Bend It like Magnus: Simulating Soccer Physics

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The aim of the experiment was to investigate the forces acting on a soccer ball through its flight in air. This was done by treating the soccer ball as a sphere moving in a viscous fluid and then applying fluid dynamics to derive the relationships between the forward movement of the ball, the Magnus effect and rotational velocity, drag and linear velocity and gravity. Upon establishing the relationships, a simulation model was created and run to observe the effect of several parameters on ball flight. Finally, the model was used against a famous goal by Roberto Carlos and the physics behind the goal was established. The model predicted that the ball would curve about 2.6 m under theoretical circumstances where only the drag, gravity and Magnus forces were acting on the ball.

I. INTRODUCTION

In 1997 during the Tournoi de France, Brazilian left back Roberto Carlos scored a goal that for years was called a wonder goal by his fans and a freak goal by his critics. Carlos took a free kick from about 35m in front of the goal, initially hitting the ball so far to the right that it cleared a wall of defenders approximately 15 meters in front of him by about a meter and caused a ball boy nearly 10 meters to the right of the goal to duck. The ball then seemingly inexplicably swept to the left and entered the top right corner of the goal, to the amazement of the goalkeeper, players and fans alike [1]. Many fans and critics alike claimed that the goal defied the laws of physics. Similarly, throughout his career David Beckham has become famous for his control over curve kicks thus becoming the inspiration for the phrase “bend it like Beckham”.

Physicists soon proved that Carlos’ kick did not defy the laws of physics, but rather highlighted the beauty of physics in real world instances [2]. Indeed the ball had some erratic behavior due to external conditions such as wind blowing against the ball but the curving of the ball was not erratic nor due only to these uncontrollable conditions. Under normal circumstances the ball should have moved in a straight path with the major forces resisting this motion being gravity and drag force. In a simplified but accurate version of the movement, the ball can be treated as any spherical object moving in the viscous fluid, air. Considering a three dimensional coordinate system where the z-axis represents the vertical height the ball reaches, the y-axis represents the forward displacement of the ball and the x-axis represents the lateral displacement of the ball, it was expected that the ball would have had no significant lateral displacement if only the forces mentioned were acting on the ball. However, what physicists highlighted was that this kick allowed the introduction of a new force on the ball due to the introduction of spin on the ball as it moved. This force is called the Magnus force and it acts perpendicular to the motion of the ball. The effect is named after Heinrich Magnus who described it in 1852; however there is proof that the effect was discovered by Isaac Newton in 1672 while observing tennis players playing a game [4]. In a spinning ball moving at fast speeds, the spin causes the velocity of the ball relative to the air surrounding it to vary at different points on the ball depending on the speed and orientation of the spin [5]. The Magnus effect can cause a significant force to be experienced on the ball, allowing for situations such as Carlos’ goal to occur. Thus, although Beckham has been given the title “Bend it like Beckham”, the true title should be “Bend it like Magnus” since anybody can kick a ball and curve it with a significant deviation as Carlos or Beckham’s kicks due to the Magnus effect, given that certain conditions work in their favor. The true challenge lies not so much in making the ball curve, but rather in learning exactly how to judge the necessary factors and curve the ball precisely as desired. The effect of Magnus force on the lateral movement of the ball is illustrated in FIG1.

This goal was the primary inspiration for this paper, which covers the process behind creating a simulator that was able to model the trajectory of a soccer ball kicked with some exit velocity v_0 and rotational velocity ω.

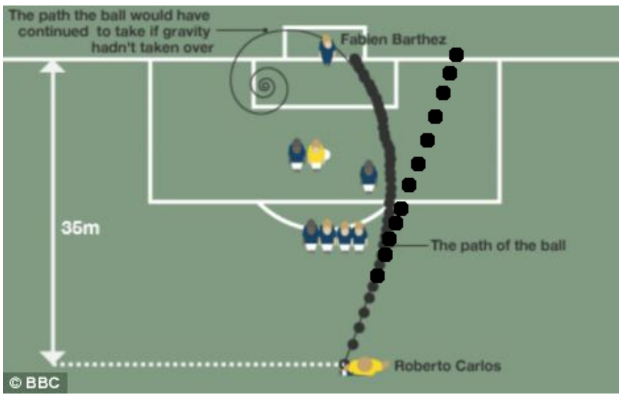

FIG. 1: The figure above shows the x and y directional trajectory of the ball from Carlos’ shot. It also shows the straight line trajectory the ball was expected to take. The straight trajectory shows the path the ball would have taken had there been no Magnus Forces and wind acting on it and thus helps to visualize the effects of the Magnus forces on the ball. The image was taken from [1] and edited to include the straight trajectory.

