The random walk of radiation from the sun

Lisa May Walker
Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
(Dated: May 11, 2006)

A simulation was run to find the average escape time of photons from the radiation zone of the Sun. An escape time of was found for a constant-density Sun and for a linear density gradient Sun by extrapolation from smaller radii. These values are respectively less than and greater than the expected value by an order of magnitude.

I. INTRODUCTION

Energy is created in the core of the Sun in the form of neutrinos and radiation, via the proton-proton (p-p) cycle. The neutrinos escape into space a few seconds after being created [1], due to the fact that they interact extremely weakly with matter, while the photons take 100,000–1 million years to escape [2], instead of the naïvely expected $R_\odot/c$. If no massive particle can travel faster than the speed of light, how is this possible? In addition, the photons created by the p-p cycle are in the form of high energy γ-rays [3], but the spectrum shown in figure 1 is that of an almost perfect blackbody at a temperature of $5900K$. Something must be happening to the radiation on its way from the core to the surface of the Sun.

The energy created in the core of the Sun is primarily from the p-p cycle. This reaction follows the pattern [6]

$$\begin{aligned} {}^1\mathrm{H} + {}^1\mathrm{H} &\rightarrow {}^2\mathrm{D} + \mathrm{e}^+ + \nu \\ {}^2\mathrm{D} + {}^1\mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\ {}^3\mathrm{He} + {}^3\mathrm{He} &\rightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H}. \end{aligned} \tag{1}$$

Thus liberating one γ-ray directly from the reaction, as well as the γ-rays that are created from annihilation of

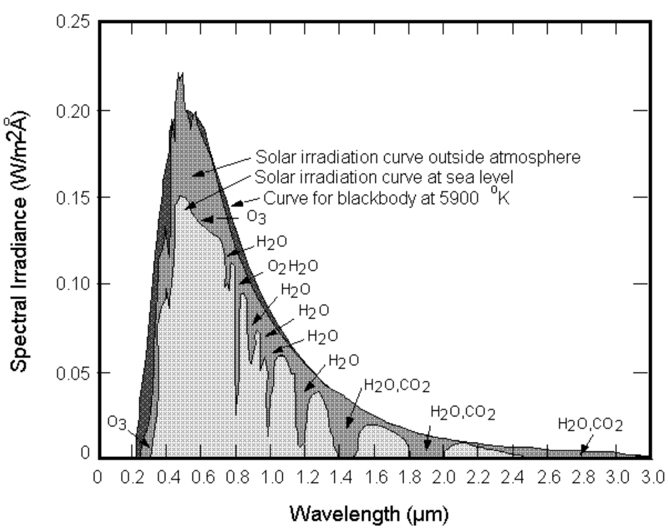

FIG. 1: The spectrum of the Sun as compared with a perfect blackbody spectrum. Figure courtesy of [5].

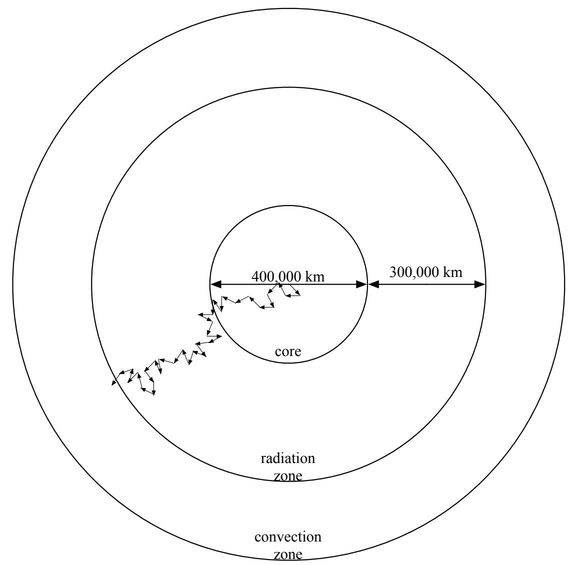

FIG. 2: (b) An illustration of a random walk of radiation from the core through the radiation zone of the Sun. As shown, scattering can cause a change in direction by a small amount or by an angle close to π. This is a simplified version, as the random walk can also cause the radiation to double back on itself, crossing through where it has already been.

