

# Measurement of the Hall Coefficient in a Germanium Crystal

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This experiment measured the Hall Coefficient of a sample of germanium to be  $(1.55 \pm 0.15) \times 10^{-2} \text{ m}^3/\text{C}$ . It could not be compared to a standard value as the exact composition of the semi-conductor is unknown.

From the Hall Coefficient it was deduced that the charge carriers are positive. Additionally, the density of the charge carriers was determined to be  $2.5 \times 10^{21} \text{ carriers/m}^3$ .

A small adjustment made to the equation modeling the situation was required in order to obtain reasonable results. This adjustment was to compensate for a slight misplacement of the electrodes onto the germanium sample.

## INTRODUCTION

The Hall Effect, discovered in 1879<sup>1</sup>, is the well established effect seen in any electrical conductor. In general, when a current is passed through a conductor in the presence of a magnetic field, a voltage called the Hall Voltage is generated perpendicular to both the flow of the current and the magnetic field. It is easily explained using classical electricity and magnetism laws, although it can be more fully described using quantum mechanical laws.

This experiment will attempt to measure the Hall Coefficient, a variable associated with the equations describing the Hall Effect. This coefficient describes the mobility of electrons within the conductor, and thus it varies depending on the substance being measured.

## THEORY

In this experiment the Hall Effect is observed in a small crystal of germanium. However, the Hall Effect itself can occur in any conductor or semi-conductor.

Suppose a current,  $I$ , is induced in a conductor perpendicular to a magnetic field of strength  $B$ . The magnetic field will exert a force on the current, comprised of moving charges assumed to be positive, in the  $z$  direction. This will create an electric potential,  $E$ , also in the  $z$  direction. This potential in turn will exert a force on the charges in the opposite direction. The directions of these values are shown in Figure 1.

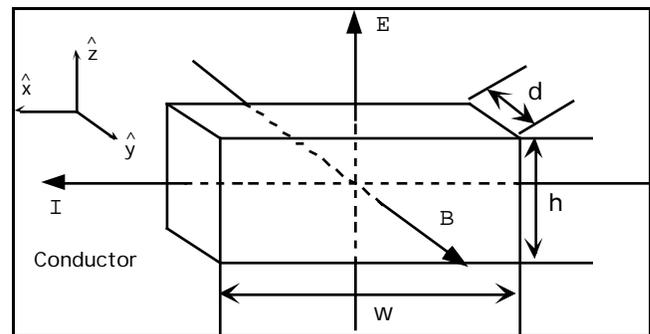


Figure 1 – General Hall Effect Diagram

Very quickly after the current and magnetic field are introduced, the system will reach an equilibrium such that the force due to the magnetic field is equal and opposite to the force due to the electric field. In addition, if the charges move only in the  $x$  direction, and the magnetic field is strictly in the  $y$  direction, we can be assured that both the forces and the electric field will be in only the  $z$  direction.

The current,  $I$ , is equal to the product of the total charge in an  $xy$  plane of the conductor times the velocity,  $v$ , of that charge. In addition, the total charge is equal to the product of the unit charge,  $e$ , the charge density,  $N$ , and the cross sectional area of the conductor,  $hd$ .

If one were to measure the potential difference from the top of the conductor to the bottom, that voltage,  $V$ , would be the product of the electric potential,  $E$ , and the height of the conductor,  $h$ .

Solving this for  $V$  will give the measured voltage as a function of the rest of the variables.

$$V = \frac{B}{Nde} I \quad (1)$$

In this experiment the voltage is measured as a function of current, so one could expect a linear relationship between the two parameters. The slope of that line will be given by  $\frac{B}{Nde}$ .

Since both the magnetic field and the depth of the conductor can be measured with other means, and since the unit charge is known, both the charge density,  $N$ , and the sign of the charge carrier can be determined from the slope. The Hall Coefficient itself,  $R_H$ , is defined<sup>2</sup> to equal to the inverse of the product of  $N$  and  $e$ .

$$R_H = \frac{1}{Ne} \quad (2)$$

It is generally known that an electrical current is made up of negatively charged electrons passing through a conductor. Yet for certain substances, the Hall Coefficient dictates that the charge carriers are positive. Positive charged particles are likely not moving, as they are most frequently found in the form of protons bound in the nuclei of the atoms.

One can think of the flow of charges as either being negatively charged electrons, or positively charged holes where electrons are missing.