II. THEORY

There are several forces acting on a ball moving through a fluid that were considered in the model. The forces considered were: drag on the ball, gravity, The Magnus force which all opposed the force driving the ball to move forward.

From Newton's second law, it is taken that while in the air, the ball's velocity changes due to the applied force according to the equation

$$F_{net} = ma \tag{1}$$

where F_{net} is the force on the ball, m is the mass of the ball and a is the acceleration of the ball. The force can be rewritten as a second order parametric function of displacement with respect to time:

$$F = m\frac{dv}{dt} = m\frac{d^2s}{dt^2} = ms''(t) \tag{2}$$

where dv/dt is the velocity function with respect to time, $s(t)$ is the displacement defined as a function of t, the time. This was done to better track the displacement of the ball over time. The components of force were considered separately in each dimension of three dimensional motion since the movement of a curving ball is along all three coordinate axes. Further, certain resistive forces such as gravity only act in one dimension, in this case the z direction since that represent vertical displacement. The forces acting on the ball are visually described in FIG 2.

Once again, considering the components of force while in the air are

$$F_{mx} = mx''(t) \text{ (lateral direction)}$$

$$F_{my} = my''(t) \text{ (forward direction)}$$

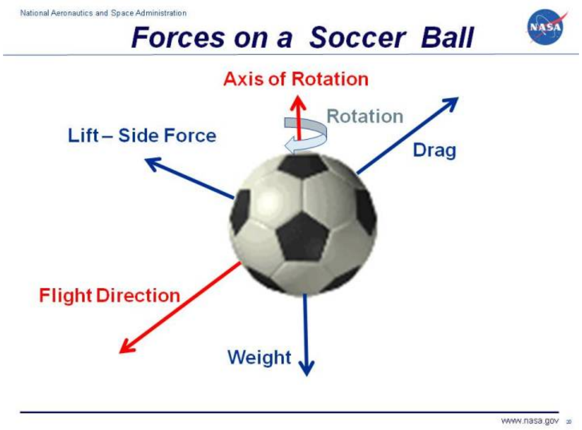

FIG. 2: The forces acting on a ball. The ball moves forward with some velocity. This is opposed by drag, weight and the lift of the ball. The lift is the Magnus force experienced by the ball. [6]

$$F_{mz} = mz''(t) \text{ (vertical direction)} \tag{3}$$

so that each component could be considered individually. The movement of the ball was opposed by the respective drag forces (and gravity in the z direction), the net forces acting on the ball could be calculated as:

$$F_{net} = F_G + F_D + F_S + F_L \tag{4}$$

where F_G is the downward force experienced by gravity, F_S is the sideways component of the Magnus force and F_L is the lifting component of the Magnus force. To further explain F_S and F_L consider a ball that is rotating strictly with topspin or backspin. The ball will have no sideways rotation and hence $F_S = 0$. Likewise, consider a ball that has strictly sideways spin. The Magnus force now has no component in the z direction and thus $F_L = 0$. However when a ball is rotating in more than one axis, F_S and F_L must be considered.

From Eq. 3 the components of F_{net} have been described. The next thing to be considered was the drag force, which was defined as:

$$F_D = 0.5C_D\rho Av^2 \tag{5}$$

where C_D is the drag coefficient of the ball, ρ is the deinsity of the fluid in which the ball is travelling (air in this case), A is the cross sectional area of the ball, and v is the velocity of the ball [5]. This equation simply says that a sphere moving in a fluid experiences a drag force proportional to the density of the medium, the cross sectional area of the sphere and the square of the velocity with which it is travelling by some constant C_D. In the case of our model, F_D is considered to have the same form in all three dimensions and hence can be equated to all three components of Eq. 3. However since the model was in three dimensions the velocity had to be broken up into the components. The drag force then became:

$$F_{Dx} = 0.5C_D\rho Av^2\hat{v}_x$$

$$F_{Dy} = 0.5C_D\rho Av^2\hat{v}_y$$

$$F_{Dz} = 0.5C_D\rho Av^2\hat{v}_z \tag{6}$$

but since we know that $\hat{v}$ is simply the unit vector for a vector $\mathbf{v}$, and is defined as

$$\hat{v} = \frac{\mathbf{v}}{|v^2|}$$

we can substitute this into Eq. 6. Before this is done, it was considered that $|v^2| = (\sqrt{v_x^2 + v_y^2 + v_z^2})^2$. Thus the equation of drag became:

$$F_{Dx} = 0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}\mathbf{v}_x$$

$$F_{Dy} = 0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}\mathbf{v}_y$$

$$F_{Dz} = 0.5C_D\rho A\sqrt{v_x^2+v_y^2+v_z^2}\mathbf{v}_z \tag{7}$$

Next we considered the force due to gravity, $F_G = mg$ which can simply be applied to the z component of Eq. 3. Finally, the Magnus force was considered. The Magnus force equation started out analogous to the drag equation as:

$$F_M = 0.5\rho Av^2C_L \tag{8}$$

where

$$C_L = \frac{C_M r\omega}{v}$$

and thus

$$F_M = 0.5C_M\rho Ar\omega v \tag{9}$$

where F_M is the Magnus force, C_M is the coefficient of proportionality, ρ is the density of air, A is the cross sectional area of the ball, ω is the rotational velocity of the ball and v is the linear velocity of the ball [5]. This equation thus implies that the Magnus force is proportional to the density of the medium, the cross sectional area of the ball, the radius of the ball, the angular velocity of the ball and the velocity of the ball by some constant C_M.

This introduction of ω required some investigation, since ω was stated as a vector in the form $\{\omega_x, \omega_y, \omega_z\}$ where this first component represented a rotation about the x axis, the second about the y axis and the third a rotation about the z axis. For example, strictly sideways spin (with no lift force) is represented as a rotation about the z axis so only $\omega 3$ would have a non-zero value and $\omega_x = \omega_y = 0$. For simpler models, it was assumed that ω was constant. A ball that was kicked with an initial rotational velocity would experience no damping but would continue to spin at the same pace. This is not true in reality and ω was defined by a function. To derive this function, the forces acting on a rotating ball were considered. A rotating sphere would experience two extremities with respect to the drag force. By the definition of the Magnus effect it can be taken that the side of the ball where the tangential velocity v_t was in the same direction as the drag, the ball's velocity relative to the air would be minimal. On the side where the tangential velocity was opposing the drag, the ball's velocity relative to the air would be maximum [2]. Thus the tangential velocity could be added to and subtracted from the linear velocity to represent the the maximum and minimum net velocity of the ball on either side and hence the maximum and minimum drag forces experienced on either side of the ball. The maximum drag force is

$$F_{Dmax} = (0.5\rho AC_D)(v+v_t)^2$$

and the minumum is

$$F_{Dmin} = (0.5\rho AC_D)(v-v_t)^2$$

and the difference of the forces becomes

$$F_{net} = (0.5\rho AC_D)\left((v+v_t)^2-(v-v_t)^2\right) \tag{10}$$

which simplified to give:

$$F_{net} = 2\rho AC_D v v_t$$

From this net drag force, the torque could be equated since it is the product of the radius and linear force so [3]

$$\Gamma = F_{net}r$$

and also $v_t = r\omega$ so

$$\Gamma = 2\rho AC_D v\omega r^2$$

It is also known that $\Gamma = I\omega'(t)$ where I is the inertia which for a sphere is defined as

$$I = \frac{2}{5}mr^2$$

which upon substituting and simplification into the equation

$$\omega'(t) = \frac{\Gamma}{I}$$

gives the result

$$\omega'(t) = \frac{5\rho AC_D v\omega}{m} \tag{11}$$

Considering the equation in the respective axes and expanding the velocity in terms of the components give the following equations

$$\omega_x'(t) = \frac{5\rho AC_D\sqrt{v_x^2+v_y^2+v_z^2}\omega_x[t]}{m}$$

$$\omega_y'(t) = \frac{5\rho AC_D\sqrt{v_x^2+v_y^2+v_z^2}\omega_y[t]}{m}$$

$$\omega_z'(t) = \frac{5\rho AC_D\sqrt{v_x^2+v_y^2+v_z^2}\omega_z[t]}{m} \tag{12}$$

