quickly during the first year until we were using the capacity of the 250-seat hall. Although the prospect of doing so many presentations calls for a significant commitment from the presenter, it also offers him/her the opportunity to work the bugs out of the initial presentation and to "perfect" the program in a way that can be a source of satisfaction. There are some other advantages for the repeated programs. The prospect of several presentations justifies an effort in developing demonstrations, greater than that for a single performance. It also protects the presenter from the disappointment of a poor attendance at the sole presentation due to the vagaries of weather, competing programs, and all the unknown factors that determine the attendance on any particular night.

A few years ago we started experimenting with making videotape versions of selected programs. After spending some time climbing up the learning curve, we now produce video versions of approximately two of the programs each year. This is done with the services of the TV department of our Educational Communication Division. Royalties from the sales of these tapes provide funds for the making of future programs. Information about the programs now available may be received on request.[1]

It is impossible to measure the impact of such a program. We do get much anecdotal information: A mother says, "We raised three children on the Science Bag." University students recall coming to the university as kids to see Science Bag programs when they were growing up. A parent mentions her daughter's fascination with a program, given years ago, on Fibanocci numbers and suggests some relation to the fact that the daughter in now an undergraduate majoring in mathematics. Another parent reports the whispered comment of her eight-year old boy after the final applause has died down, "Someday I'm going to give a talk like that."

Those of us who are involved with the program know that it produces some good results.

[1]Information about Science Bag videotapes that are available can be obtained from Blue Sky Associates, P. O. Box 349, Winchester, MA 01890.

Self-organized criticality: An experiment with sandpiles

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In 1987, Bak, Tang, and Wiesenfeld introduced the notion of self-organized criticality (SOC) in the guise of a computer simulation: a "sandpile cellular automaton machine." They supposed that a real, many-bodied, physical system in an external field assembles itself into a critical state. The system then relaxes about the critical state creating spatial and temporal self similarities which give rise to fractal objects and $1/f$ noise. Their computer modeling was of a system like a sandpile at its critical angle of repose. This provided a new paradigm for many-body dynamics. Understanding SOC may well allow substantial strides to occur in the theory of flow and transport. The simplest model system, one for which computer simulations and corresponding real experiments are feasible, is a "sandpile" near its critical angle of repose. The size and duration of avalanches occurring as subsequent "sand" grains are added can provide detailed information about the "sandpile" as a model of SOC, and for SOC as a basis for many-body dynamics. This article describes a fairly simple, junior-level experiment in this new field involving the measurement of the distribution of avalanche sizes which occur as grains of sand are added to a "sandpile." The universality of the phenomena can be observed and a power law relationship can be deduced.

I. INTRODUCTION

Self-organized criticality (SOC), a new paradigm for understanding complicated dynamical systems, has recently been proposed by Bak, Tang, and Wiesenfeld.[1] SOC may change the fundamental way scientists approach such diverse problems as earthquakes,[2] the stock market,[3] forest fires,[4] ecology,[3] and weather,[3] to name just a few. All of these phenomena are large, composite, dynamical systems whose many parts influence each other with a short range interaction. It was proposed that such a system will naturally evolve to a critical state where small perturbations could lead to either minor or catastrophic events. The system attains a self-organized critical state, and hence is referred to as self-organized criticality. Such a critical state can be modeled after critical points that appear in thermal physics[5] except that no controlling parameter (such as temperature) is needed to bring the system to its critical point. The advantages of using the framework of critical point phenomena is that concepts such as scaling, power law relationships, and universality could be anticipated in these large composite systems as well.

Our intuition from mechanics can lead us astray when studying large composite systems. Small systems may be well understood by a simple causality: if an object has an acceleration, then a net force acts on it: the larger the force, the larger the acceleration. When many objects interact with short-range forces, it is much harder to identify the cause that leads to an observed effect. Large fluctuations can occur without any discernibly large perturbation. Bak and Chen,[3] in describing SOC, state that "large interactive systems perpetually organize themselves to a critical state in which a minor event starts a chain reaction that can lead to a catastrophe." While more minor events occur than major ones, the same mechanism can produce both.

The implications of SOC are profound and make it an important area for students to investigate. Large fluctuations in the stock market or large earthquakes can be understood as a catastrophe that began as a minor perturbation of the system. Moreover, SOC is a "holistic theory"[3] where the global features, such as the relative number of large and small events, do not depend on the microscopic mechanisms: one observes "universal" behavior.

