Using light scattering method to find the surface tension of water

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The goals of this experiment are to confirm the relationship between angular frequency ω and wave vector q and to determine the surface tension σ of water. By applying a known frequency on the surface of the water, cylindrical waves are formed. When a laser beam illuminates the surface of the water, the incidence rays are scattered and a diffraction pattern appears on the distant wall. By varying the frequency of the speaker, different values of the distance between bright spots, Δx are measured. According to the simplified dispersion relation for surface tension waves, $\omega \propto q^{1.5}$. The experimental relationship between ω and q in this experiment was found to be $\omega \propto q^{1.5\pm0.1}$. The surface tension of the water was found to be 88 ± 1 mN/m which was larger than the expected value of 72.8 mN/m by 20%.

INTRODUCTION

The surface of liquid seems complete smoothness if there is no external force acting upon it. It is due to the liquid molecules sticking up together so that it forms as a film and we would see a water strider moving on the surface of the water as if it is on ground. However, when a stone is throwing into the water, we see the waves that are formed because of the potential energy due to the surface deformation caused by the gravity and surface tension.[1] Potential energy is then turned into kinetic energy which can be catastrophic when the energy is huge as seen in the occurrences of tsunamis.

In this experiment, the energy formed from the waves was explained and the simplified dispersion equation was used to determine the relationship among the angular frequency, wavevector and surface tension.

By using Lagrange equations of an oscillator, the dispersion relations for waves on liquid surfaces are derived with gravity and surface tension as restoring mechanisms.[1] A wave formed on the surface of the liquid is due to the gravity and surface tension of the liquid. The vertical rise or the fall of liquid from the surface requires the energy with density ρ, acceleration due to gravity g, the surface tension σ and wavenumber q as important factors. Accounting for the kinetic energy results from the potential energy, the general dispersion relation for surface waves on liquids of any depth with both gravity and surface tension as restoring forces was derived by Klemens.[1]

The dispersion formula used in this experiment to determine the relationship between ω and q was derived by G. Weisbuch and F. Garbay[2] for $gq \langle\langle \sigma q^3/\rho$ as,

$$\omega^2 = (\sigma/\rho) q^3 \, , \qquad (1)$$

and q is expressed as,[2]

$$q = \frac{2\pi}{\lambda} \sin(\frac{r}{2}) \left[\sin(\theta - \frac{r}{2}) + \sin(\theta + \frac{r}{2}) \right] \qquad (2)$$

where λ is the wavelength of the light source, θ is the angle of incidence and r is the angle between the reflected ray and the first order diffraction spot.

Since the angle of incidence in this experiment is relatively small such that $\sin(\theta) \approx \theta$, eq.(2) can be simplified as:

$$q = 2\pi r\theta/\lambda, \qquad (3)$$

where the unit of θ and r are in radian.

EXPERIMENT

A Uniphase model 155 SL He-Ne laser with wavelength of 632.8 nm was used to emit a laser beam to the surface of the distilled water in a cylindrical container with a depth of 1.5cm and diameter of 10cm. The reflected beam appeared on the distant wall. A speaker was connected to Pasco PI 9587 function generator. Waves of the known frequency were created by a pointed, stiff wire connected with the speaker via a rod touched the surface of the water. The basic set-up of the apparatus was shown in Fig.1.

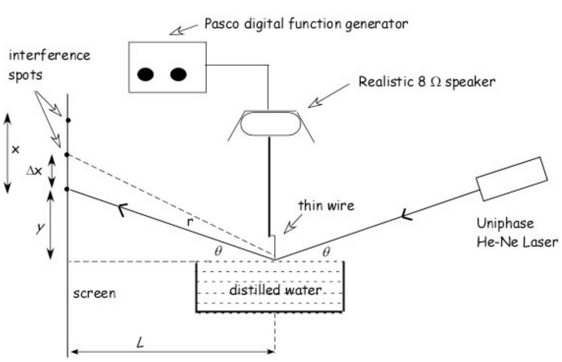

Fig.1. The schematic of the apparatus.

To determine the angle of incidence θ, the amplitude knob from the function generator was set so that only one bright spot appeared on the screen. The frequency of the function generator was set to 84 Hz. By varying the distance L, the height y, the distance between the bright spot and the surface of water, was measured and y versus L was plotted whose slope was the angle θ in radian for small angles. (Note: As the water is filled up to the top of the container, y can be measured from the top edge of the wall of the container).

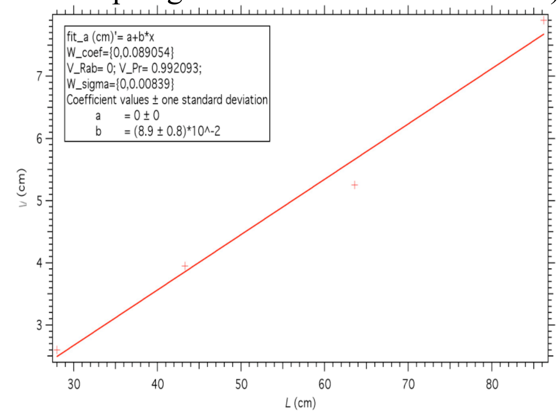

Fig.2. The plot of y versus L whose slope is the angle θ in radian.