With these equations derived, the Magnus force could be better defined. Since both the rotational velocity and linear movement were involved in determining the Magnus force and since both these elements had x, y and z components, it was determined that the direciton and the magnitude of the force had to be determined by a cross product of the two vectors [5]. The advantage of using the cross product in this situation is that it combined the elements F_S and F_L through the individual components. Thus the Magnus force equations for each individual component became:

$$F_{Mx} = 0.5\rho C_M Ar(\omega_y'(t)z'(t) - \omega_z'(t)y'(t))$$

$$F_{My} = 0.5\rho C_M Ar(\omega_z'(t)x'(t) - \omega_x'(t)z'(t))$$

$$F_{Mz} = 0.5\rho C_M Ar(\omega_x'(t)y'(t) - \omega_y'(t)x'(t)) \quad (13)$$

which allowed all the components that were necessary to be acquired. Thus by equating F_{net} as the sum of forces causing the ball to slow down over time, the following parametric system of equations were determined for calculating the curve of a soccer ball in flight.

$$mx''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}x'(t)$$

$$-0.5C_M\rho Ar(\omega_y'(t)z'(t) - \omega_z'(t)y'(t)),$$

$$my''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}y'(t)$$

$$-0.5C_M\rho Ar(\omega_z'(t)x'(t) - \omega_x'(t)z'(t)),$$

$$mz''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}z'(t)$$

$$-0.5C_M\rho Ar(\omega_x'(t)y'(t) - \omega_y'(t)x'(t)) - mg \quad (14)$$

III. SIMULATION

For the simulation modeling, Mathematica was selected as a tool. This is because mathematica had a lot of built in functions that could be called to simplify certain parts such as solving the system of differential equations. This also allowed the code to be kept simple and neat and allowed for more time to play around with different values of the input parameters. The NDsolve function was crucial in solving the differential equations and only required simple initial conditions to be provided to solve the differential equations.

The modeling was adapted and allowed for several builds that progressively got more complicated in a single mathematica notebook. This was advantageous in the case that one wanted to backtrack and explore different aspects of the simulation, so that the model would not have to be reconstructed from scratch.

A. 2D Model

The model took parameters:m, g, γ, ρ, A, θ and v where m was the mass of the ball, g was the effect of gravity, γ was the linear drag coefficient, A was the cross sectional area of the ball, θ was the launch angle of the ball, and v was given as a launch velocity of the ball which was then divided into vsin θ for the z(t) component initial condition and vcos θ for the y component initial condition. This model was simple and produced plots of z(t) vs. y(t) that agreed with the theory.

B. Single NDSolve

This model got more intricate and attempted to model the drag forces involved in three dimensions and added the Magnus forces to the previous model. All three variables were solved in a single NDSolve command and the parameters remained the same with the addition of C_M for the Magnus coefficient, ω_0 for rotational velocity, r for the radius of the ball and the renaming of γ to C_D. This model also considered ω as a vector of ω values in all three dimensions and used the cross product so that both lift and sideways movements would be considered due to the Magnus effect.

C. Manipulator

This third model included the addition of the decaying ω function and thus added a new level of complication to the model. The ball was no longer assumed to have a constant angular velocity and thus the curvature trajectories became significantly less distinct but at the same time more realistic. This model also added the functionality to create a manipulated function that would allow the initial launch velocity and also all three components of rotational velocity to be altered and seen in a dynamic plot that reflected the changes.

IV. ASSUMPTIONS

Although many assumptions had to be made in creating the model, some that may have major effects are listed as follows: The effect of wind is very prominent in real soccer. The model assumes that wind more or less acts uniformly on the whole ball and is thus only somewhat reflected in the drag coefficient of the ball. In reality, wind in a single constant direction may affect the curved flight of the ball significantly depending on the situation. The drag coefficient is taken to be constant throughout the flight of the ball. In reality the drag coefficient varies with the decreasing speed of the ball, especially around critical velocity with turbulent and laminar air flow around the ball [5] [7] [8]. This would affect the accuracy of the results The coefficient of the Magnus force was also assumed to be constant and equal to 1 [5]. In reality this constant would also vary, possibly even more greatly than the drag coefficient since it is affected not only by linear velocity but also rotational velocity. The air density varies depending on day and time. Ball conditions such as wetness or how well it is pumped should also affect the results The ball is treated as a sphere in some cases (Intertia of a sphere, fluid dynamics of a sphere) and not in other cases (drag coefficient was considered for an average soccer ball [5]. This relationship could be further investigated.

