Contraction of a Free Falling Liquid Jet

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This experiment tested the theory that as a liquid stream falls it contracts in response to gravitational acceleration. This was observed by analyzing digital photographs of a stream falling from a rectangular trough. The theory was found to accurately describe the shape of the falling stream. It was also found that viscosity, flow rate, and the width of the trough does not effect this gravitational contraction. It was also found that the initial contraction of the stream is caused by the convergence of streamlines, which may rely heavily on those three factors. This convergence can also have an overriding affect on the shape of the falling stream.

INTRODUCTION

When pouring a liquid from a container the width of the stream decreases as it gets further from the lip of the container.

Suppose we have a free falling stream of an incompressible liquid flowing with a constant flow rate Q such that the continuity equation holds:

$$Q = Av = const. \qquad (1)$$

where A is the cross-sectional area of the stream and v is the velocity with which the stream is moving. As the fluid falls, the cross section of the stream becomes circular in shape. The area A can then be written as

$$A = \pi R^2 = \frac{\pi w^2}{4} \qquad (2)$$

where w is the width of the stream. The velocity can be found using the conservation of energy $mgh = \frac{1}{2}mv^2$ where h is the distance down from the edge of the trough. The equation can be manipulated such that

$$gh = \frac{1}{2}v^2 \Rightarrow v = \sqrt{2gh} \qquad (3).$$

Substituting equations 2 and 3 into equation 1 yields and rearranging we find that

$$w^4 = \frac{8Q^2}{g\pi^2}\frac{1}{h} \Rightarrow w \propto h^{-1/4} \qquad (4).$$

In addition to the gravitational effect on the stream, the fluid dynamics of the system will also play a role in the shape of the contracted stream.[1] Fluid flow can be visualized using streamlines, which are everywhere tangent to the local velocity vector.[2] The effects of fluid dynamics on the stream contraction were observed but not investigated due to time constraints.

EXPERIMENT

The contraction of the falling stream was studied as it spilled over the edge of a rectangular trough. The contraction was further studied by varying the width of the trough, the flow rate, and the viscosity. The smaller trough had a width of 1.15 cm and the larger had a width of 1.91 cm. Rectangular troughs were used instead of circular in an effort to simplify the contraction effects caused by converging streamlines. The interior surfaces of the troughs were smooth in order to reduce as much disturbance in the fluid as possible.

The fluid used in the experiment was stored in a large glass carboy. The fluid flowed from the carboy into the reservoir via a rubber hose (see FIG. 1). As the reservoir filled, the fluid flowed through the trough and spilled over the edge to free-fall into a plastic catch bucket. The flow rate was regulated with a Whitey swagelock valve. The end of the hose was placed in the bottom of the reservoir to minimize the perturbation of the fluid as it entered the trough. To increase the flow rate though the system, the carboy was placed on a wooden block on top of a table.

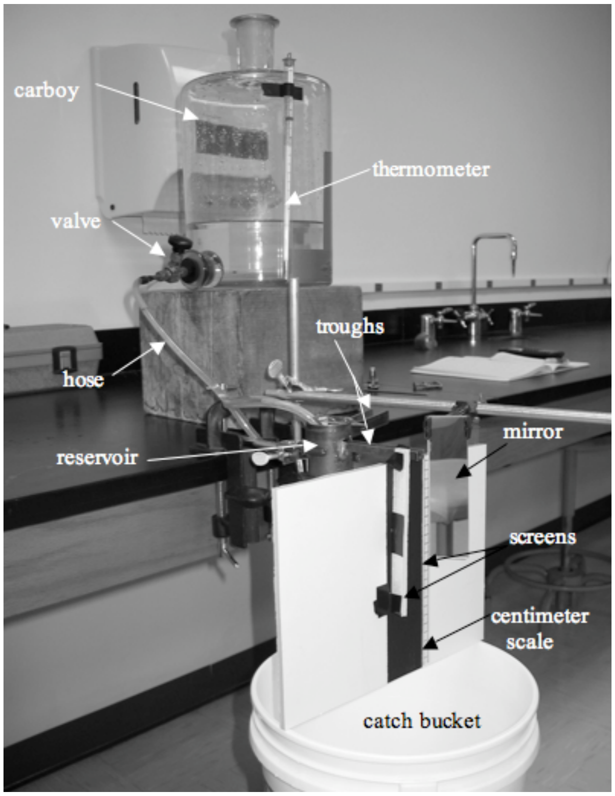

FIG. 1 – The setup used in the experiment.

To better observe the falling stream, a black plastic screen was placed just behind it. The bottom of the screen rested on the top of the catch bucket and its top was wedged under the trough. A piece of centimeter-scaled tape was placed along the length of the screen to provide a known scale that could be used when analyzing. A rectangular mirror was mounted at a 45° angle near the stream so that both a frontal and side view could be observed simultaneously. A narrow (~2cm) black screen was taped perpendicular to the main screen and across the stream from the mirror in order to provide a distinct background for that image as well.