the positron and an electron. Once a γ-ray is created, via the p-p cycle, it must travel $R_{\odot\mathrm{rad}} = 700,000$ km, the radius of the Sun, before it can escape. This happens in two different ways, depending on how far it is from the center of the Sun. In the core and the radiation zone, shown in figure 2, the energy travels via radiative transfer. At a radius $R_{\odot\mathrm{rad}} = 500,000$ km, the energy leaves the radiation zone and begins traveling upward via convection in the aptly named convection zone [6].

Investigating how energy makes its way from its creation in the core to the surface of the Sun means looking at its travel through the various zones. The energy spends far more time as a photon in the radiation zone of the Sun than the convection zone [7], so the radiation zone is the crucial region to examine, as well as the region that has received the least study. The final path that a photon travels through the radiation zone is modeled by a random walk, a simplified example of which is shown in figure 2.

In the core and radiation zone, energy is transported in the form of photons, which interact with electrons as they travel. The radiation zone consists of an equal number of both protons and electrons, however the electrons play the dominant role in the scattering process. The cross-section for an interaction is given by [8]

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar E_3}{8\pi\, m_2\, c\, E_1}\right)^2 S\,|\mathcal{M}|^2 \qquad (2)$$

where E_1 is the energy of the photon, E_3 is the energy of the resultant particle, m_2 is the mass of the interacting particle, S is a statistical factor, and $|\mathcal{M}|$ is the scattering amplitude. Since electrons have a mass of [9]

$$m_e = 9.11 \times 10^{-31}\ \text{kg}$$

and the mass of a proton is [9]

$$m_p = 1.67 \times 10^{-27}\ \text{kg},$$

the cross section for a proton is a factor of 10^8 smaller than that of an electron. Thus, interactions with protons occur infrequently enough that their effect on the scattering process can be ignored. The photons created in the core through the p-p cycle, traveling outward in a random direction.

In addition, the only possible interaction between free electrons and photons in the radiation zone is scattering. Free electrons cannot absorb photons, as it would violate conservation of energy and momentum, so the process is

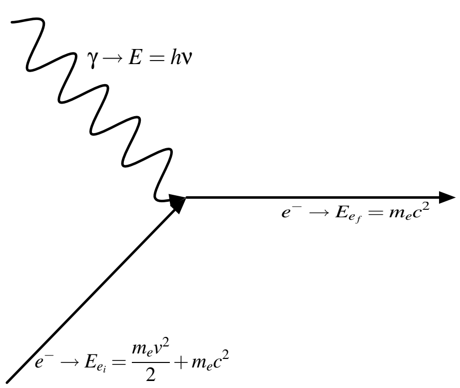

FIG. 3: The Feynman diagram for absorption of a photon by an electron.

not allowed [8]. Thus in the radiation zone, the photon travels until it scatters off an electron, at which point it may lose energy and begins traveling in another random direction.

When looking at the scattering of photons in the core and radiation zone, the most important value is the mean free path, or average path length, between interactions. This mean free path f comes from the density of the radiation zone of the Sun $\rho_{\odot\text{rad}}$ and the Thomson cross section σ_T. The mean free path is calculated by [10]

$$\begin{aligned} m_{\odot\text{rad}} &= \rho_{\odot\text{rad}} V_{\odot\text{rad}} \\ N &= \frac{m}{m_e + m_p} \\ f &= \frac{V}{\sqrt{2}\sigma_T N}, \end{aligned} \qquad (3)$$

where N is the number of particles. Thus the mean free path is given as

$$f = \frac{m_e + m_p}{\sqrt{2}\sigma_T \rho}. \qquad (4)$$

II. THE PROGRAM

For the simulation, a photon was placed at $r = 0$, and started traveling with speed c in a random direction. After each time step, a probability factor was used to determine whether or not the photon had interacted with an electron. If it had not collided, it continued in its path. If it had collided then it was given a new, random direction of travel, still traveling with speed c.