There are probably several assumptions that are made but those listed might have the most significant effect on the data and should be investigated in future work.

V. RESULTS AND ANALYSIS

The model proved succesful in providing accurate flight trajectories for a ball in air. Despite the assumptions listed in the previous section, the model was able to accurately simulate the curved trajectory of a ball in air. In the Roberto Carlos goal, the ball is speculated to have curved with a deviation of approximately 4-5 meters away from the goal. When run in the model with a launch speed of 30 m/s, a launch angle of about 15 degrees with pure sidespin, the ball was projected to curve about 2.6 m as shown in FIG 3. FIG 4 shows an alternate view of the kick as if seen along the direction in which the kick was taken. FIG 5 looks at kick considering only the linear drag and not the curve by rotating the image to focus on the y and z axis. The parameter values were attained from [2] and [9]. One would think this is not completely accurate but it should be considered that Carlos' kick did not have purely sidespin as assumed in the calculations. This is shown in the figures below. Also, several factors were judged from videos after the kick occurred and these values may have been inaccurately measured. The model succeeds in verifying that curvature would occur and makes a close guess to what that deviation would be and is expected to be much more accurate if all the values were relatively known. Further, it calculates exactly what would happen, if only the forces considered were acting on the ball. One interested thing that was noted was that this model showed curved movement occuring instantly whereas in reality there is a slight delay before the ball begins to curve. This might be due to the changing drag and Magnus coefficients as ball speed varies but more investiation is needed into this discrepancy.

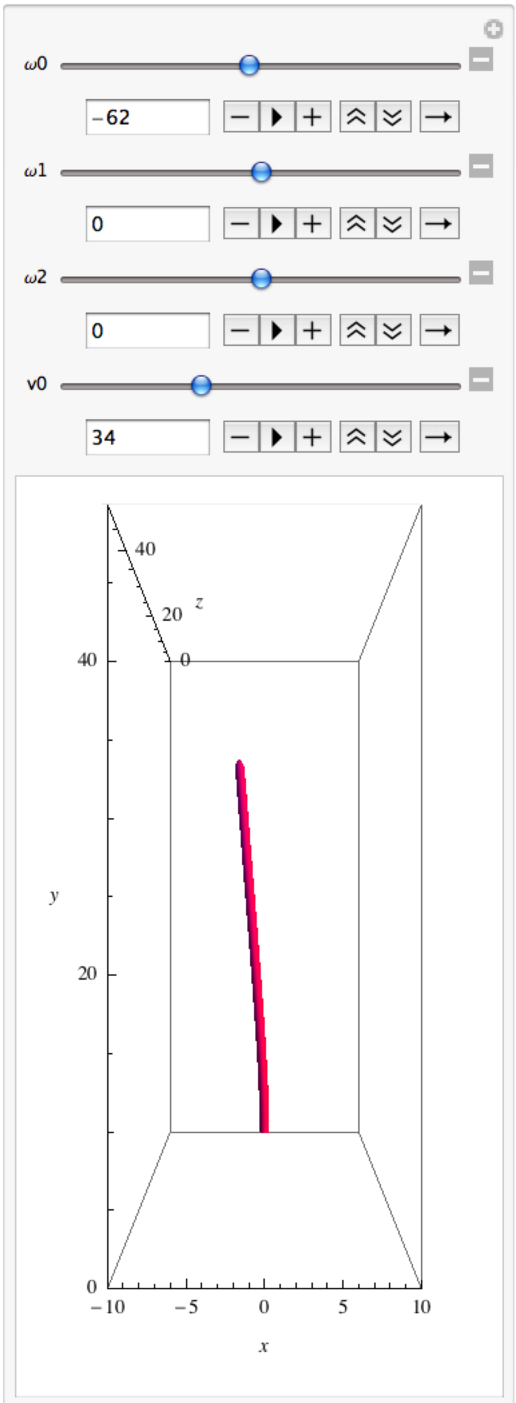

FIG. 3: Diagram showing top view of simulation for Roberto Carlos' free kick, the deviation along the x axis is seen to be about 2.6m. This result shows that the simulation is able to predict curvature even for very low sarting values. The x axis shows the lateral deviation and the y axis shows the distance the ball is expected to travel in 2 seconds.

