Lead Zeppelin: An Investigation into the Feasibility of a Modern Vacuum Balloon

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The concept of a vacuum lift cell makes use of Archimedes principle to obtain lift in a solid shell through displacement of air. In order to be successful, such a cell must be light enough to maintain neutral buoyancy while remaining strong enough to resist atmospheric pressure. Three models for spherically shelled vacuum lift cells were created mathematically to simulate the material properties of aluminum 6061-T6, standard modulus carbon fiber composite, and multi-layer graphene. A condition for neutral buoyancy was created for each model followed by a condition for critical stress and critical buckling pressure. The relationships between these conditions were then analyzed to determine the functionality of each model. Aluminum 6061-T6 failed to resist compressive stress or buckling while maintaining neutral buoyancy, and was able to resist 0.2% of atmospheric pressure before buckling. Standard modulus carbon fiber composite and multi-layer graphene were able to resist compressive stress, but were unable to maintain neutral buoyancy without buckling. Standard modulus carbon fiber composite and multi-layer graphene resisted 16% and 82% of atmospheric pressure at sea level respectively.

I. INTRODUCTION

Archimedes' principle states that an object within a medium will experience an upward force equal to the weight of medium displaced. This principle has been used to achieve flight for centuries in the form of lighter-than-air craft such as hot air balloons and hydrogen airships. While they still see use in certain industries, lighter-than-air craft have fallen out of favor as a form of aerial transportation in the last century due to the inherent design challenges and dangers associated with the concept. In addition, airplanes are faster, more durable, and can carry a more significant payload for a given size. However, as they do not require fuel to achieve lift, airships have great potential in the areas of fuel efficiency and environmental friendliness, two factors which are becoming increasingly desirable in modern transportation industries. A possible solution to the difficulties associated with traditional airships would be to develop a functional vacuum balloon. Such a craft would make use of evacuated chambers to achieve significant buoyant force within Earth's atmosphere. The concept of a vacuum airship has existed for centuries, with the first prototype presented by Arthur De Bausset in 1884. He proposed suspending a transport compartment from an enormous, thinly walled, steel cylinder, but was unable to eventually construct his invention [1]. Unfortunately, De Bausset's design and any subsequent proposals have failed due to the challenges of creating a chamber strong enough to resist atmospheric pressure.

The purpose of this study was to mathematically determine the feasibility of creating a chamber capable of achieving net lift through the use of vacuum induced buoyancy while possessing the strength required to resist stress and buckling due to atmospheric pressure through the use of modern materials. Such a chamber is referred to as a vacuum lift cell within this document and could be applied to a variety of uses, from transport to communication and advanced construction [2].

II. THEORY

In order to be functional, a vacuum lift cell must fulfill two criteria:

1. The buoyant force acting upon it due to Earth's atmosphere must be strong enough to equal or overcome the force of gravity.

2. Its structural integrity must be strong enough to withstand the pressure generated by Earth's atmosphere without collapsing.

In order for an object on Earth to float within a medium, the mass of the displaced air Ma must be greater than the total mass of the object, or equal to the mass of the object for neutral buoyancy. For this study, a spherical shell design was chosen due to its inherent structural integrity and radial symmetry.

As the mass of an object is equal to its volume times its density, the mass of an empty spherical shell M_s can be modeled by the equation

$$M_s = \frac{4}{3}\pi\rho_s(R^3 - r^3) \tag{1}$$

where ρ is the density of the material composing the shell, and R and r are the outer and inner radii of the shell respectively. Similarly, the mass of the air displaced by

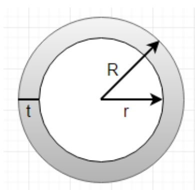

FIG. 1: Cross section of a vacuum lift cell with inner radius r, outer radius R, and thickness t.

the sphere can be modeled by the equation

$$M_a = \frac{4}{3}\pi\rho_a r^3 \tag{2}$$

where ρ_a is the density of air. The volume of the air displaced by the shell itself will be negligible compared to the volume displaced by the vacuum and can safely be ignored. Thus, setting Eq. 1 and Eq. 2 equal to one another yields the condition

$$\frac{4}{3}\rho_a r^3 = \frac{4}{3}\rho_s(R^3 - r^3) \tag{3}$$

for the neutral buoyancy of an empty spherical shell. Simplifying this relationship yields

$$\rho_a r^3 = \rho_s R^3 - \rho_s r^3 \tag{4}$$

for the lift requirement of a spherically shelled vacuum cell.

