inside and outside the crystal. Inside the crystal the magnitude of the wavevector $|k_{p,s,i}|$ of a particular beam, p , s or i , is given by

$$|k_{p,s,i}| = \frac{2\pi}{\lambda_{p,s,i}} = \frac{\omega_{p,s,i} n_{p,s,i}}{c}, \quad (3)$$

where $n_{p,s,i}$ are the refractive indices of either the pump, signal or idler beams. The wave vector k_p points in the direction of propagation of the incident beam.

Outside the crystal both the longitudinal momentum and the transverse momentum are conserved. The longitudinal momentum conservation is given by

$$|\vec{k}_p| = |\vec{k}_s| \cos\theta_s + |\vec{k}_i| \cos\theta_i \quad (4)$$

and the transverse momentum conservation is given by

$$|\vec{k}_s| \sin\theta_s = |\vec{k}_i| \sin\theta_i, \quad (5)$$

where θ_s and θ_i are the angles the signal and idler beams make with the direction of propagation of the pump beam respectively.

Substituting the values for $|\vec{k}_s|$ and $|\vec{k}_i|$ in Eqns. 4 & 5 with their corresponding values from Eqn. 3 we get

$$n_p \omega_p = n_s \omega_s \cos\theta_s + n_i \omega_i \cos\theta_i \quad (6)$$

and

$$n_s \omega_s \sin\theta_s = n_i \omega_i \sin\theta_i. \quad (7)$$

Requiring that the signal and idler beams have the same angular frequency such that $\omega_s = \omega_i$ implies that the signal and idler beams also make the same angle with the direction of propagation of light so that $\theta_s = \theta_i$. Imposing these conditions on Eqns. 2, 4 & 5 we get

$$n_p = n_{s,i} \cos\theta_{s,i}, \quad (8)$$

B. Coincidence theory

$$n_p \omega_p = 2n_{s,i} \omega_{s,i} \cos \theta_{s,i} \quad (9)$$

and

$$n_s = n_i \quad (10)$$

respectively. Eqns. 8, 9 & 10 are known as the ‘phase matching’ equations.

In spontaneous parametric down-conversion, we use the plane formed by the the optical axis of the crystal and the direction of propagation of the the incident pump beam to distinguish ordinary and extraordinary rays. A ray is ordinary if its polarization is perpendicular to this plane and is extraordinary if its polarization lies within this plane.

In type-I SPDC each of the twin photon beams produced by the crystal, namely the signal beam and the idler beam, have correlated polarizations. This type of down-conversion corresponds to the pump beam e_p being extraordinary and the signal and idler beams, o_s and o_p , being ordinary and is described as

$$e_p \rightarrow o_s + o_p. \quad (11)$$

Using Eqn. 17, Eqn. 15 can be rewritten for type-I SPDC as

$$n_e \lambda_p = 2n_o \lambda_{s,i} \cos \theta_{s,i}, \quad (12)$$

where n_e is the extraordinary index of refraction and n_o is the ordinary index of refraction.

In SPDC, n_e depends on the wavelength λ_p of the beam as well as the angle ϕ that the direction of propagation of the beam forms with the optical axis of the non-linear crystal. However, n_o only depends on the wavelength of the down-converted beam.

The relationship between the $n_{e,o}$ and $\lambda_{p,s,i}$ is different for different crystals and is described using Selmeier’s empirical equations.

For a non-linear crystal, Selmeier’s equations, which are used to determine the phase matching conditions for spontaneous parametric down-conversion for a non-linear crystal, are given by

$$n_o(\lambda) = \left[2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2 \right]^{\frac{1}{2}}, \quad (13)$$

and

$$n_e(\lambda) = \left[2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2 \right]^{\frac{1}{2}} \quad (14)$$

where λ is expressed in micrometers [8].

The entangled photons, the signal and the idler, produced during SPDC under ideal conditions are perfectly synchronized in time and space. If the pump beam is in a coherent state, the statistics of the photons in each of the entangled beams obey a Poisson process [9].

A common scheme of measuring the coincident photons involves the use of two photon detectors, namely the signal-beam detector and the idler-beam detector, and an electronic counter.