A particularly simple, yet instructive, model of SOC is a sandpile. Consider an inverted cone of sand that has been created by pouring sand through a funnel onto a platform. The sand self-organizes in the presence of the external gravitational field to a critical state called its critical angle of repose. Once there, additional grains of sand sprinkled on the pile can either settle or produce an avalanche. The avalanches can be as small as one grain rolling down the surface to a catastrophe where the entire pile collapses. The number of small avalanches is very much greater than the number of large avalanches. SOC predicts that the distribution of avalanches of size S will be a power law

$$N(S) = N_0 S^{-\lambda}.$$

The value of the power λ and the region of validity of a simple power law is currently being debated. However, the appearance of such a power law is common in all the example composite systems cited above where SOC may yield dramatic insights. For example, the appearance of fractal spatial objects may be a result of SOC.

The time it takes for an avalanche to occur can also be considered. A small avalanche can take a long or short time to occur, similarly for a large avalanche. When the pile is near its critical angle of repose, the distribution of times during which an avalanche occurs is also expected to be a power law. In frequency space, the distribution of avalanches of frequency f would look like

$$N(f) = N_0 f^{-\mu},$$

where μ is an exponent whose value may be one, although various simulations and experiments have found different values (usually between one and two) in different systems. A value of one corresponds to the ubiquitous "$1/f$" noise observed in so many physical processes from noise in a resistor to music.[1,6]

The field of SOC, though new, is very accessible to undergraduates. In articles ranging from the "Game of Life"[7] and whether it exhibits power law relationships to sliding blocks on a table as a stick-slip model for earthquakes[8] to bubble formation of magnetic domains,[9] the literature is quite readable. Since review articles[3,10] have been written, this article focuses on an experiment that readily allows an observation of a power-law relationship in describing the distribution of the size of avalanches on a "sandpile." This experiment is a somewhat simplified version of the one developed by Held, *et al.*[11] A pile of uniform size beads is constructed on the pan of a digital balance which monitors the mass of the pile as additional beads are added one at a time. The avalanches are observed as a decrease in the pile's mass as the beads fall off. A distribution of avalanche sizes shows a power-law relationship that is independent of the bead diameter, material, or size of the pile. This universality gives an appreciation of SOC as a paradigm for understanding nature, where fractals and $1/f$ noise abound.

II. THEORY

In 1987, Bak, Tang, and Wiesenfeld[1] introduced the notion of self-organized criticality (SOC) in a computer simulation: a "sandpile cellular automaton machine." A real physical system supposedly assembles itself in a critical state which depends on an external field. The system then relaxes about the critical state creating spatial and temporal self similarities which give rise to fractal objects and $1/f$ noise. Their model system was like a sand pile at its critical angle of repose but modeled in a very simple way on a computer.

Traditionally, the principles of physics have been formulated and utilized in the form of partial differential equations, or PDE's. Cellular automata machines, or CAMs, have received much attention lately as an alternate way of expressing and utilizing these principles. A CAM can be thought of as an array of cells that evolve according to local rules. CAMs are typically realized on computers, especially parallel processors. One advantage of this type of simulation over, say, discretizing a PDE, is that CAMs accumulate no round-off error. Toffoli and Margolus[12] provide a comprehensive introduction to CAMs.

The original[1] CAM illustrated SOC by updating the integer field z ("slope") synchronously according to the local rules,

$$\text{If} \quad z(x,y) > K \quad \text{then} \quad z(x,y) \to z(x,y) - 4,$$

$$z(x \pm 1, y) \to z(x \pm 1, y) + 1,$$

$$z(x, y \pm 1) \to z(x, y \pm 1) + 1.$$

The CAM is started with $z \gg K$ and allowed to relax ("self organize") to $z \ll K$ where K is the critical angle of repose. The system is randomly perturbed by incrementing one of the z's and the avalanches are observed on all scales, large and small.

Mandelbrot[13] focuses attention on the many examples of self similarity around us. Something is "self similar" if it "looks" similar at any scale. Self-similar phenomena imply "scale-free physics" and are characterized by "power-law correlations". *Spatial self similarity* is described by *fractals* which are sets with anomalous, non-Euclidean, scaling. These sets frequently have fractional dimensions; these dimensions specify the exact scaling. Fractals well describe the spatial patterns of mountain ranges, river basins, coastlines, soot particles, galactic clusters, etc.

