

A Study of the Motion of a Free Falling Shuttlecock

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In order to conclusively determine whether the resistive force on a feather shuttlecock in free fall is proportional to the velocity or the velocity squared, more data needs to be collected and analyzed after the shuttle has fallen larger distances. From the current data, the velocity of the shuttle after falling approximately two meters has been calculated. Data acquired for the shuttle after falling three meters should be sufficient to distinguish between a resistive force proportional to velocity or velocity squared.

INTRODUCTION and THEORY

Aristotle, (384-322B.C.) was one of the first men to publish his views regarding laws of motion and the fall of bodies. He felt that all bodies have a natural place in the universe, and that every body has an inherent tendency to move toward this natural position. The velocity of an object is dependent on only two parameters; the character of the medium through which it is traveling and the "inherent impetus", which is the lightness or heaviness of the object.¹ Therefore, the natural motion is described by $s = F/R$, where F is the impetus due to the weight or lightness of the body and R is the resistance due to the density and viscosity of the medium. According to this doctrine, "an iron ball of one hundred pounds falling from a height one hundred cubits reaches the ground before a one pound ball has fallen a single cubit".¹ In a vacuum, since the resistance of the medium goes to zero, all falling bodies would fall instantaneously.²

Very few people voiced any opposition to Aristotle's views until the time of Galileo. "In 1591, Galileo, repeating an experiment of Stevinus, dropped a ten-pound weight and a one-pound weight together from the top of the leaning tower of Pisa, and showed the incredulous onlookers that, heavy or light, they struck the ground simultaneously."³ Although this legendary account may be nothing more

than a myth, Galileo believed that the motion of objects should be explicable in mathematical terms, and attempted to discover how, rather than why, things moved. He introduced mathematical form to the old concepts of space and time, allowing him to solve his dynamical problem.⁴ Through experimentation he found that a body moving down an inclined plane acquired the same velocity as one dropped from the same vertical height. He determined that velocity varied with time, and that the vertical distance fallen increases with the square of time. Galileo introduced the concept of acceleration and asserted that the acceleration of falling bodies is constant. He also asserted that it was not motion, but a change in velocity or direction that required the action of a force. "Galileo's great discovery that terrestrial movement can be described in mathematical terms opened the tremendous advances in science of the Newtonian epoch".⁴

Newton supported the findings of Galileo and introduced three laws of motion which extended Galileo's ideas. The first law claims every body continues its state of rest or uniform motion in a straight line unless it is compelled to change this state by forces acting on it. The second law reads; the effect of a force F on the motion of a body of mass m is given by the relation $\vec{F} = \frac{d\vec{p}}{dt}$, where \vec{p} describes the momentum of the object. This law is commonly known for the special case in the form, $\vec{F} = m\vec{a}$,

where \bar{a} is the acceleration and the mass is constant. The third and final law states, every body exerting a force on another, experiences a force exerted by the second body equal in magnitude and in the opposite direction.

When objects fall through the atmosphere, the velocity is slowed as a result of air resistance. It is generally accepted that for a free falling object, the resistive force resulting from air resistance is proportional to either the first or second power of the instantaneous velocity.⁵

A badminton shuttlecock is a good object to use in order to study the effects of air resistance. It has a small mass, hence a small gravitational force, but a large area, which will result in a large resistive force. These two factors will allow position and velocity measurements to be made using standard laboratory equipment. It is hopeful that analysis of the data will provide conclusive evidence of the terminal velocity for the shuttlecock as well as the nature of the resistive force.