The time step was determined so that the path length $l = ct$ was significantly less than the mean free path f for a Sun with constant density $\rho = 15,000 {}^{kg}/_{m^3}$, which is the actual maximum density for the radiation zone [2]. The mean free path was determined using equation 4 and found to be $f = 1.2 \times 10^{-4}$ m. Thus the time step used was $t = 10^{-13}$ s, yielding a step size of $l = 3 \times 10^{-5}$ m.

The probability determination whether a collision occurred was a two-step calculation. For the Sun with a constant ρ, there was a static probability of collision given by [11]

$$P = 1 - e^{-l/f},$$

and a randomly-generated probability factor $prand$ on the same range $[0, 1]$ after each time step. These were compared, and if $prand < P$ then the photon had collided, and a new direction was randomly generated. Otherwise, it continued in its path.

The probability calculation for a Sun with a linear density gradient is more complicated. The density gradient was described by

$$\rho = -\left(3 \times 10^4\ \frac{\text{kg}}{\text{m}^3}\right)\frac{5r}{R_{\text{rad}}} + (1.5 \times 10^5\ \frac{\text{kg}}{\text{m}^3}) \qquad (5)$$

where R_{rad} is the radius of the radiation zone of the Sun in the simulation and r is the radius of the photon's current location. Since the density was no longer constant, rather a function of r, the probability of collision was no longer constant. Thus, ρ, f, and P had to be recalculated after each time step.

Initially, the program used $R_{\text{rad}} = R_{\odot\text{rad}}$ to calculate the escape time. However, it ran for over a day without

	Constant Density		Linear Gradient	
$R_{\rm rad}$ (m)	$t_{\rm mode,scale}$	$\Delta t_{\rm mode,scale}$	$t_{\rm mode,scale}$	$\Delta t_{\rm mode,scale}$
5	9.4	4.4	44	21
0.5	113	52	470	220
0.05	1520	680	10000	2000

TABLE I: The most common escape times $t_{\rm mode,scale}$ for both a Sun with a constant ρ and a linearly-decreasing ρ. The error in $t_{\rm mode,scale}$ is also given as Δt.

escaping. Since the intent of the simulation was to find an average escape time for many different runs, a more efficient method was needed. Thus, the simulation was run for several $R_{\rm rad} \ll R_{\odot{\rm rad}}$, each returning 1,000 escape times, in an effort to extrapolate an escape time for $R_{\odot{\rm rad}}$.

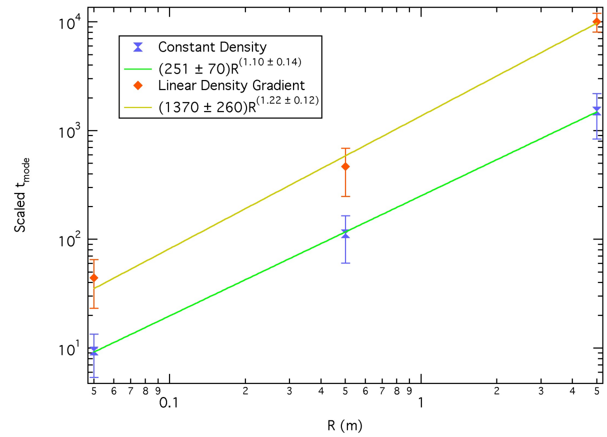

FIG. 5: A log-log plot of $t_{\rm mode,scale}$ vs. $R_{\rm rad}$ for both a constant ρ and a linearly decreasing ρ, according to equation 5. These were fit by a power-law and extrapolated up to $R_{\odot{\rm rad}}$.