To determine the wavevector q, the angle r first needed to be calculated. This time, the amplitude was adjusted with the particular frequency so that x was larger and the distance between bright spots Δx could be measured. The angle r was calculated using the equation: $\tan(r) = \Delta x/L$. Applying small angle approximation, the angle $r \approx \Delta x/L$.

The calculated angles r and θ were substituted into eq.(3) to yield the wavevector q. By plotting log(ω) versus log(q) using eq.(1), the relation $\omega \propto q^{1.5}$ can be verified using curve fit equation: $y = Ax^{pow}$, where $A = \sqrt{\sigma/\rho}$. Then, from the value of A, the surface tension of water can be calculated.

ANALYSIS AND INTERPRETATION

The plot of y versus L to determine the angle θ is shown in Fig.2. The angle θ is the slope of the line which is $(8.9 \pm 0.8)\times 10^{-2}$ radian.

By varying the frequency while the fixed distance L=3.86 m and measuring Δx for different frequency, the angle r and wavevector q were calculated. The uncertainty in the measurements of Δx was 0.1 cm and L was 0.01 m. The wavelength of the He-Ne laser was 632.8nm and the angle θ was $(8.9 \pm 0.8) \times 10^{-2}$ rad. The room temperature was measured to be about 21°C.

The plot of log(ω) versus log(q), with the line weighted fit to the data points, is shown in Fig.3.

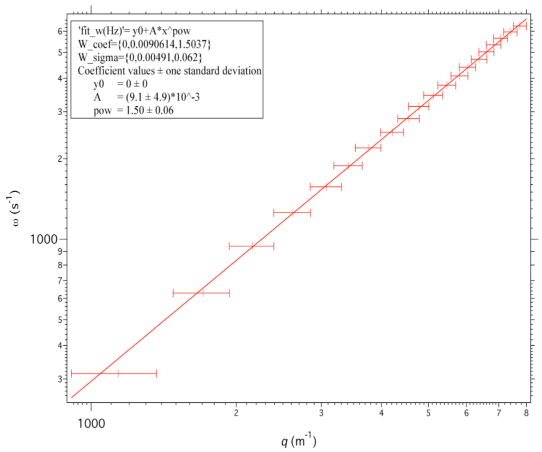

Fig.3. This plot shows the relationship: $\omega \propto q^{1.50 \pm 0.06}$.

The plot of log(ω) versus log(q) was curve fitted by the equation: $y = Ax^{pow}$ which confirms the theoretical relationship of $\omega \propto q^{1.5}$ with the percent error of less than 1% and percent uncertainty of 7%.

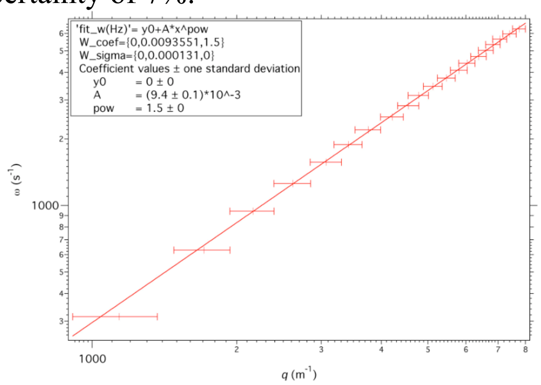

Fig.4. This plot shows the value of *A* with uncertainty when the power is fixed at 1.5.

The value A in Fig.4 was used in the equation: $A = \sqrt{\sigma/\rho}$ to get the surface tension of water σ=88 ± 1 mN/m using the value[3] of density of water at 20°C which was 998.2 kg/m^3. The percent error for the surface tension of water was within 21% from the accepted value[3] of 72.8 mN/m at 20°C. It might be due to the result of a large uncertainty in wavevector.

CONCLUSION

To conclude, this experiment showed the interaction between light and surface tension waves. Also, it gave insight into a light scattering method. The simplified dispersion equation is useful to conduct this kind of experiment rather than general dispersion equation.

The experimental relationship between angular frequency and wavevector was found to be $\omega \propto q^{1.50 \pm 0.06}$ which was within experimental error from the theoretical value of $\omega \propto q^{1.5}$. The surface tension at room temperature 21°C was found to be 88 ± 1 mN/m which was within the accepted value of 72.8 mN/m at 20°C by 20%.

ACKNOWLEDGMENTS

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[3]D. R. Lide, *CRC Handbook of Chemistry and Physics*. 86th ed. (Taylor & Francis Group, Boca Raton, 2005) p.6-2, 6-4