The signal-beam detector D_s is used to trigger a counter and the first photodetection by the idler-beam detector D_i following the D_s detection is used to stop the counter. If the time between the start and the stop of the counter is less than a pre-described threshold known as the coincidence resolving time, the counter is incremented by one, and a coincidence event is registered by D_i . At this point the search for further coincidences starts afresh.

If no photons are detected by D_i within the coincidence resolving time, the coincidence counting mechanism starts afresh and the counter is re-triggered as soon as D_s detects a photon. If in a given resolving time only a single photon is detected by D_s but multiple photons are detected by D_i , only one coincidence is registered by the counter.

In the process of photon counting using the phenomenon of SPDC, N = number pairs of twin photons produced in a given resolution time, N_s = number of counts registered by D_s in the same resolution time, N_i = number of counts registered by D_i in the same resolution time and N_c = number of coincidences between D_s and D_i in the same resolution time. Therefore, N_i , N_s and N_c are given by

$$N_i = N\eta_i, \quad (15)$$

$$N_s = N\eta_s \quad (16)$$

and

$$N_c = N\eta_i\eta_s, \quad (17)$$

where η_i and η_s are the quantum efficiencies of D_i and D_s respectively.

Dividing Eqn. 17 by Eqn. 16, the quantum efficiency η_i of the idler detector D_i is given by the relation

$$\eta_i = \frac{N_c}{N_s}. \quad (18)$$

Similarly, by dividing Eqn. 17 by Eqn. 15, the quantum efficiency η_s of the D_s is given by

$$\eta_s = \frac{N_c}{N_i}. \quad (19)$$

Therefore, using SPDC, no previously calibrated detector is required in order to determine the quantum efficiency of D_i or D_s .

However, in a given finite resolving time, several accidental coincidences occur. These coincidences generally occur as a result of coincidence registration due to a correlated photon in the signal beam and a non-twin photon in the idler beam or vice-versa, or due to noise photons. Therefore, these accidental coincidences must be corrected for when determining the efficiency of the detector.

III. EXPERIMENTAL SETUP & PROCEDURE

A diagram of the experimental setup is illustrated in Fig. 2. The setup consists of a pump laser diode that was used to emit a photon beam having wavelength 405 nm which was directed using a mirror, which was designed to reflect light having wavelengths between 400 nm and 750 nm, through a uniaxial, non-linear and birefringent beta-barium borate (BBO) crystal having dimensions $3 \times 3 \times 5$ mm which split the photon beam into a pair of entangled photon beams having wavelengths 810 nm each. A half-waveplate was used to vertically polarize the incident pump beam. For normal incidence of the pump beam on the crystal, as in our experiment, the twin photons split at a half angle of 3 degrees. A beam blocker was used to block the light from the pump beam that was transmitted straight through the crystal. Each of the split photons were directed to fiberports using mirrors. Separate fiber optic cables were used to direct the two photons from the fiberports through longpass filters, which filtered out all light that did not have a wavelength between 780-1800 nm, and finally to avalanche single photon counting modules (SPCM), one for each entangled photon beam. The filters were used to ensure that only light having wavelength 810 nm reached the SPCM. Once the SPCM detected photons a TTL pulse was sent from the SPCM to an adaptor box which connected to the development and education circuit board (DE2) via a 40 pin cable. The DE2 board contained a field programmable gate array chip programmed to read the TTL pulse and was connected to a LabVIEW-computer interface in order to read the signals received by the SPCMs [10]. The entire apparatus was placed on a vibrationally isolated, optical bread-board table in order to reduce the effects of surrounding movement.

In the experiment, the incident 405 nm beam represents the pump beam, the two 810 nm entangled photon beams represent the signal and idler beams.

Before single photon counts or coincidences could be determined, it was essential to fully prepare and align the