III. ANALYSIS

The scaled escape times for various radii of a constant-density Sun are shown in figure 4. The scaling was done by dividing the escape times by $R_{\odot{\rm rad}}/c$, to obtain a ratio of actual escape time to the expected escape time without scattering, or "interaction-free escape time," which allows for a better comparison between radii. The shape of the histogram is very similar for all three radii, indicating that as the radius increases, there are no drastic changes in what is happening, simply a scaling increase. All three have a $t_{\rm mode,scale}$ that occurs shortly after $t_{\rm min}$ and a much slower decay. The hourglass data points in figure 5 show $t_{\rm mode,scale}$ for a constant ρ that were the most common for each radius. These escape times were fit by a power-law, which gives

$$t_{\rm mode,scale} = (251 \pm 70)\, R_{\rm rad}{}^{1.10\pm0.14}$$

yielding a scaled $t_{\rm mode}$ for $R_{\odot{\rm rad}}$ of

$$t_{\rm mode,scale} = (9.30 \pm 2.59) \times 10^{11}$$

and thus a real escape time of

$$\begin{aligned} t_{\rm mode} &= t_{\rm mode,scale}\frac{R_{\odot{\rm rad}}}{c} \\ &= (1.55 \pm 0.43) \times 10^{12}\ {\rm s} \\ &= (4.9 \pm 1.4) \times 10^{4}{\rm yrs.} \end{aligned}$$

The error for these values was obtained by judging the width of the peaks in figure 4. The peaks for a constant ρ did not decay quickly, so they had rather large errors.

Figure 6 shows $t_{\rm mode,scale}$ for various radii of a Sun's radiation zone with a linear density gradient given by equation 5. These $t_{\rm mode,scale}$ have also been scaled by $R_{\odot{\rm rad}}/c$ for purposes of comparison. The histograms for the linearly decreasing ρ also all share a characteristic shape. However, they do not peak as closely to $t_{\rm min}$ as the histograms for a constant ρ, but their peaks seem to be narrower. This is indicative of a few photons escaping relatively quickly, but the vast majority escaping over a small range in time. The photons that escape with times close to $t_{\rm min}$ most likely escape the core quickly, but spend similar amounts of time in the radiation zone as the photons with longer escape times. These escape times as a function of radius for a linearly decreasing ρ were fit by a power-law, which gives

$$t_{\rm mode,scale} = (1370 \pm 260)\, R_{\rm rad}{}^{1.22\pm0.12}$$

yielding a scaled $t_{\rm mode}$ for $R_{\odot{\rm rad}}$ of

$$t_{\rm mode,scale} = (5.6 \pm 1.1) \times 10^{13}$$

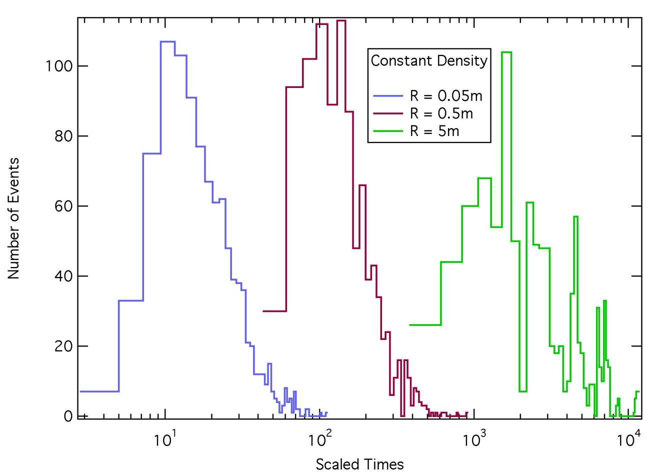

FIG. 4: The escape times for a Sun with constant ρ plotted on a log-log plot to reveal what is happening for smaller values of t.

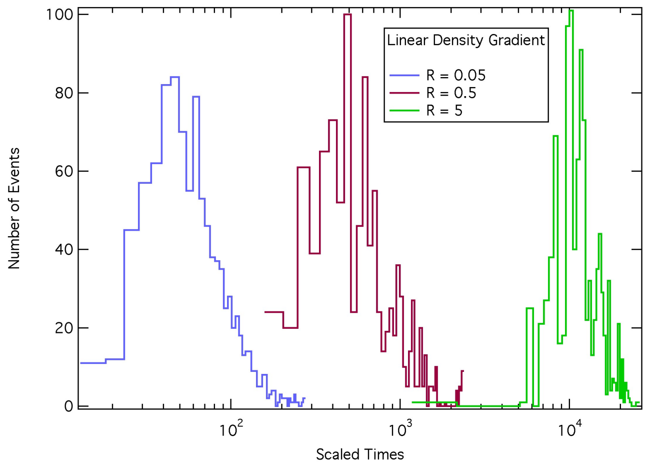

FIG. 6: (a) The escape times for a Sun with a linearly increasing ρ plotted on a log-log plot to reveal what is happening for smaller values of t.

