

Determining the Ratio C_p/C_v using Ruchardt's Method

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The ratio of heat capacity at constant pressure to the heat capacity at constant volume was measured using the Ruchardt method. Values were taken for air, Helium, and Nitrogen, and were found to be 1.31 ± 0.01 , 0.46 ± 0.08 , and 0.617 ± 0.088 respectively. These values for Helium and Nitrogen disagree with the theoretical values, 1.667 and 1.4, respectively.

INTRODUCTION

The ratio of heat capacity at constant pressure to heat capacity at constant volume is a value that occurs frequently in thermodynamics. Heat capacity is a proportionality constant between the amount of heat added to a system, and its subsequent change in temperature. This constant can be measured in an isobaric or an isochoric process, to find heat capacity at constant pressure and volume, respectively. In this experiment, this ratio between these two heat capacities, γ , is measured using an essentially mechanical technique. A metal piston, placed between two columns of the same gas, is made to oscillate using a magnetic coil. By changing the frequency of oscillation of the piston using a function generator, one can search for the resonant frequency of the piston. This resonant frequency is the frequency at which the piston vibrates fastest inside of the glass tube. This very elegant technique was developed by Ruchardt. Ruchardt had originally placed the gas columns vertically, with a large reservoir of the gas being studied. In this experiment, the gas columns are oriented horizontally, to minimize any affect due to gravity. After measuring the resonant frequency, finding γ involves a simple calculation.

THEORY

Before we begin to analyze the mechanics of the Ruchardt apparatus, it is useful to see the underlying thermodynamics of the system. Using the definition of chemical potential¹ for an ideal monatomic gas, we find the familiar *Sackur-Tetrode* equation:

$$= N \log \frac{n_Q}{n} + \frac{5}{2} \quad (1)$$

$n_Q = \frac{2 M^{3/2}}{h^3}$ is the quantum concentration of the gas, and $n = \frac{N}{V}$ is the concentration of the gas.

Now that we have an expression for the entropy of an ideal, monatomic gas, we can find the heat capacity at constant volume. Since $C_v = \left(\frac{\partial U}{\partial T} \right)_v$, we can apply this to the Sackur-Tetrode equation to find that

$$C_v = N \log \frac{n_Q}{n} + \frac{5}{2} = \frac{3}{2} N \quad (2)$$

The heat capacity at constant pressure is defined in terms of the heat capacity at constant volume, $C_p = C_v + N$. This is found to be $\frac{5}{2} N$. Now that we know these two values, we can find γ , the ratio between the heat capacities:

$$\frac{C_p}{C_v} = \frac{5}{3} \quad (3)$$

For diatomic molecules, the development of γ is the same. The only difference is the number of degrees of freedom in the molecule. In the case of a monatomic molecule, there are three degrees of freedom. For diatomic molecules, there are 5 degrees of freedom, taking into account the 2 rotation degrees. It can be shown that, for a diatomic gas, $C_p = \frac{7}{2} N$, $C_v = \frac{5}{2} N$, and $\gamma = \frac{7}{5}$.

To determine γ for the given gas in the Ruchardt apparatus, we must apply Newton's second law to the vibrating mass inside of the glass tube. The force in this case can be defined in terms of the cross-sectional area of the glass tube, as well as a

small change in the pressure of the gas, dP . From Newton's second law,

$$\ddot{x} - 2 \frac{A(dP)}{m} = 0 \quad (4)$$

To find dP , we use the fact that for an adiabatic process² $PV = \text{const}$. This gives us dP in terms of dV . Then we can use the fact that³ $A/V=1/L$, and $dV=Ax$, so the final equation of motion becomes

$$\ddot{x} + \frac{2PA}{mL}x = 0 \quad (5)$$

This is the familiar equation of motion for a simple harmonic oscillator, with frequency

$$f_0 = \frac{\sqrt{2PA}}{mL} = (2f)^2$$

the linear frequency of oscillation of the metal mass inside of the glass tube,

$$= \frac{4f^2 mL}{2PA} \quad (6)$$

EXPERIMENT

The Ruchardt apparatus that is used is shown in Figure 1.

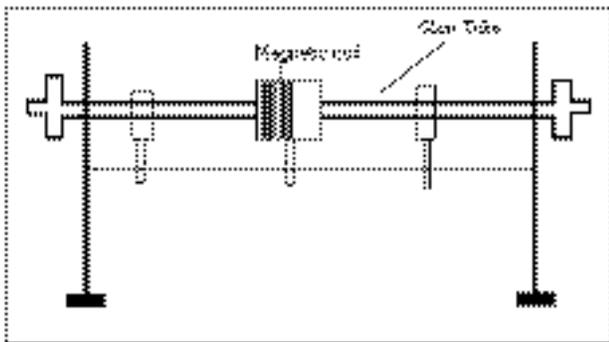


Figure 1. Ruchardt apparatus. Note that the magnetic coil is centered on the glass tube to equally split the tube.

There is an iron mass inside of the glass tube that vibrates in presence of a magnetic field. The metal mass fits snugly inside of the glass tube, to minimize leakage of gas from one end of the tube to the other. A Tektronix function generator, model CFG253, is wired into an Optimus car power amplifier, which in turn sends the signal to the magnetic coil and the HP oscilloscope, model 54501A. The amplifier is wired simultaneously to an oscilloscope and a variable power resistor, used to control the amount of voltage that enters the coil.

A vacuum is applied to the Ruchardt apparatus before each run, to assure that only the

gas being tested is present inside the glass tube. The power applied to the coil causes the metal mass inside of the tube to vibrate. At a certain frequency, one can hear the resonant frequency by the fast vibration and loud noise coming from the metal mass. The value of this frequency is determined using the oscilloscope. Using the time markers on the oscilloscope, one can place these on successive peaks, and find the frequency of oscillation of the metal mass. The length of each gas column is measured using a ruler, and this data collection process is repeated to garner more data.