Semiconductors such as germanium can conduct via either holes or electrons depending on what trace elements it is doped with. If it is doped with an element that contains loosely bound outer electrons such as arsenic, electron conduction will be more likely. Such a semiconductor is said to be n-type. Doping the semiconductor with an element such as Boron which has a more complete outer orbital, will cause positive hole conduction to occur. This type of semiconductor is called p-type.<sup>1</sup>

## EXPERIMENT

For this experiment, a small crystal of the semiconductor germanium was used. It was likely doped with some small amount of an unknown substance. This crystal was mounted in a plastic frame and had four leads attached. Two leads were designated to provide the current, while the other two were for measuring the voltage perpendicular to the current. Those wires were wrapped around a metal support beam so that the current in them would be antiparallel and any magnetic field that they might induce would be minimized.

The current leads were attached to an HP Power Supply 6216C used as a current source, and a Keithley 175 Autoranging DMM used to monitor the current through the germanium. The voltage in the perpendicular direction was monitored via the voltage leads with a Tektronix CDM250 DMM.

Three sets of data were taken in this experiment. The first set used the apparatus as described above. The current was increased slowly from zero to approximately 15 mA, and the voltage perpendicular to the current was recorded at each step.

The second two sets of data were taken when the Ge crystal was in the presence of a magnetic field. This magnetic field was supplied by a large C-shaped magnet placed so that the north and south poles of the magnet were parallel to the front and back faces of the Ge crystal.

The same current range was introduced and the resulting voltage values were recorded twice, one time each with the magnetic field oriented both positively and negatively with respect to the positive  $B$  shown in Figure 1. The strength of the magnetic field at the center between the two poles was measured in order to obtain a value for  $B$ .

The depth or thickness of the crystal was also needed. Due to the fragile nature of the Germanium crystal, and the fact that it was permanently soldered into its housing, a traveling microscope was used to measure this.

The distance between the two focal planes,  $t$ , of the edge of the crystal was measured directly with the microscope. The actual depth,  $d$ , was then calculated from  $t$  and the angle that the scope formed with the horizontal,  $\theta$ .

$$d = \frac{t}{\cos \theta} \quad (3)$$

The angle,  $\theta$ , was found by measuring the heights of the front and back and the length of the scope.

$$\theta = \arcsin \frac{h_1 - h_2}{l} \quad (4)$$

## DATA AND CALCULATIONS

Measuring the magnetic field yielded the following average value for magnetic field strength and its associated uncertainty.

$$B = 1.728 \text{ kG} \quad B = 0.016 \text{ kG}$$

Measuring the thickness and the various heights and lengths of the microscope yielded the following values and uncertainties.

$$\begin{aligned} t &= 1.34 \text{ mm} & t &= 0.13 \text{ mm} \\ h_f &= 142 \text{ mm} & h_b &= 1 \text{ mm} \end{aligned}$$

$$h_2 = 128 \text{ mm} \quad h_2 = 1 \text{ mm}$$

$$l = 127 \text{ mm} \quad l = 1 \text{ mm}$$

Using these values the angle the microscope forms and the depth of the crystal were calculated.

$$= 0.1104 \pm 0.0009$$

$$d = 1.38 \pm 0.13 \text{ mm}$$

The three data sets of voltage vs. current are shown in the graph in Figure 2.

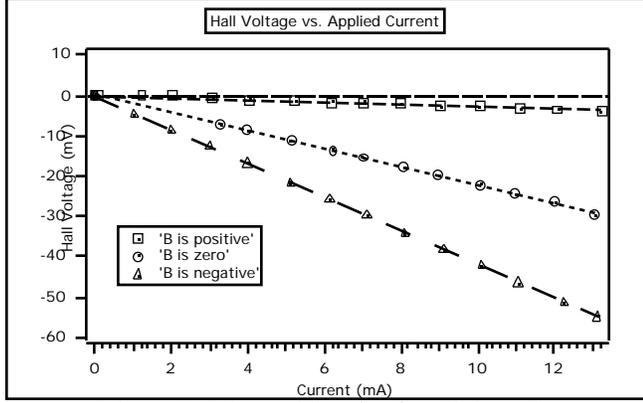


Figure 2 - Plot and line fits for all three data sets

The slope,  $S$ , of each line and their corresponding uncertainties are as follows:

$$\text{Positive } B \quad S = -0.298 \pm 0.013 \text{ V/A}$$

$$\text{Zero } B \quad S = -2.2474 \pm 0.0048 \text{ V/A}$$

$$\text{Negative } B \quad S = -4.1739 \pm 0.0020 \text{ V/A}$$

## ANALYSIS AND INTERPRETATION

The first thing one notices about the graph of the data in Figure 2, is that all slopes are negative. In addition, the case where  $B=0$ , where one would expect a zero slope, still has a distinctly negative constant of proportionality between the current and the measured perpendicular voltage. If Equation 1 were correct in describing the situation, we would expect one line to lie along the x-axis, one above and one below.

