Verification of the Compton Effect

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The primary goal of this experiment was to confirm the characteristics of the Compton effect. This effect states that when electromagnetic radiation interacts with a free electron, the shift in wavelength of the scattered radiation is independent of the intensity of the incident radiation and the time of exposure to the radiation, and dependent only upon the scattering angle. This experiment examined only the 180^{O} scattering angle for ^{137}Cs and ^{22}Na radioactive sources. The backscatter energy peaks for ^{137}Cs and ^{22}Na were found to be 0.196 ± 0.026 MeV and 0.190 ± 0.044 MeV respectively. The ^{137}Cs peak had a 6.5% error while the ^{22}Na peak had a 10% error. The digital method of isolating the backscatter peak proved to be more accurate and precise than the analog method.

INTRODUCTION

Heinrich Hertz's production and discovery of electromagnetic waves in 1887 simultaneously led to the particle description of light. Hertz demonstrated the photoelectric effect, the phenomenon that occurs when light incident on the metal begins to eject surface electrons from the metals. Millikan and Einstein went on to discover that there is a minimum, or threshold, frequency of incident radiation for which the photoelectric effect is unobservable. Also, no time lag ever occurs because the energy of a single photon is enough to eject an electron, a concept previously unheard of.

In 1900, J.J. Thomson developed the classical theory describing the scattering of electromagnetic radiation by electrons in matter.[1] According to Thomson's hypotheses[2], the incident radiation of frequency f_O should accelerate an electron in the direction of propagation of the incident radiation, and the electron should undergo forced oscillations and re-radiation at frequency f, where $f \leq f_O$. In the classical theory it was also thought that the frequency of the scattered radiation should depend upon the length of time the electron was exposed to the incident radiation as well as the intensity of the incident radiation.[3]

Arthur Holly Compton surprisingly discovered that the wavelength shift of x-rays scattered at a given angle is independent of both the intensity of the incident radiation and the length of exposure to the incident radiation, and depends only upon the scattering angle.[1] Compton bounced x-rays off of a graphite target using three different scattering angles; 45^{O}, 90^{O}, and 135^{O}. The wavelength was measured with a rotating crystal spectrometer, and the intensity was determined by an ionization chamber that generated a current proportional to the x-ray intensity.[3] Compton's discovery was quite possibly one of the most important in modern physics and it ultimately gave rise to the wave-particle duality theory that still prevails today.

THEORY

Consider an incident photon colliding with a free electron at rest. The energy of the incident photon can be described by the equation $E_0 = h\nu_0$, with momentum $p_O = \frac{h}{\lambda_0}$. After the incident photon collides with the rest electron in the material, the photon causes the electron to recoil, sending it off with a velocity *v*. The photon is deflected away from the electron at an angle θ, with energy $E = h\nu$ and momentum $p = \frac{h}{\lambda}$.

Energy is conserved in the collision, and the resulting conservation equation looks like

$$E_O = E + \frac{p_e^{\,2}}{2m_e} \tag{1}$$

Momentum is also conserved throughout the collision, but the momentum of the electron after the collision is not immediately known.

The law of cosines relates the momenta magnitudes,

$$p_e^{\,2} = p^2 + p_0 - 2pp_0\cos\theta \qquad (2)$$

Substituting hc/λ for E, and hc/λ_0 for E_0 in Eqn. 1, then solving for $p_e^{\,2}$ and substituting into Eqn. 2 gives

$$p_e^{\,2} = 2m_ec\left(p_0 - p\right) \qquad (3)$$

Since Eqn. 2 and Eqn. 3 are both equivalent to $p_e^{\,2}$, it is possible to equate these expressions:

$$p^2 + p_0 - 2pp_0\cos\theta = 2m_ec\left(p_0 - p\right) \qquad (4)$$

Substituting $p = h/\lambda$ and further simplifying leads to the equation that describes the shift in wavelength that the incident photon undergoes, also known as *Compton's Equation* (Eqn. 5):

$$\lambda - \lambda_0 = \frac{h}{m_ec}\left(1 - \cos\theta\right) \qquad (5)$$

The change in wavelength is thus predicted to be independent of the incident energy of the photon, and dependent only upon the scattering angle.

The quantity h/m_ec is called *Compton's Wavelength of the Electron* and is known to be 0.00243 nm.

Due to Eqn. 5, it can be assumed that *Compton's Shift* in wavelength is only observable when λ is very small, because the value of $(\lambda - \lambda_0)/\lambda$ must be appreciable. For this reason the Compton effect is generally observed for x-rays and gamma radiation.