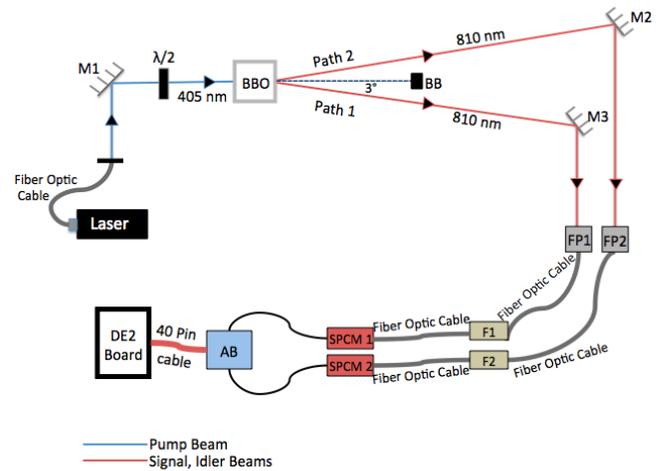


FIG. 2: A diagram illustrating the application of the phenomenon of spontaneous parametric down-conversion in our experimental setup. The apparatus consists of a blue laser beam, a linear polarizer, a half-wave plate $\lambda/2$, three mirrors M1, M2 and M3, a non-linear beta-barium borate (BBO) crystal, a beam blocker BB, two fiberports FP1 and FP2, fiber optic cables, two filters F1 and F2, two single photon counting modules SPCM1 and SPCM2, an adaptor box AB, a forty pin cable and a development and education circuit board (DE2).

apparatus. Due to reasons such as optical losses resulting from absorption of light within the crystal and the mirrors, only a small percentage of the photons produced by SPDC can actually be analyzed. Therefore effort was made to efficiently align the apparatus in order to maximize the light traveling through the system.

Upon completion of the alignment of the apparatus, the phenomenon of type-I spontaneous parametric down-conversion was used to analyze the photon counting rates, the accidental coincidence rate and the true coincidence rate of the two single photon counting modules. Care was taken to prevent the incidence of light having high intensity on the SPCMs as they are sensitive to light.

In addition, the phase matching equations described in Eqns. 8, 9 & 10, along with Selmeier's equations described in Eqns. 11 & 12 were used in a FORTRAN program developed by the National Institute of Standards and Technology in order to explore the phase matching problems for our experimental setup [8].

IV. RESULTS AND ANALYSIS

A. Dark Count Test

The dark count rate of a photon detector is the average rate of registered counts without any incident light. This determines the minimum count rate at which the signal is dominantly caused by real photons [9]. In order to determine the dark count rates of the two detectors, SPCM 1 and SPCM 2, used in our experiment, the de-

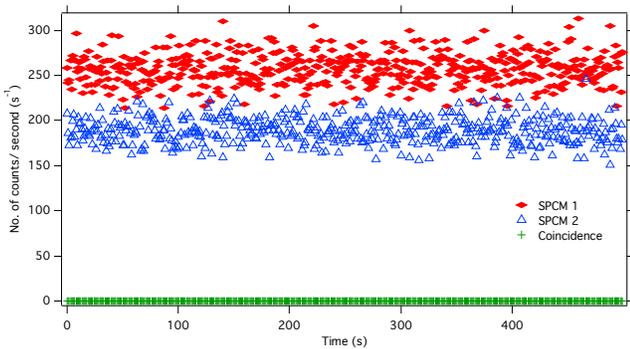


FIG. 3: A diagram showing the number of counts/s registered by SPCMs 1 & 2 in a time interval of 500 s illustrating the dark count rates of the two single photon detectors.

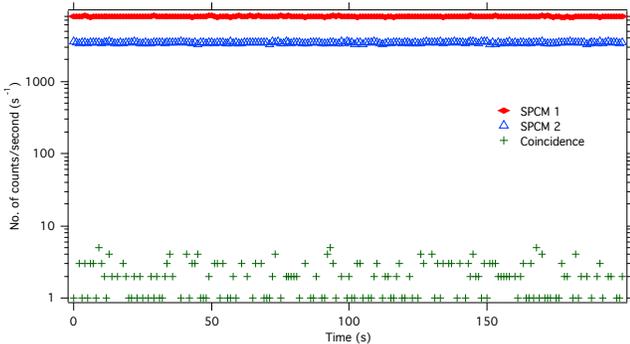


FIG. 4: A semi-log plot of the number of counts/s registered by SPCMs 1 & 2 over a time interval of 200 s illustrating the accidental coincidence rates of the two single photon detectors.

tectors were run in complete darkness in the absence of incident light.