If the resistive force in the motion of the shuttle is proportional to the velocity, ($F_{res} = kmv$) the position of the shuttle will be described by the equation $x(t) = v_T \left[t - \frac{v_T}{g} (1 - e^{-gt/v_T}) \right]$, where v_T is the terminal velocity and g is the acceleration due to gravity. The position as a function of velocity is

$$x(v) = -\frac{v \cdot v_T}{g} - \frac{v_T^2}{g} \ln \left[g \left(1 - \frac{v}{v_T} \right) \right] + \frac{v_T^2}{g} \ln(g).$$

If on the other hand, the force due to air resistance is proportional to the second power of velocity ($F_{res} = kmv^2$) the equation of motion reads;

$$x(t) = \frac{v_T^2}{g} \ln \left[\cosh \left(\frac{gt}{v_T} \right) \right],$$

with the position related to the velocity according to the equation,

$$x(v) = -\frac{v_T^2}{2g} \ln \left[g \left(1 - \left(\frac{v}{v_T} \right)^2 \right) \right] + \frac{v_T^2}{2g} \ln(g).$$

EXPERIMENT

In taking measurements of the motion of the shuttle, two different setups were used. In the first setup, a Pasco motion sensor was used to measure the distance of the

(5025.2 ± 0.2) $\times 10^{-6}$ kg feather shuttle from the ground as it was dropped from rest at a variety of heights. The equipment was unable to measure distances above 0.7 meters, therefore this method was not very useful in analyzing the data.

The second setup used to collect data involved recording the motion of the falling shuttle with a high-speed camera, as seen below in Figure 1.

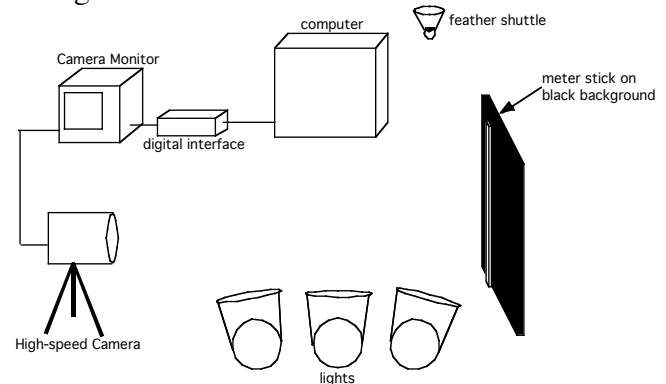


Figure 1: High-speed Camera Setup

The high-speed camera is positioned to view approximately 0.4 meters of a meter stick. The meter stick is set against a black background so that when the white shuttle passes by it is clearly visible. Although it would be nice to have a larger view of the falling shuttle, when the camera is positioned farther away, the objects become extremely fuzzy. The camera records 250 frames per second, and the recorded data is viewed using the camera monitor. The data recorded on the high-speed camera monitor is imported into iMovie and edited before being transferred to VideoPoint for analysis.

In VideoPoint, a scale can be set (otherwise all distances are given in pixels). The picture below is an example of what is seen on the VideoPoint screen. The scale is set by selecting two points and assigning a known distance between the two of them. In the analysis, the distance of 0.3 meters was used to set the scale. The center of the black band on the shuttle is used for all frames to determine the position of the shuttle. In the first frame, an origin (as well as point S1) are set to the middle of this band. In the following frames, the origin

does not move, but point S1 follows the motion of the shuttle. The data is then exported into Excel.

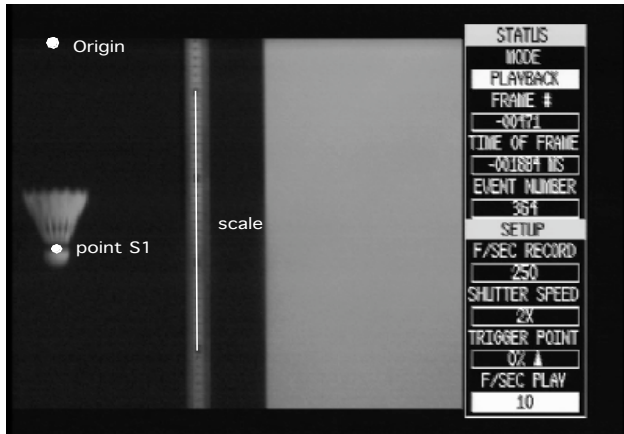


Figure 2: A clip from VideoPoint

In Excel, the only data necessary is the distance from points S1 to the origin (in meters) with respect to time. It was also found that the time recorded using VideoPoint did not coincide with the correct time recorded from the high-speed camera. The time increments needed to be divided by 25 to obtain the correct time scale. Also in VideoPoint, of 3 successive frames, only 1 displayed any motion, so the 2 extra were unnecessary and deleted. Once these corrections were made, the data was analyzed using IgorPro.