In order to derive the equation for the energy E of the photon after the collision, θ is set to 180°, the angle appropriate for this particular experiment. Substituting for λ and λ_0, then solving for E gives:

$$E = \frac{E_0}{1 + \dfrac{2E_0}{mc^2}} \qquad (6)$$

The shift in energy of the photon after the collision is found by simply subtracting E from E_0. The resulting equation is:

$$E - E_0 = E_0\left(\frac{2E_0}{mc^2 + 2E_0}\right) \qquad (7)$$

EXPERIMENT

An aluminum block was set at an 180° angle from the edge of the NaI scintillating crystal. Two different sources were used for different data collection runs; Cesium, ^{137}Cs, and Sodium, ^{22}Na. Some gamma events were incident on the NaI crystal directly from the source, while others were first deflected off of the Al block at an 180° angle. A Bicron photomultiplier tube surrounded by a thick lead collimator translated the photoemissions of the crystal into current pulses. The pulses were then sent to a Nucleus 800 multi-channel pulse height analyzer (MCA) connected to a video monitor. In order to digitally analyze the data collected by the MCA, a serial cable was connected from the MCA to a computer, where LabVIEW was used to collect the data.

The MCA display used for this experiment was only capable of displaying the channel number (1 to 256) where a gamma count was detected. These channels actually correspond to energy values, but the display leaves it up to the user to determine the conversion equation necessary to extract the energy data. In order to develop this equation, the incident energy peaks were analyzed for both ^{137}Cs and ^{22}Na, noting the respective channels for the peaks (see Table 1).

	Energy Peak	**Observed Channel #**
For Cs	32 keV, 0.6612 MeV	19, 234
For Na	0.511 MeV	185

Table 1. Three data points used to determine the conversion equation from channel # to energy in MeV.

A linear equation was fit to the three data points from Table 1 results in the equation:

$$\text{Energy} = (-0.024 \pm 0.006) + (0.00292 \pm 0.0003)\times(Ch) \qquad (8)$$

where Ch represents the channel number from the MCA display and E is in MeV. This equation was used on all of the data sets to more explicitly show where the peaks occurred in terms of their energy.

In order to determine the peak of the 180° backscatter energy for ^{22}Na and ^{137}Cs, two different methods were employed. The first method, analog as it were, was used to determine the backscatter for ^{22}Na. The analog MCA worked in this manner: A "data addition" function allowed for the counting of gamma events in different channels corresponding to certain energy values. A "data subtraction" function then allowed the user to subtract counts from a channel whenever a gamma event corresponding to that energy was detected.

With the Al block in place and the addition function turned on, the MCA collected incident as well as backscattered counts. Then the Al block was removed, being careful not to move the source, and the "data subtract" function on the MCA was used to eliminate the incident radiation's energy peaks. The resulting peaks on the display were then mostly due to backscatter. A plot in Fig. 3 of the resulting counts for ^{22}Na gave evidence to a backscatter peak at 0.190 ± 0.044 MeV, a value very close to the predicted peak of 0.170 MeV.

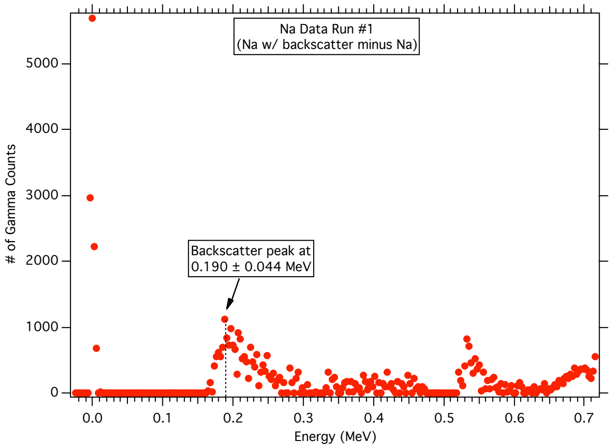

FIG. 3. ^{22}Na plot with 'incident' radiation subtracted from 'backscatter + incident' radiation. The peak determined from the 180^0 backscattered gamma events is labeled.

The other method used to determine the backscatter peaks was done digitally using Igor Pro Version 4.05 Carbon. To isolate the backscatter energy due to the ^{137}Cs source, two separate data collection runs were made that both used the "data addition" function on the MCA for 225 seconds. Fig. 4 shows the first data run that was made using an Al block to create scatter of the gamma events.

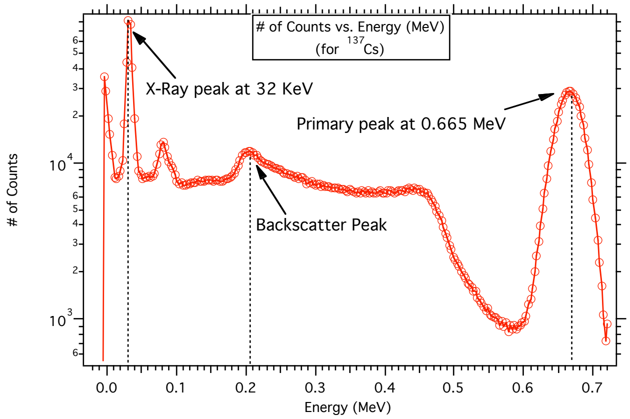

FIG. 4. Plot of ^{137}Cs with Al present to create backscatter.