The stress σ generated on an empty spherical shell by Earth's atmosphere can be derived in the following fashion. The compressive stress on a spherical shell is equal to $\sigma = F/A_c$ where F is the atmospheric force. As presented by Akhmeteli [3], A_c is the cross sectional area of the shell and is represented by the equation

$$A_c = \pi R^2 - \pi r^2 \tag{5}$$

which can be rewritten as

$$A_c = \pi R^2 \left(1 - \left(1 - \frac{R-r}{R}\right)^2\right). \tag{6}$$

Since $(R - r) \ll R$, Eq. 6 can be simplified using a first order approximation

$$\left(1 - \frac{t}{R}\right)^2 = 1 - \frac{2t}{R} + \left(\frac{t}{R}\right)^2 \approx 1 - \frac{2t}{R} \tag{7}$$

where $t = (R-r)$ is the thickness of the shell. This yields the relationship

$$A_c \approx 2\pi R t_s \tag{8}$$

for the cross-sectional area of a spherical shell.

The force acting on a spherical shell due to Earth's atmosphere will be symmetric about the entire sphere. Thus, one can determine the force F_z acting on the shell by integrating the force over the area of a hemisphere

$$F = \int_0^{2\pi} \int_0^{\pi/2} P_a \cos\theta \sin\theta R^2 d\theta d\phi \tag{9}$$

where R is the outer radius [3]. Solving this integral for F yields the result

$$F = P_a R^2 \pi \tag{10}$$

for the force acting on a hemisphere of radius R [3]. Inserting Eq. 8 and Eq. 10 into the stress relationship yields

$$\sigma = \frac{P_a R^2 \pi}{2\pi R t} = \frac{P_a R}{2t} \tag{11}$$

for the atmospheric stress on a spherical shell of outer radius R.

However, stress is not the only structural factor to take into consideration. A spherical shell will experience buckling before reaching the limit of its compressive strength. The critical buckling pressure for a spherical shell of inner radius r and outer radius R has been derived as

$$P_{cr} = \frac{2E(R^3 - r^3)}{R^2\sqrt{3(1-\nu^2}} \tag{12}$$

where E is the elastic modulus of the material, and ν is Poisson's ratio [4]. The full derivation for this relationship goes beyond the scope of this report and can be found in Timoshenko's *Buckling of Spherical Shells* [4].

Thus, in order to construct a functional vacuum lift cell at sea level, the material used would have to have a density low enough to satisfy Eq. 4 for a spherical shell of inner radius r. It would simultaneously need a yield strength that is higher than the output of Eq. 11 and an elastic modulus and Poisson's ratio that yeild a critical buckling pressure that is lower than atmospheric pressure at sea level.

III. PROCEDURE

Three materials were considered for each virtual model: aluminum 6061-T6, standard modulus carbon fiber composite, and multi-layer graphene. The constants used for each material are presented in Table 1. Aluminum 6061-T6 is an alloy of high strength and low weight commonly used in the construction of aircraft. Standard modulus carbon fiber composite is a commonly available composite composed of epoxy-bonded carbon filaments. Note that the density provided in Table I does not take into account the epoxy used to bind the layers of carbon fiber together, which has a density of 1174 kg/m [9]. Hence, the total density of the composite would be approximately 1500 kg/m depending on the proportion of fiber to epoxy. The term graphene as used in this report refers to the the stacked form of the traditionally single-layer carbon allotrope. The material is still very new, and while many values for its material constants have been theoretically and experimentally derived, none have been universally agreed upon. Some of the values used in the models presented in this report were the mid-range values chosen from among various results.

Each material was simulated in a single layer shell. First, the material's density was used determine the dimensions of the shell required for neutral buoyancy using Eq. 4. An arbitrary radius of 1 m was used for each shell model. Next, Eq. 11 was used to determine the stress acting on the shell due to Earth's atmosphere. The result

	Density [kg/m]	Yield strength [MPa]	Elastic modulus [GPa]	Poisson's ratio
Aluminum 6061-T6	2800 [5]	248 [6]	68.9 [5]	0.33 [5]
Standard modulus carbon fiber Composite	1800 [10]	210 [10]	137 [10]	0.77 [7]
Multi-layer graphene	1550 [12]	361 [13]	1000 [11]	0.25 [11]

TABLE I: Table of the material constants for aluminum, carbon fiber, and graphene used for the simulation of a vacuum lift cell presented in this report.

of this calculation was compared to the yield strength of the material to determine whether or not the shell could resist compressive stress. Finally, Eq. 12 was solved for the material's elastic modulus and Poisson's ratio to determine the critical buckling pressure of the shell. This value was then compared to atmospheric pressure at sea level to determine whether or not the shell would buckle.