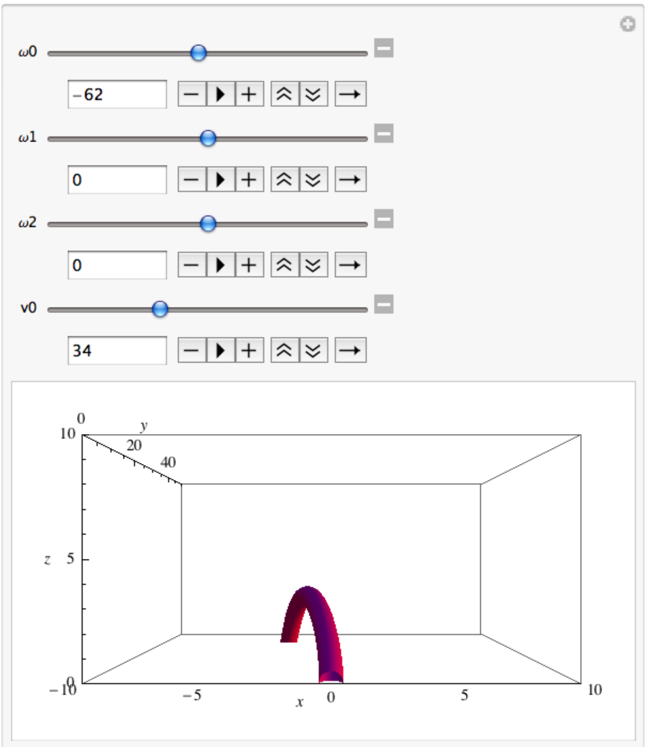

FIG. 4: Alternate view of Roberto Carlos' kick showing a 3 dimensional view. The model assumes that the launch angle in terms of x (lateral) displacement is 0. Thus the figure represents the curvature seen in the ball if one looked along the x component of the angle of launch for the ball.

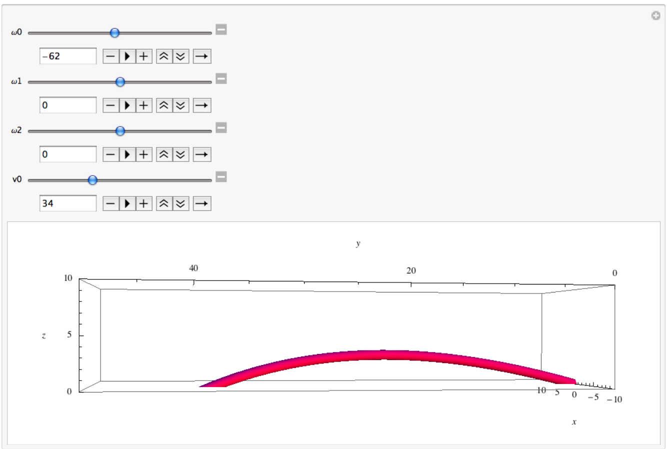

FIG. 5: Alternate view of Roberto Carlos' kick. This view shows only the linear drag being reflected in the z and y axes. It can be seen that from a side view the ball seems to follow a trajectory similar to the output of model 1 verifying the purely sideways spin does not affect either axes component.

In analyzing this figure the strengths of the model can be seen. It does accurately simulate the motion of a ball but in a theoretical manner. The model does not include external factors such as wind and can be further expanded to even better simulate the curvature of a soccer ball through the air. Considering purely sideways spin, a plot of the effect of varying the rotational velocity is included in FIG. 6 and a three dimensional view is shown in FIG 7. FIG 7 has the same initial conditions as FIG 6 (Which takes the initial parameters from the Carlos' kick and simply varies the angular velocity) but angular velocity values from 0 to -1000 rad/s in increments of 100 were used:

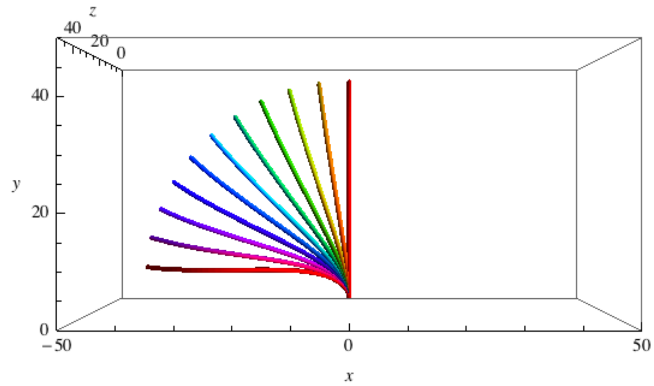

FIG. 6: This figure looks at a 2 dimensional top view of several different trajectories looking at purely sideways spin in increments of 100 rad/s and ranging from 0 to 1000 rad/s. The decaying rotational velocity shows that the greatest curving occurs in the earlier stages of flight instead of their being a slight delay before curving as seen in reality.

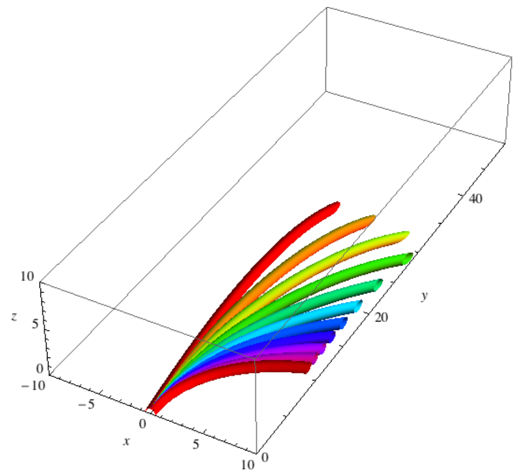

FIG. 7: 3 Dimensional view of different curvatures. Different angular velocities not only affect the amount the ball curves but also directly affect the distance traveled, at a high enough angular velocity it is expected that the ball would spiral in until the angular velocity reaches 0, in which case the ball would continue to move with the tangential velocity assuming that it does not hit the ground.

It is interesting to note that this model can allow for some very interesting special cases. One such case is when the ball is given high enough backspin, the ball will actually curve up until it starts moving backwards

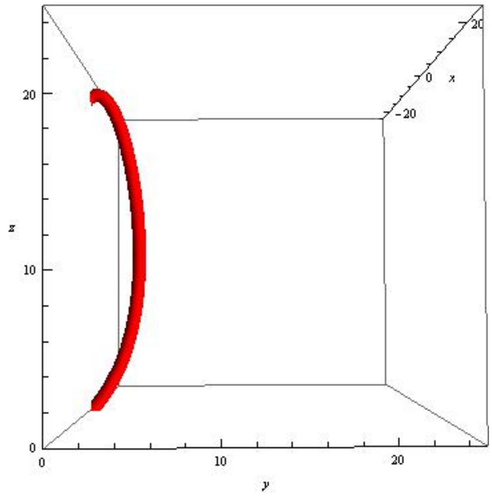

FIG. 8: Side View of ball with enough spin to go backwards and up. This is the special case mentioned above.

as shown in FIG 8. This would be the case where the rotational velocity of the ball provides enough lift and spin to overcome both gravity and the forward flight of the ball and instead move in the direction opposite to launch.

VI. CONCLUSION

In conclusion it can be said the flight trajectory of a soccer ball moving through the fluid air was successfully simulated. The models were able to accurately model the effects of different rotational velocities and exiting velocities. The model did make certain assumptions which leaves room for future work, to create the most accurate possible model however for purposes of a general idea, this model will accurately simulate flight trajectories. Due to the ease of use with the code, the model can also be adapted to simulate trajectories of any spherical object moving through the air by simply tweaking different parameters. The aim of the model was achieved in that Roberto Carlos' goal was accurately simulated and the model can be used to determine that the goal was not a fluke of physics but rather a calculated effort by the player and similar goals could be reproduced by applying the correct forces in the correct spots. Roberto Carlos' kick specifically was calculated to have 2.6 m of curvature, which represented the value that the ball would have curved if only drag, Magnus force and gravity acted on it.

VII. ACKNOWLEDGMENTS

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