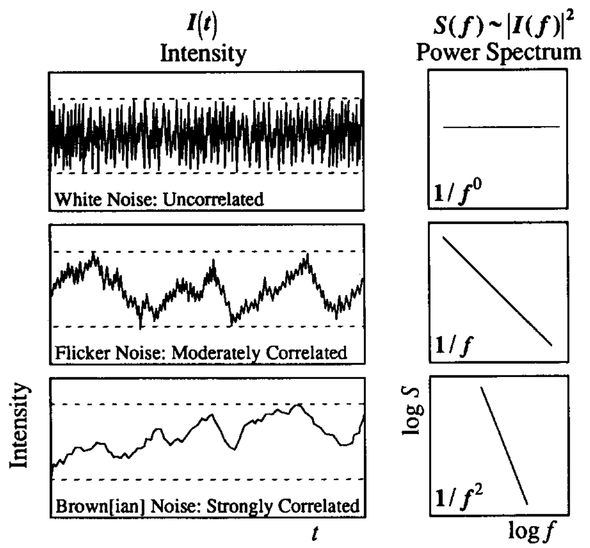

Fig. 1. Three common types of noise depend on the exponent in the power-law behavior. White noise is uncorrelated and has a power spectrum $S(f)$ that is independent of the frequency. Flicker noise has a power spectrum that varies as $1/f$ and is moderately correlated. Brownian noise is strongly correlated and has an exponent of -2.

Temporal self similarity is described by flicker noise, which is a fluctuation with a $1/f$ power spectrum. (The power spectrum of a fluctuating quantity is a measure of the power in each bandwidth or the "intensity per Hertz." It is calculated by squaring the normalized Fourier transform of the time intensity function.) Flicker noise characterizes the temporal sequences of earthquakes, current fluctuations in resistors, star luminosity, traffic flow, music, etc. Three kinds of "scaling" noise are shown in Fig. 1 corresponding to integer values of μ. Each has a power spectrum $S(f) \sim 1/f^{\mu}$ and each sounds the same when played back at any speed, as long as the volume is adjusted appropriately. But flicker noise is special. It has a $1/f$ power spectrum,

$$S(f) \sim 1/f \Rightarrow S(f)df \sim \frac{df}{f} = \frac{dt}{t},$$

and thus equal amounts of noise in all bandwidths $df \sim f$. This means that the power contained in slow oscillations, $T \pm 0.01T$ (say), is the same as the power contained in fast oscillations, $t \pm 0.01t$ (where $t < T$).

The original paper by Bak, Tang, and Wiesenfeld[1] introduced the concept of SOC as an explanation for the ubiquity of flicker noise (and fractals) in physics. The authors reported "the discovery of a general organizing principle" of many-body systems. They demonstrated a "toy" model of a pile of sand realized as a CAM (as described above) that exhibits SOC. Although they suggested that a real sandpile at its angle of repose exhibits SOC, they emphasized that SOC is much more general than sandpiles. (The idea being that if indeed the sandpile CAM is operating at a critical state it should exhibit universal properties that are identifiable and generalizable to more complicated real-world systems.) However, the authors distanced themselves from concrete interpretation, and the connection between idealized model and reality was not well drawn. In their 1989 article,[1] they wrote that it would be nice if sandpiles exhibit SOC and it would be disappointing if they do not, but the theory was not primarily one of granular systems.

Many investigators have attempted to draw upon experience with scale-free physics near equilibrium critical points in order to provide insight into SOC. One group[14] investigated interesting scaling properties of different classes of "far from trivial" CAMs exhibiting SOC. These studies were abstract in the sense that these CAMs were intended to have the property of SOC rather than designed to better represent actual sandpiles. Further, in a recent review article,[15] Leo Kadanoff hopes and expects that new ideas are needed to understand SOC: "My own hope is that sand slides ... are essentially new and have some—as yet unknown—formation, combining some elements of scaling, universality, and renormalization group."

Most physical theories work because the relevant scales can be isolated. (One need not consider water molecules when describing ocean tides.) In SOC, this is not possible as all scales are important. Fortunately, we have encountered scale-free physics before and, in the study of equilibrium critical point phenomena, have developed appropriate concepts and techniques. The vocabulary and techniques of equilibrium critical point phenomena (scaling, universality, renormalization group,...,) can be adapted to study SOC,[16] although new ideas will likely be needed to fully understand SOC. It is also important to remember a distinction between the two: the former state requires the fine tuning of some parameter (the temperature, say) while the latter state is self organizing.