IV. RESULTS

A neutrally buoyant aluminum 6061-T6 sphere with an inner radius of 1 m was found to experience atmospheric stress equal to 347 MPa and a critical buckling pressure of 179 Pa. Using the same parameters, a carbon fiber composite sphere with was found to experience atmospheric stress equal to 186 MPa and a critical buckling pressure of 17 MPa. The multi-layer graphene sphere experienced an atmospheric stress of 191 MPa and a critical buckling pressure of 83 MPa. A plot of the critical buckling pressure for each material as a function of thickness and outer radius is shown in Fig. 2. Note that this plot does not take into account the neutral buoyancy condition necessary for a functional vacuum lift cell.

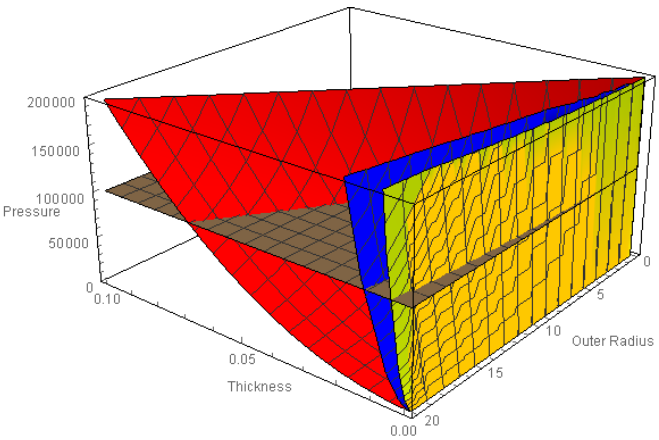

FIG. 2: A plot of the critical buckling curves for aluminum 6061-T6 (red), standard modulus carbon fiber composite (blue), and graphene (yellow). Atmospheric pressure at sea level is represented in gray. All points above this plane show combinations of thickness and radius that would be able to resist Earth's atmospheric pressure at sea level, while points below the plane show combinations that would fail to do so.

V. ANALYSIS

The results for each material have been plotted in Fig. 3. Each plot compares the outer radius and thickness of the materials' neutral buoyancy condition with

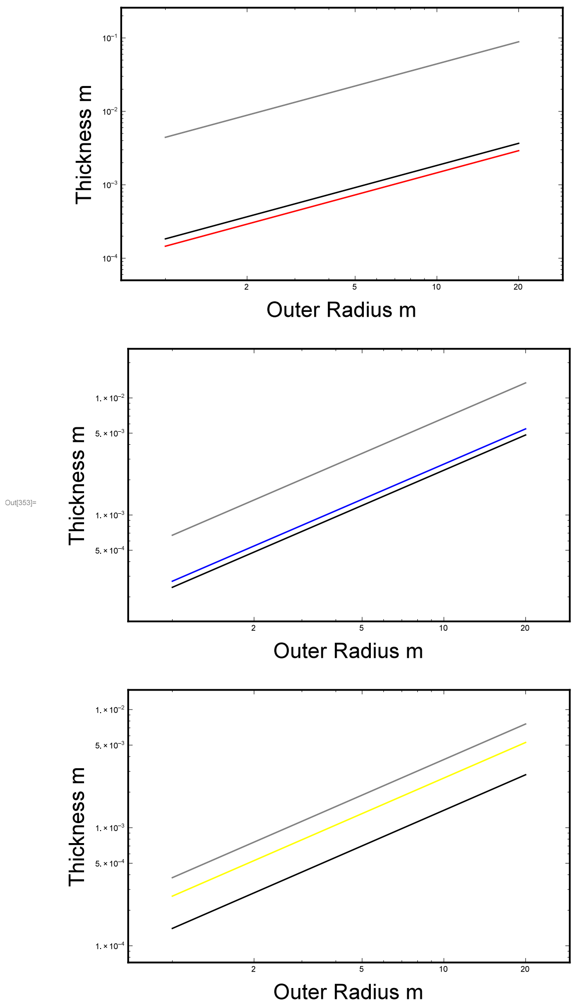

FIG. 3: logarithmic plots of thickness as a function of radius for the neutral buoyancy of aluminum 6061-T6 (red) , standard modulus carbon fiber composite (blue), and multi-layer graphene (yellow). The neutral buoyancy condition for each material is plotted with its respective critical stress (black) and critical buckling pressure (gray).