Since this was not observed, a modification was made to the equation modeling the system. The modification consists of the addition of a constant,  $K$ , and the new equation is shown below.

$$V = \frac{B}{eNd} + K I \quad (5)$$

This modification is justified by the fact that the relationship between voltage and current is still reasonably linear. The value of  $K$  was determined from the case where there is no magnetic field.

$$K = -0.2986 \text{ V/A} \quad K = 0.013 \text{ V/A}$$

The units of Volts/Amperes have been adopted in order to satisfy the rest of the equation.

If we accept Equation 5, then the slope values that we calculated can be defined as follows.

$$S = \frac{B}{eNd} + K \quad (6)$$

Since we now know the value of  $K$ , we can define  $S'$  to be the value we originally intended to measure in the slope.

$$S = \frac{B}{Nde} \quad (7)$$

By subtracting we can find the two values for  $S'$  in the remaining two cases.

$$\text{Positive } B \quad S' = 1.9488$$

$$\text{Negative } B \quad S' = -1.9265$$

The remaining discrepancy between these two values could be due to other small sources of error such as imperfections in the crystal structure, or the magnetic field being slightly askew. In any case, a more helpful value would be the average of the magnitudes of those two which has the value of  $\bar{S} = 1.93 \pm 0.010$  Volts/Amperes.

With that value it is now possible to find the value of the Hall Coefficient.

$$R_H = (1.56 \pm 0.15) \times 10^{-2} \frac{m^3}{C}$$

This yields a charge density of  $2.5 \times 10^{21}$  carriers/ $m^3$ . In addition, the fact that it is positive indicates that the charge carriers are also positive.

While the physical source of  $K$  is likely several factors, the most influential could be the fact that the voltage is not being measured exactly perpendicular to the flow of the current. This is something that could not be adjusted as it was the result of a slight misplacement in the soldering of the leads to the germanium crystal.

Because the voltage was not measured exactly perpendicular to the current, a component of the voltage generated across the path of the current is also being measured. From this one can see that in the absence of a magnetic field there should be a measured voltage,  $V_m$ , that is a component of the voltage across the current path,  $V$ .

$$V_m = V \cos \theta \quad (8)$$

Several voltage readings were taken in both directions simultaneously, and the ratio of  $V_m/V$  was a constant  $-0.0263$ . This corresponds to an angle of  $1.597$  radians.

Because of the width of the attachments points, the actual path of the current when not in the presence of a magnetic field was not able to be determined visually. However, it is likely to be between two extreme cases where the current

leaves from the one edge of an attachment point and travels to the far edge of the other. A quick measurement and calculation shows that that the angles in those two cases both would enclose the previously calculated value.

$$\theta_{max} = 1.70 \text{ radians} \quad \theta_{min} = 1.13 \text{ radians}$$

It can be safely stated that most of the variable  $K$  is likely due to the misplaced electrodes. Other factors that could contribute to a skewing of the data include irregularities in the germanium crystal, and thermoelectric effects.

## ERROR

This experiment has the potential to have systematic errors which could skew the final calculations. These include a slight misalignment of the magnetic field, irregularities in the grain of the germanium crystal, and stray magnetic fields generated by nearby electrical equipment. When the two the positive and negative slopes were averaged, it was hoped that any effect these causes would have on the data would be negated.

## CONCLUSION

This experiment measured the Hall Coefficient,  $R_H$ , of a particular sample of germanium to be  $1.55 \times 10^{-2} \pm 1.5 \times 10^{-3} \text{ m}^3/\text{C}$ . A reference source<sup>3</sup> lists the Hall Coefficient for Ge to be  $-8 \times 10^{-2} \text{ m}^3/\text{C}$ , which is of the same order of magnitude. Because the origins of the sample and its exact composition are unknown, the standard value could not be compared directly with ours to determine the accuracy. However, one would expect the numbers to be approximately the same, which they are.

From our value of  $R_H$  it can be deduced that the charge carriers are positive, and thus the semiconductor is p-type. Additionally, the density of the charge carriers was determined to be  $2.5 \times 10^{21} \text{ carriers}/\text{m}^3$ .

## ACKNOWLEDGMENTS

<sup>1</sup> Shockley, William, Electrons and Holes in Semiconductors, (D. Van Nostrand Co. Inc., Princeton New Jersey, 1950), pp204-215

<sup>2</sup> Pugh, E. M. and Pugh E. W., Principles of Electricity and Magnetism, (Addison-Wesley Publishing Company, Reading Massachusetts, 1960), pp. 212-213

<sup>3</sup> Ed. Lerner, Rita G. and Trigg George L., Encyclopedia of Physics, 2nd Ed., (VCH Publishing Inc., New York, 1991), p474