A figure illustrating the number of photons detected per second over a 500 s time interval is shown in Fig. 3. The experimentally found average dark count rates of each of the SPCMs 1 & 2 were 257 counts/s and 189 counts/s having a standard deviation of 17 counts/s and 14 counts/s respectively. The average dark count rates for SPCMs 1 & 2 were 4 % and 18 % less than the dark count rates determined by the manufacturer.

The coincidence rate between the two detectors was experimentally found to be zero during this test.

B. Accidental Coincidence Test

In order to determine and analyze the accidental coincidences between SPCMs 1 & 2, the detectors were lightly and uniformly illuminated with a 120 V non-dimming green light and were run for 200 s. The number of counts/second of the two detectors as a function of time is illustrated in Fig. 4.

The average number of photons detected per second for each of the SPCMs 1 & 2 were 7930 counts/s and 3427

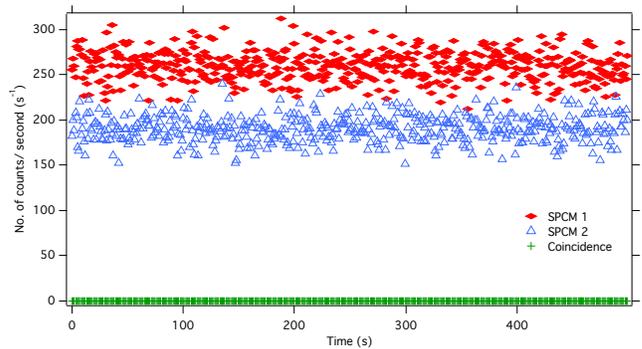


FIG. 5: A graph of the number of counts/second registered by SPCMs 1 & 2 over a time interval of 500 s when spontaneous parametric down-conversion was used to produce entangled photons. No coincidences or entangled photons are seen as the detectors did not register the down-converted photons.

counts/s with a standard deviation of 85 counts/second and 63 counts/second respectively. The average coincidence rate between SPCMs 1 & 2 was experimentally found to be 1.6 counts/s having a standard deviation of 1 count/s. The coincidence counts between the two detectors are accidental because they are not resulting from the coincidences between the twin photons emitted during SPDC.

C. Coincidence Test for the Entangled Photons produced by SPDC

In order to analyze the coincidence rate between SPCMs 1 & 2 produced by the spontaneously down-converted signal and idler beams, the detectors were run for 500 s and the corresponding photon count rates as a function of time were recorded, as illustrated in Fig. 5. The run was taken in completely darkness. The only light present was down-converted infrared light produced by the normal incidence of the blue light on the crystal.

The average count rate of each of the SPCMs 1 & 2 were experimentally found to be 260 counts/s and 190 counts/s having a standard deviation of 16 counts/s and 14 counts/s respectively. The experimentally found coincidence rate between the two detectors was zero. These results are very similar to the results from the dark count tests despite different conditions, implying that the down-converted light was not registered by the detectors.

D. Qualitative Analysis of Coincidence Results

The orientation of the half-waveplate in the experimental setup is crucial to determine the polarization of the incident pump beam on the crystal. An incorrect orientation results in the incident beam not being linearly polarized, resulting in the incident beam being ordinary and hence the absence of down-conversion.

TABLE I: A table describing the crystal specifications such as crystal type, temperature T , length, the wavelength λ_{signal} of the incident pump beam, and the half-angle θ between the direction of propagation of the incident beam and the optical axis of the crystal specified.

Crystal Specifications			
Type	BBO	BBO	BBO
Length (mm)	5	5	5
$T^{\circ}\text{C}$	22	22	22
λ_{signal} (nm)	405	405	405
ϕ°	30	27	28.9
θ°	0	5	3

The orientation of the crystal is also crucial for down-conversion to take place, as the angle between the optical axis of the crystal and the incident pump beam determines the half-angle split that the down-converted photons follow.

In addition, the position of the fiber ports FP1 and FP2 are important in determining the coupling efficiency of the down-converted light through the fiber optic cables. The use of a linear translator to increase the distance between FP1 and mirror M3, and FP2 and mirror M2 would improve the coupling efficiency of the down-converted photons, thereby resulting in a higher probability of the photons being registered by the detectors.

The alignment of the optics along the down-converted path as described in the experimental setup & procedure section was performed using a 808 nm laser. However, the down-converted light produced by the crystal have a wavelength of 810 nm each. The coupling efficiency during alignment was determined using the 808 nm laser, which could be different for the 810 nm light from the down-converted photons.