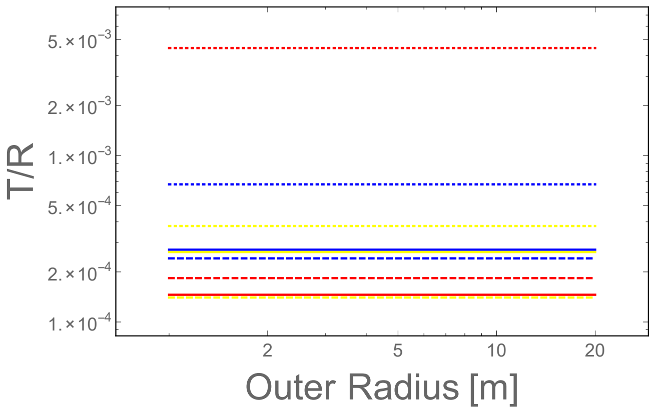

FIG. 4: A logarithmic plot of thickness divided by outer radius as a function of outer radius for aluminum (red), carbon fiber composite (blue), and graphene (yellow). The buoyancy conditions, stress conditions, and buckling conditions for each material are represented by solid, dashed, and dotted lines respectively.

the respective critical stress and critical buckling conditions. As the neutral buoyancy curve lies below both the critical stress and critical buckling curves, Fig. 3 indicates that aluminum 6061-T6 does not have the ratio of durability and weight to satisfy either condition assuming neutral buoyancy. Both standard modulus carbon fiber composite and multi-layer graphene were able to satisfy the critical stress conditions. However, neither was capable of satisfying the critical buckling condition while maintaining neutral buoyancy. The plot in Fig. 4 consolidates the data for each model. As the T/R neutral buoyancy curve for graphene is the nearest to its buckling curve when compared to the buoyancy and buckling curves for the other materials, it can be concluded that graphene would be the best material candidate for a functioning vacuum lift cell. However, while the multi-layer graphene cell was able to resist a significant 82% of atmospheric pressure at sea level, its structural integrity would still not be sufficient to resist atmospheric pressure at sea level while following the conditions required for neutral buoyancy. Standard modulus carbon fiber composite and aluminum 6061-T6 also displayed insufficient structural integrity at neutral buoyancy, being able to resist 16% and 0.2% of atmospheric pressure at sea level respectively. As none of the presented models could simultaneously resist atmospheric pressure at sea level while remaining neutrally buoyant, we can conclude that a single layer spherical shell design will not function as as an effective vacuum lift cell using materials currently available.

VI. FUTURE WORK

While this research rules out the construction of single layer spherical vacuum lift cells using modern materials, it does not cover all possible designs. A single layer sphere is not necessarily the optimal design for a vacuum lift cell. Future study could include simulations of symmetrical polyhedrons or designs with an internal supporting structure such as the multi-layered shell proposed by Akhmeteli and Gavrilin [3]. A sphere with internal support such as the ribbing visible inside SpaceX's Dragon 2 spacecraft could also be worth investigating [14]. In terms of materials, multi-layer graphene demonstrated the best resistance to stress and buckling at neutral buoyancy, prompting additional investigation.

VII. CONCLUSION

In order to determine the feasibility of a spherical vacuum lift cell, spherical shells of aluminum 6061-T6, standard modulus carbon fiber composite, and multi-layer graphene were simulated mathematically using experimentally derived material constants. A neutral buoyancy condition was established for each of the three materials using mass and displaced volume, along with a condition for critical stress and critical buckling pressure. These conditions were then compared to one another for each model in order to determine whether a combination of radius, thickness, and material would allow for a cell that could resist atmospheric pressure at sea level while maintaining neutral buoyancy. By plotting the logarithm of each shell thickness vs logarithm of each outer shell radius, it was determined that aluminum was the least effective material as it was unable to maintain neutral buoyancy while resisting atmospheric stress, and was only able to withstand 0.2% of atmospheric pressure before buckling. Both standard modulus carbon fiber composite and multi-layer graphene were able to resist compressive stress, but were only able to resist 16% and 82% of atmospheric pressure at sea level respectively. The failure of these models prompts the conclusion that a spherically shelled vacuum lift cell is not possible with the use modern materials. The functionality of other shapes and structures remains unclear and presents the possibility of future work on the subject of functional vacuum lift cells.

VIII. ACKNOWLEDGMENTS

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