RESULTS

Three separate trials were captured using the high-speed camera. In the first, the shuttle began from rest while in the view of the camera. The second fell 1.33 meters before being recorded by the camera, and the third fell 1.88 meters before being recorded. The data sets are shown below.

From the calculated value for the slope in Figure 5, the terminal velocity must be at least $5.27 \frac{m}{s}$. Although the data sets provide some insight towards a value for terminal velocity, they provide no conclusive evidence of a value or the nature of the resistive force.

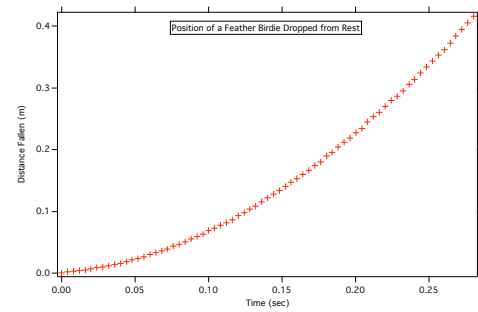


Figure 3: Shuttle dropped from rest

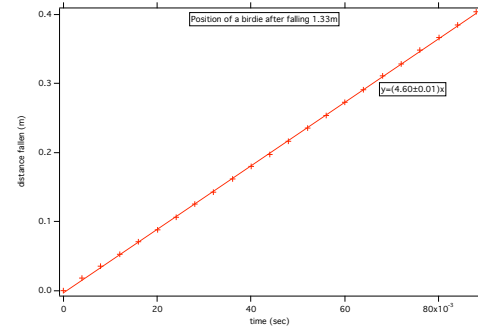


Figure 4: Shuttle after 1.33m

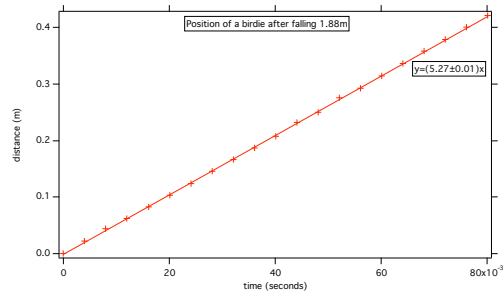


Figure 5: Shuttle after 1.88 m

Graphing the three sets of data on the same time axis would be extremely helpful for further analysis. Unfortunately, this is not possible since there was no way to measure or calculate the time required for the shuttle to fall 1.33 and 1.88 meters. Therefore, the velocity at each time was calculated for all three data sets using Excel. We know that $v = \frac{\Delta x}{\Delta t}$, therefore, to calculate the velocity, the position at the time previous was subtracted from the position at one time increment after, and then divided by the change in time. As a result of capturing 250 frames per second using the high-speed camera, two successive frames are separated by $\frac{1 \text{ frame}}{250 \text{ frames/sec}} = .004 \text{ sec}$. In calculating the velocity, there is a two frame separation, hence $\Delta t = 0.008 \text{ sec}$.

The first thought was to graph velocity as a function of the distance fallen and fit the data using Igor. If the resistive force is proportional to the velocity squared then, $v(x) = \sqrt{-v_T^2 e^{(-2xg/v_T^2)} + v_T^2}$. But, an explicit function cannot be calculated for the velocity as a function of position if the resistive force is proportional to the first power of the velocity. Therefore, the two resistive forces cannot be compared and a new method is required to analyze the data.