Finally, an improvement in the alignment of the optics along the predicted down-converted paths would result in a higher probability of the detectors registering photons.

E. A Computational Exploration of the Phase Matching Conditions in SPDC

By changing the angle ϕ between the optical axis of the BBO crystal and the direction of propagation of the 405 nm blue light incident on the crystal, for a given crystal temperature and crystal length, the half angle θ that each of the down-converted signal and idler beams make with the direction of propagation of the pump beam were computationally determined using a FORTRAN program developed by the National Institute of Standards and Technology.

For three different angles ϕ , three different half angles θ were formed, requiring that $\theta_{signal} = \theta_{idler}$. The crystal specifications and corresponding results are summarized in Table. I. For the crystal type, length, temperature and

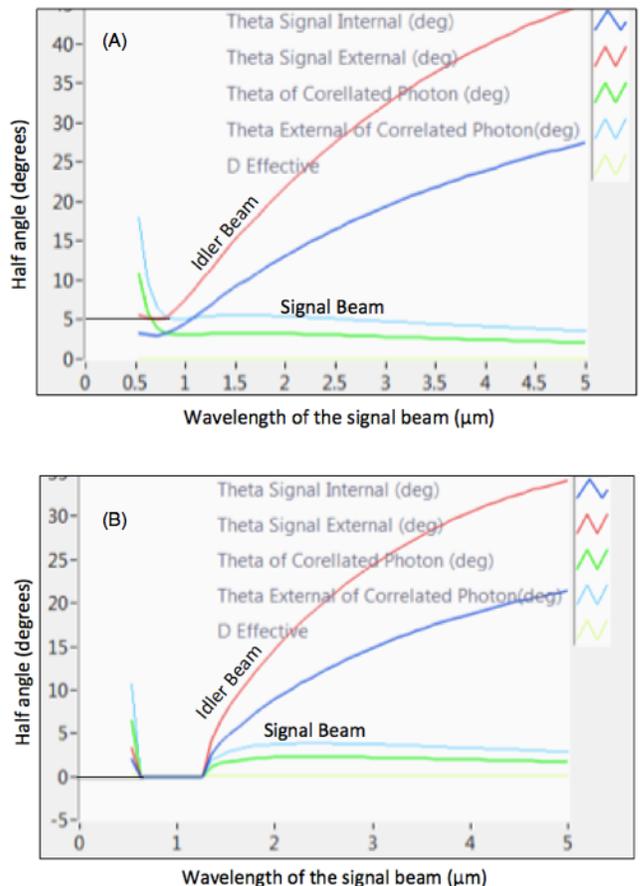


FIG. 6: A diagram illustrating the relationship between the half-angle formed by the signal and idler beams and the wavelength of the signal beam for (A) $\phi = 27^{\circ}$ and (B) $\phi = 30^{\circ}$, corresponding to a half-angle split of $\theta = 5^{\circ}$ and $\theta = 0^{\circ}$ respectively for the down-converted photons.

the wavelength of the pump beam described in Table. I, as used in our experiment, Fig. 6 illustrates the relationship between the half-angle θ and the wavelength of the signal beam λ_{signal} for (A) $\phi = 27^{\circ}$ and (B) $\phi = 30^{\circ}$. Since in our experimental setup, for a pump beam having wavelength 405 nm, both the resulting twin photons have the same wavelength, the angle corresponding to the point of intersection of the signal and idler beams is the half-angle θ that the twin photons make.

A half-angle of $\theta = 5^{\circ}$ is formed when $\phi = 27^{\circ}$. When $\phi = 30^{\circ}$, a half-angle of $\theta = 0^{\circ}$ is formed corresponding to the signal and idler beams being collinear.

Figure 7 illustrates the relationship between θ and λ_{signal} for $\phi = 28.9$, resulting in a half angle of $\theta = 3^{\circ}$ approximately, which is what is required in our experiment.