Fig. 4 shows a plot of the second run, which had no Al present, thus allowing the MCA to detect mostly incident radiation. Random scattered radiation may have also been collected, but previous versions of this experiment have determined[4] that extra Pb shielding around the photomultiplier tube yields no better results, so the process was not attempted.

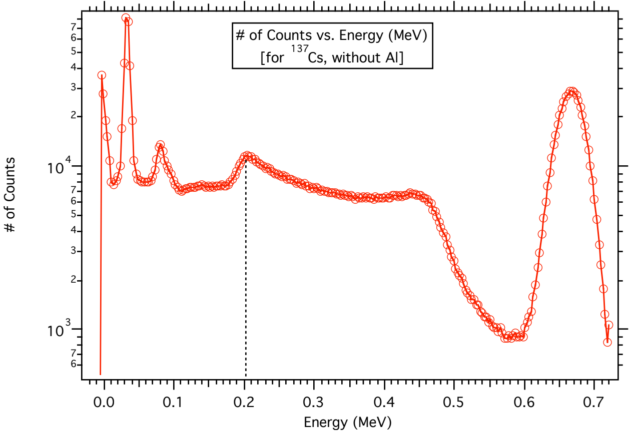

FIG. 5. ^{137}Cs without Al for backscattering. Notice the slightly less prominent peak near 0.205 MeV.

The data run with only the incident radiation (Fig. 5) was then subtracted digitally from the data run with backscatter (Fig. 4). Fig. 6 shows the plot of this subtraction technique.

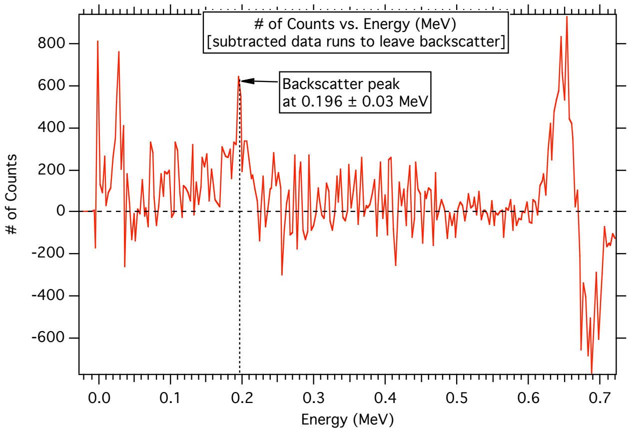

FIG. 6. The plot of the digitally subtracted data runs for ^{137}Cs.

As one can see the backscatter peak is very clear and it appears at 0.196 ± 0.026 MeV, which is very close to the expected peak. The negative energy values that appear in Fig. 10 might be due to a slight shift in the gain of the amplifier within the MCA between the two data sets used

ANALYSIS AND INTERPRETATION

When identifying the channel in which the peak energy lies, a degree of error was present in the measurement. The digital method was more precise with a $\Delta Ch_{Cs} = 4$ channels, while the analog method yielded a $\Delta Ch_{Na} = 12$ channels.

Error was calculated by taking the square root of the sums of squares of the partial derivatives of the energy equation. The errors in the peaks for ^{22}Na and ^{137}Cs were found to be 0.044 MeV and 0.026 MeV respectively.

Using Eqn. 6, the theoretical backscatter energy peaks can be mathematically determined. For ^{22}Na the incident energy E_0 is known to be 0.511 MeV, so the energy E_{Na} of the scattered photon is found to be 0.170 MeV. The value determined experimentally for E_{Na} was 0.190 ± 0.044 MeV which yields an experimental error of 10%, extremely high for an experiment of this sort. This speaks to the very low accuracy of the analog method used to isolate the backscatter energy from the incident energy. For ^{137}Cs the incident energy is known to be 0.6612 MeV and from this the theoretical value for E_{Cs} is determined to be 0.184 MeV. Experimentally, the value for E_{Cs} was found to be 0.196 ± 0.026 MeV, with a relative error of 6.52%. This is relatively error free when compared to the error found in the E_{Na} value.

CONCLUSION

In examination of the 180^{O} scattering angle for ^{137}Cs and ^{22}Na radioactive sources, the backscatter energy peaks were found to be 0.196 ± 0.026 MeV and 0.190 ± 0.044 MeV respectively. The ^{137}Cs peak had a 6.5% error while the ^{22}Na peak had a 10% error. The error found in the experimental value for E_{Na} is certainly systematic and can be attributed to a limitation in the MCA used to isolate the backscatter energy. The digital method of measuring the energy should be used in future runs of this experiment due to its superior accuracy in comparison to the analog method. This experiment verified Compton's theory that photons do indeed possess energy and momentum.

[1] R.A. Llewellyn & P.A. Tipler, Modern Physics, 3rd ed, (W.H. Freeman and Company, New York, 2000), pp. 146-149.

[2] R.A. Serway, C.J. Moses, & C.A. Moyer, Modern Physics, (Saunders College Publishing, Philadelphia, 1989).

[3] McGraw-Hill Encyclopedia of Science & Technology, 6th ed. 4, CLI-CY. (McGraw-Hill Book Company, New York, 1987).