If instead the data is graphed as distance versus velocity, explicit equations for both a resistive force proportional to the velocity squared and also a resistive force proportional to the velocity can be calculated. The data was fit to the two equations $x(v) = -\frac{v_T^2}{2g} \ln\left[g\left(1 - \left(\frac{v}{v_T}\right)^2\right)\right] + \frac{v_T^2}{2g} \ln(g)$, for force proportional to the second power of velocity, and $x(v) = -\frac{v \cdot v_T}{g} - \frac{v_T^2}{g} \ln\left[g\left(1 - \frac{v}{v_T}\right)\right] + \frac{v_T^2}{g} \ln(g)$, for force proportional to the velocity.

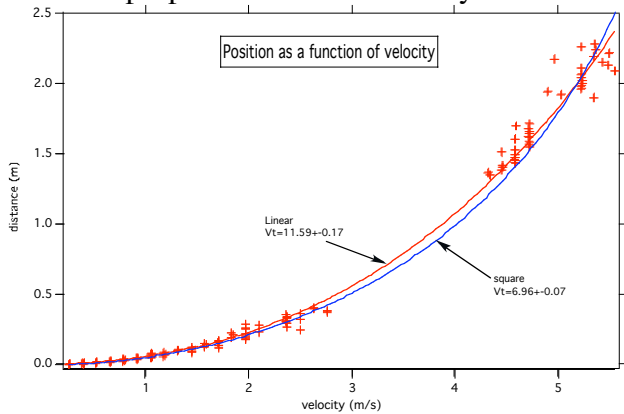


Figure 6: Position vs. Velocity for a shuttle

Both the fits describe the data well. It is not possible to determine whether a resistive force proportional to the velocity or the velocity squared fits the data best.

When looking closely at the data graphed in Figure 6 it appears as if the velocities can only take on discrete values. This is most apparent at higher velocities where several data points for different positions are aligned vertically at intervals of velocity which are approximately evenly spaced. This is most

likely a result of the resolution of high-speed camera and contributes to the noise present at higher velocities. Even if the velocities could be calculated with greater precision, the type of resistive force acting on the shuttle most likely could not be determined without collecting more data.

CONCLUSION

More data for larger distances fallen is needed in order to make a conclusive argument that the resistive force is either proportional to the velocity or the velocity squared. Previous research done by Peastrel, Lynch, and Armenti⁵ found that the resistive force for a shuttlecock during free fall was proportional to the velocity squared, with a terminal velocity of $6.8 \frac{m}{s}$.

In their research, the time required for the shuttlecock to fall a given distance was measured using a millisecond timer. The distances fallen ranged from 0.61 to 9.50 meters. A detailed description of the shuttle used in the Peastrel, Lynch, and Armenti experiments was not provided, so the dimensions and mass cannot be compared to the feather birdie used in my experiment. These small differences may alter the terminal velocity as well as the proportionality factor, k , in the resistive force, but I assume that they will not be able to change the type of resistive force—whether it is proportional to the velocity or velocity squared.

The conclusion of Peastrel, Lynch, and Armenti, support that the resistive force on the shuttle in this experiment is also proportional to the velocity squared rather than the velocity. Therefore the terminal velocity is $6.96 \pm 0.07 \frac{m}{s}$, which is similar to the value obtained by their research group. The mass of the shuttle used is $(5025.2 \pm 0.2) \times 10^{-6} kg$. As derived earlier, for a resistive force proportional to velocity squared,

$$v_T = \sqrt{\frac{g}{k}}, \text{ making } k = \frac{g}{v_T^2} = (0.202 \pm 0.004) m^{-1}.$$

Therefore the resistive force is $F_{res} = v^2 (102 \pm 2) \times 10^{-5} \frac{kg}{m}$ when v is in $\frac{m}{s}$.

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⁵ M. Peastrel, R. Lynch, A. Armenti, Jr.
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