ANALYSIS AND INTERPRETATION

Twenty-three runs of data were taken; though only 16 were used in the analysis of the data. Runs one through three, and sixteen represent air inside of the glass tube. Runs four through eight and twelve through fifteen were taken with Helium in the tube. Finally, runs twenty-one through twenty-three were taken with Nitrogen in the tube. As for the other variables in the equation to find gamma, m is the mass of the piece of metal inside of the tube, in kilograms, l is the length of the column of air, in meters. l was measured from the end of the glass tube to the end of the metal piston, for both sides. Since the average difference in the lengths of each end of the glass tube was usually one to two millimeters, the average length was calculated for each run. P is the pressure of the gas inside of the tube. To find this pressure, one needs to know the atmospheric pressure, measured from a barometer, and the gauge pressure of the gas tank. The error in measurements for the barometer readings took into account the parallax. Error in the gauge measurement was also taken into account, estimated at around ± 2 KPa. The pressure P can be found by $P = P_{\text{atm}} + P_{\text{gauge}}$. A is the cross-sectional area of the inside of the glass tube.

Table 1 gives the average values used in the analysis of gamma. Note that the pressure for air is just the atmospheric pressure, while Helium and Nitrogen includes terms for the gauge pressure.

Table 1. Values of pressure, mass, length, diameter, and frequency that are used in the analysis of gamma. Errors in pressure for the gases took into account the error in barometric and gauge readings.

	value	std. dev.
m (g)	8.8392	0.0003
L (cm)	25.28	0.04
P air (N/m ²)	9.75E+04	4.0E+02
P He (N/m ²)	1.44E+05	2.4E+04
P N ₂ (N/m ²)	1.70E+05	2.4E+04
d (cm)	1.405	0.006
f air (Hz)	21.3	0.15
f He (Hz)	14.93	0.36
f N ₂ (Hz)	18.84	0.08
P gauge He (N/m ²)	4.70E+04	2.0E+03
P gauge N ₂ (N/m ²)	7.20E+04	2.0E+03

The errors in the frequencies are the standard deviations of several runs, the ones listed earlier in this section. Error in gamma was propagated using a fractional error approach for equation (10). Using this approach, we arrive at a value of $\gamma = 1.31 \pm 0.01$. This value makes sense theoretically, since the air around us is made up of mostly diatomic gases, and theoretically gamma for a diatomic gas is 1.4. Table 2 gives the values for gamma found in this experiment.

Table 2. Experimental values of γ for air, Helium, and Nitrogen.

	value	std. dev.
air	1.31	0.01
Helium	0.46	0.08
Nitrogen	0.62	0.09

The experimental values of gamma for Helium and Nitrogen do not agree with theoretical values. For a monatomic gas, such as helium, $\gamma = 5/3 = 1.667$. The experimental value found in this experiment is not within the error bounds dictated by the standard deviation. A possible explanation for this discrepancy is the pressure inside of the tube. The pressure was corrected for the gauge pressure of the regulator on the Helium tank, which is around 4.7×10^3 N/m². If the gauge that this pressure was read from is not correct, this would definitely influence the value for gamma. To get a gamma for Helium of around 1.667, the pressure would have to be less than the value that has been used in the analysis. Assuming the pressure that was used is correct, the resonant frequency would have to be higher.

For Nitrogen, the experimental gamma found was 0.617 ± 0.088 . This differs from the

theoretical value of gamma for a diatomic gas, $\gamma = 7/5 = 1.4$. This experimental value is also not within error bounds of the theoretical value. The most likely explanation for this difference is also the pressure inside of the glass tube.

To analyze this discrepancy further, the ratio between gamma for a monatomic gas gamma for a diatomic gas was taken. Normally, this ratio would only depend on the respective frequencies of Nitrogen and Helium, except that the pressures for each of the gases were different also. The ratio of the two gamma's turns out to be

$$\frac{\gamma_{He}}{\gamma_{N_2}} = \frac{f_{He}^2}{f_{N_2}^2} \frac{P_{N_2}}{P_{He}} \quad (7)$$

The ratio of the experimental values turns out to be 0.637, while the theoretical value is around 1.19. A value of 1.19 means that the frequency for Helium should be greater than the frequency of Nitrogen. In the experiment, the values of the frequency of Helium were consistently smaller than that of Nitrogen. Here, the pressure can not be a reason for this difference, since it is accounted for in the ratio equation. It was postulated that the resonant frequency that was being picked up was actually a lower harmonic of the actual resonant frequency. The higher harmonic was searched for, but nothing was found, to verify this suspicion. No other possible explanation has been thought of for this effect.

CONCLUSION

Gamma for air, Helium, and Nitrogen were measured using the Ruchardt apparatus. The experimental value of gamma for air agreed with theoretical values, since air is mostly diatomic molecules. Major discrepancies existed in the values for Helium and Nitrogen, which may be partly explained by the inexact measurement of pressure inside of the glass tube. Looking at the ratio between gamma for helium and nitrogen, this value also disagreed with the expected value of 1.19. Future work may include analyzing why Helium gave such low resonant frequencies. Also, a better way to measure the pressure inside of the tube should be found.

¹C. Kittel and H. Kroemer, *Thermal Physics*, 2nd ed. (Freeman, New York, 1980), p. 162-167.

²S. Fujita, *Statistical and Thermal Physics*, (Robert Krieger, Malabar FL, 1986), p. 315.

³*Junior Independent Study Lab Manual*, The College of Wooster, 1998, p.47 (unpublished).