A digital camera (Canon Power Shot G3, 4.0 Mega Pixels) was used to record still shots of the stream. The camera was placed on a tripod directly across the catch bucket from the apparatus. Multiple pictures were taken for each data set. After each data set the camera was moved to download the pictures and then replaced to approximately the same position. The pictures were downloaded onto a computer and analyzed using Scion Image 1.63 software, which enabled me to measure the width of the stream and the corresponding distance from the edge of the trough within ± 0.01cm. The table of these distances was imported into IgorPro (version 5.00 carbon) for further analysis. This process was repeated for each data set.

Three parameters were varied in the experiment: the width of the trough, the flow rate, and the viscosity of the fluid. The trough width was varied by rotating the reservoir such that the desired trough was above the observation area.

With the smaller trough, two flow rates were used. When using the larger trough, I removed the valve from the system and connected the hose directly to the stopper at the bottom of the carboy, giving the tubing system a larger diameter. This had a noticeable effect on the flow rate and actually caused the occurrence of a completely different stream shape, which shall be discussed later.

I initially used regular tap water to take measurements for my data sets. To increase the viscosity I added Karo syrup to the tap water. This was accomplished by putting 160mL of Karo syrup into a plastic gallon jug (3.79 L). The jug was then filled to the top with water. The mixture was shaken vigorously and then poured into the carboy. The carboy was swirled around to ensure that the water and Karo syrup were sufficiently mixed. After the data sets were recorded another 160mL of Karo syrup was added to the empty jug. It was then filled by siphoning the remaining fluid from the carboy and the catch bucket. The reservoir and all other hoses were drained before putting the new mixture into the system.

ANALYSIS AND INTERPRETATION

Water flowing from the 1cm trough with a low flow rate was considered the standard arrange-ment of variables for this experiment. A picture taken of the falling stream of water using the 1cm trough with a low flow rate (see FIG. 2), was imported into Scion Image and the stream width *w* was measured at incremental distances from the trough *h*. These data points were then plotted in IgorPro.

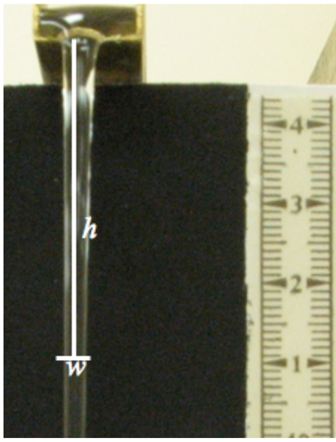

FIG. 2 – Example of a picture used to measure the contraction of a falling stream of water.

The power law function, of the form $y_o + Ax^{pow}$, fits the data fairly well (see FIG. 3). The sizes of the error bars, found to be ± 0.01cm, were determined by the limits in measurement of the software. The exponent was forced to be -0.25, making it consistent with the theory. Using the mirror mounted adjacent to the stream of water, I was able to obtain a frontal and side image of the stream simultaneously.

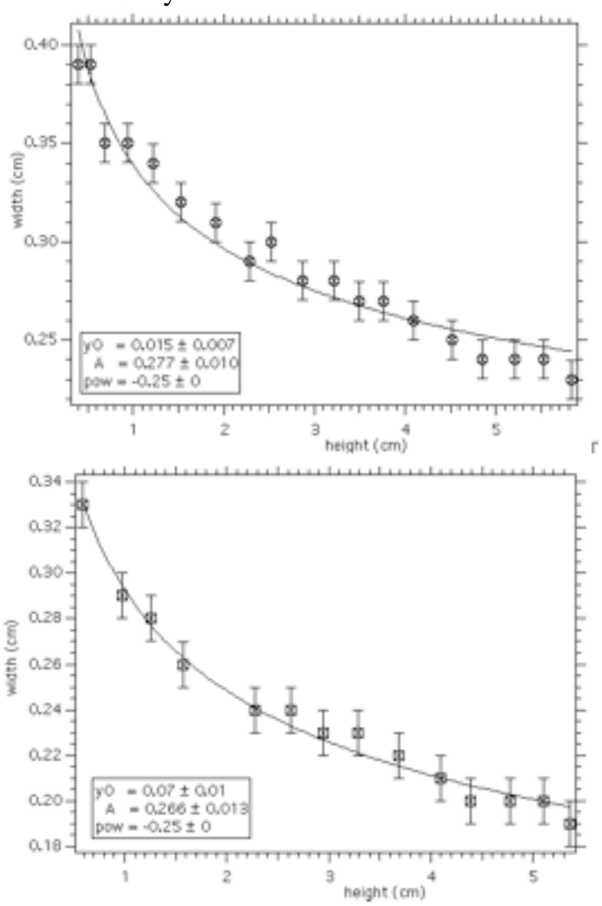

FIG.3 – Sample results using the 1cm trough. The frontal view on top and the side view is on the bottom.