and thus a real escape time of

$$\begin{aligned} t_{\text{mode}} &= t_{\text{mode,scale}} \frac{R_{\odot \text{rad}}}{c} \\ &= (9.3 \pm 1.8) \times 10^{13} \text{ s} \\ &= (2.9 \pm 0.6) \times 10^{6} \text{yrs}. \end{aligned}$$

IV. CONCLUSION

An escape time of $t_{\text{mode}} = (4.9 \pm 1.4) \times 10^4$ yrs was found for a constant-density Sun and $t_{\text{mode}} = (2.9 \pm 0.57) \times 10^6$yrs for a Sun with a linearly decreasing ρ. This was done by finding t_{mode} and its error from a histogram plot, then plotting t_{mode} vs. $R_{\odot\text{rad}}$ and using a power-law fit to extrapolate 8 decades to $R_{\odot\text{rad}}$ from smaller values of R_{rad}. The value of t_{mode} for the constant-density Sun is less than the expected value, while the value of t_{mode} for the Sun with a linearly decreasing ρ was greater than the expected value. A density function described by a gaussian curve would likely yield a time scale consistent with the expected value.

Further work in this area could be accomplished with more computing power in order to run the simulation over larger radii. It would also be possible to compute the spectrum of the light at the edge of the radiation zone in a more complicated simulation. This could be accomplished by calculating the change in momentum from each collision. In addition, a check could be added to verify that the characteristic l matches f.

An important note is the simplifications used in the program. All photons were assumed to start at $r = 0$, rather than a random point in the core. Also, the core was composed entirely of protons and electrons with no Helium ions, thus yielding a larger number of particles and therefore a smaller mean free path. In addition, the radiation zone was perfectly smooth all the way to its edge, rather than its gradual change into the convection zone.

[1] "Random Walk" *Living with a Star: Understanding Connections Between the Sun and Earth*. Berkeley University. `http://ds9.ssl.berkeley.edu/LWS_GEMS/2/random.htm`.

[2] Eric Chaisson and Steve McMillan. *Astronomy Today, fourth edition*. New Jersey: Prentice Hall, 2002.

[3] Bethe, H.A. "Energy Production in Stars." *Nobel Lectures, Physics*.

[4] "Astro 103 - Lecture 6" *The Evolving Universe: Stars, Galaxies, and Cosmology* 15 Aug 2004 University of Wisconsin `http://www.astro.wisc.edu/~mab/education/astro103/lectures/l6/l6.html`.

[5] *Processing of Hyperspectral Imagery*. University of Texas. Center for Space Research `http://www.csr.utexas.edu/projects/rs/hrs/pics/irradiance.gif`.

[6] Phillips, Kenneth J.H. *Guide To the Sun*. Cambridge: Cambridge University Press, 1992.

[7] Mitalas, R; Sills, K.R. "On the photon diffusion time scale for the Sun." *Astronomy & Astrophysics*, v.421 issue 2, 2004, p. 755-762.

[8] Griffiths, David. *Introduction to Elementary Particles*. New Jersey: John Wiley & Sons, Inc, 1987.

[9] Griffiths, David. *Introduction to Quantum Mechanics*. New Jersey: Pearson Prentice Hall, 2005.

[10] Halliday, David, Robert Resnick, and Jearl Walker. *Fundamentals of Physics: seventh edition*. New York: John Wiley & Sons, Inc, 2004.

[11] Reif, F. *Fundamentals of statistical and thermal physics*. New York: McGraw-Hill Book Company, 1965.