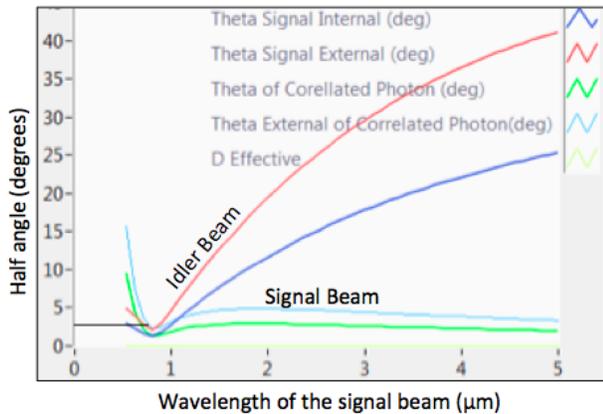


FIG. 7: A diagram illustrating the relationship between the half-angle formed by the signal and idler beams and the wavelength of the signal beam for $\phi = 28.9^\circ$. The down-converted photons split at a half angle of approximately $\theta = 3^\circ$.

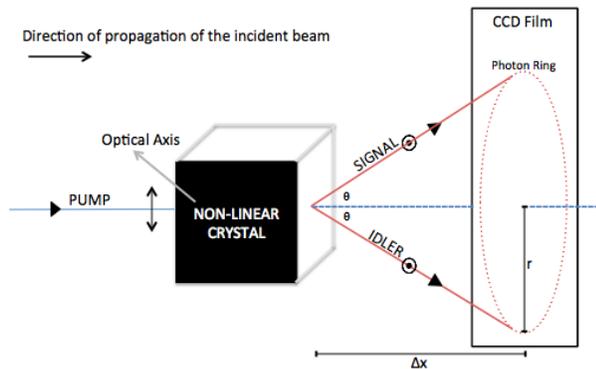


FIG. 8: A diagram illustrating the photon ring that is formed during spontaneous parametric down-conversion being captured by a CCD film.

V. FUTURE WORK

In the future, an improvement in the alignment of the current setup is essential in order for the down-converted signal and idler beams to be detected by the single photon detectors. In addition, replacement of the current crystal mount, which has two degrees of freedom of motion, with a crystal mount that has three degrees of freedom of motion, will result in a better angular adjustment of the crystal.

Once the down-converted photon beams are registered by the two photon detectors, the absolute quantum efficiency of the detectors can be determined.

In addition, the down-converted photons can be transported to asymmetric and symmetric Mach-Zehnder interferometers, manipulating spatial and polarization components of the photons in conjunction with producing Hong-Ou-Mandel interference and investigating non-uniform polarization states of light [10].

Finally, it will be interesting to visually capture the down-converted photon ring on a CCD film as illustrated in Fig. 8. In addition, the dimensions of the down-converted ring could be used to make appropriate angular adjustments to the crystal position in order to get a 3° half-angle between the down-converted photons by using the relationship

$$\theta = \tan^{-1} \left(\frac{r}{\Delta x} \right),$$

where θ is the half angle between the signal and the idler beams, r is the radius of the down-converted photon ring and Δx is the horizontal distance between the center of the photon ring and the crystal.

VI. CONCLUSION

The phenomenon of type-I spontaneous parametric down-conversion (SPDC) was studied using a linearly polarized 405 nm pump beam which was normally incident on a non-linear and birefringent Beta-Barium borate crystal. An experimental setup was designed in an attempt to use SPDC to create entangled photons to be detected by two single photon detectors, one for each entangled photon beam.

The average dark count rates of the two single photon detectors were experimentally measured to be 257 counts/s and 189 counts/s having a percent difference of 4 % and 18 % from the dark count rates determined by the manufacturers of the detectors.

The two detectors had an accidental coincidence rate of 1.6 counts/s when illuminated with a 120 V non-dimming green light.

The coincidence rate between the two detectors when down-converted entangled photons were created by the crystal was experimentally measured to be zero, indicating that the down-converted light was not registered by the detectors. An improvement in the alignment of the experimental setup, the addition of a crystal mount having three degrees of freedom of motion and an improvement in the coupling efficiency of the fiber optic cables used to transport the down-converted photon beams to the detectors will improve the chances of the entangled photons being registered by the detectors.

In addition, it was computationally found for our experimental setup, that angles of 27° , 28.9° and 30° between the direction of propagation of the incident pump beam and the optical axis of the crystal, result in 5° , 3° and 0° half-angle splits of the down-converted photons respectively.

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