The difference in image size between the frontal and side view is likely the cause of the different value for y_o in the two plots. This difference occurs because the image seen in the mirror is further from the camera than the actual stream. Thus its width relative to the scale on the stream will be different. The relationship of the width to the height in each plot should still be the same.

One of the parameters varied in the experiment was the flow rate. When it was varied using the 1 cm trough, there was no apparent change in the plot of width as a function of height. This could mean that either the flow rate does not affect this value or that the difference in the flow through the valve was not great enough to create a noticeable difference. It is interesting to note that the value for *A* is the same for not only these two plots but for all data sets taken from the 1cm trough.

The contraction of the falling stream was also studied as it left the wider, 2cm trough. The valve had to be opened completely in order to observe the stream. Similar analysis was performed on its contraction curve. It was interesting to find that the stream width from the 2cm trough was actually narrower than from 1cm trough. When the fluid reached the end of the trough the streamlines quickly converged, giving the stream a smaller initial width as it began to fall. Another possible explanation is that the screen may have been placed nearer to the stream when using the wider trough. This would have caused the centimeter-scale to appear larger relative to the stream width. The data from the wider trough, however, still supports the theory that $w \propto h^{-1/4}$. A difference between this set and the previous ones observed is that the value of the coefficient *A* was ~0.22 where *A* for the 1cm trough was ~0.27. This may be a result of the fact that the stream was initially narrower as it fell from the wider trough. The value of *A* is also different between the frontal and side views. This could mean that the stream was slightly flattened and not completely cylindrical.

When the flow rate through the trough was further increased by enlarging the diameter of the hose system another effect altogether was observed. The stream no longer had smooth, straight edges as seen in FIG. 2, rather the stream would quickly narrow and then flare back out (see FIG. 4). Morley et al[3] observed a similar effect in their study of rectangular liquid metal jets. They concluded that the contraction and expansion is caused when the stream follows its natural tendency to contract but overshoots and the sides invert. It is also possible that the increasing twist length l_t of the stream may be an effect of the gravitational acceleration. This phenomena is very interesting and would have been probed in greater detail if time had permitted.

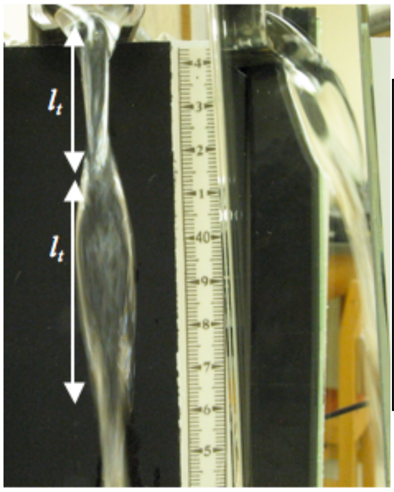

FIG. 4 – The frontal view is on the left and the side view is on the right. Notice that as the stream widens in one plane it contracts in the other.

The final variable was viscosity and its effect on the contraction of a falling stream. The 1cm trough was used at the lower flow rate for all data taken as the viscosity was varied by adding Karo syrup to the fluid. The most concentrated fluid mixture was approximately 25% Karo syrup by volume. The measurements of the stream width as a function of distance from the trough for this maximum dilution followed the same curve as before. Thus they are consistent with the previous results when there was no Karo syrup added.

This shows that the contraction of a falling fluid is not dependant on the viscosity of the liquid but is determined by the acceleration of the fluid as it falls, due to gravity.

By observing the stream in FIG. 2, it is apparent that the region of greatest contraction occurs at the instant that the stream leaves the trough. The time that it takes the fluid to fall that distance is too short for it to be caused by the effects of gravity. It appears that the initial, and greatest, contraction is actually caused by the converging of streamlines in the fluid. This also explains the effects observed in the 2cm trough when the flow rate was very high.

CONCLUSION

This experiment has shown that the contraction of a free falling stream of an incompressible fluid is caused by gravitational acceleration. The width w of the stream is related to the distance h that the fluid has fallen from the trough by $w \propto h^{-1/4}$. The viscosity of the fluid does not affect this kind of contraction. It was found that for high flow rates the falling stream exhibits a twisting shape, which may be caused by the convergence of streamlines upon exiting the trough.

ACKNOWLEDGMENTS

I thank Judy Elwell and Ron Tebbe for their assistance.

[1] Whitaker, S. *Introduction to Fluid Mechanics* (Prentice-Hall Inc, New Jersey 1968)
[2] *Encyclopedia of Physics*, 2nd Ed, edited by S.P. Parker (McGraw-Hill, New York 1993) pg 424-429
[3] A.I. Konkachbaev, N.B. Morley, K. Gulec, and T. Sketchley, Fusion Engineering and Design 51 – 52 (2000) 1109 – 1114