Bak, Tang, and Wiesenfeld's original work[1] used a simple CAM that simulated two-dimensional flow on a sandpile. They used a 50×50 array to discover power law behavior in the avalanches with critical exponents $\lambda = 0.98$ and $\mu = 1.58$. The power law behavior is expected if SOC occurs. While the values of the exponents should be universal for certain classes of CAMs, delineating the universality classes has been found to be a challenge.[14] The value of the exponents is not thought to be as important as the power law relation that they represent. One of the fascinating aspects of this emerging field is to see the scientific process in action. There are certainly scientists who do not believe in SOC because a power law relation does not hold in all regions in real physical systems, and because the exponent values vary from system to system. However, the dramatic ability of SOC to provide a conceptual framework from which to explore very diverse systems, where no previous connection was thought to exist, makes SOC one of the promising new theoretical structures today.

Physics often advances into new areas by exploiting the existence of simple examples. For example, study of the hydrogen atom facilitated the development of quantum theory. On the other hand, physics may be hampered in certain areas by the lack of such models. Sandpiles may have properties of generic importance. They are potentially rich and relatively unexplored test beds for many-body systems. Very little is known about the dynamics of nonequilibrium many-body systems. With many-body systems there is nothing to recognize, nothing to characterize, no signature behaviors, no model systems—until, perhaps, now. Sandpiles may emerge as the hydrogen atoms of many-body dynamics and SOC may play the role of quantum theory.

III. THE EXPERIMENT

Do real sandpiles exhibit SOC? The first published experimental investigation by Jaeger *et al.*[17] was negative. The predicted power-law correlations were not observed. The authors monitored spherical glass beads and rough aluminum oxide particles spilling over the edge of a rotating drum. They concluded that the avalanches were determined by the difference between static and sliding friction. However, as Jensen *et al.*[18] argued, the granular flow down a slope is very different from granular flow over a rim. Also, this experimental geometry was essentially one-dimensional, quite unlike the original two-dimensional sandpile CAM, and also unlike real, conical sandpiles.

Pierre Evesque[19] continued his earlier work and extended the results of Jaeger. Evesque studied glass beads flowing in a rotating cylinder. He found inertia-impeded avalanches that did not exhibit a power-law correlation. As with Jaeger, the experiment was more prone to large avalanches than small avalanches.

However, another experiment was more positive. Held *et al.*[11] reported power-law correlations in studying an evolving sandpile. These experimenters built a two-dimensional conical sandpile of (sieved) aluminum oxide particles. They monitored the pile's mass as a function of time as grains were added. They then constructed a power spectrum from their data which fell off like $1/f,^2$ for sufficiently small sandpiles. However, for large enough avalanches, this description breaks down. This experiment, although an improvement over the geometry of others, did not monitor the actual activity on the surface of the pile. For small piles, the spill over the edge may accurately characterize the avalanche distribution, but for large piles this is certainly not the case as an avalanche may not spill any grains at all.

The new field of SOC is so extremely exciting because it promises to allow an understanding of diverse phenomena. The possibility of understanding why our physical world is structured the way it is spatially and temporally provides a revolutionary new paradigm for physics. Sandpiles need to be explored, quantified, and understood in order to test this paradigm for the dynamics of dense fluid flow. Through numerical modeling using CAMs, a tantalizing understanding seems close, yet the experimental verification of model predictions is mixed. Power-law relationships are only observed under certain circumstances or in certain regions. The experiments have been very simplistic and have assumed that the flow of material over an edge represents the flow of material on the surface of the pile. It could be that boundary effects play a major role in what is observed both in a quantitative and qualitative sense. Yet a simple experiment of flow on a sandpile can demonstrate some of the basic issues and acquaint students with the concepts in this developing area.

Thus, we seek an experimental approach that allows a quantitative investigation of the flow of beads on a pile by measuring the flow off the edge of the pile. The beads were chosen to allow comparison with CAM models that have been separately developed: spherical balls that collide (almost) elastically and whose size and composition may vary. The initial measurements were on identical millimeter-sized glass or metallic spherical beads. Nonspherical and/or nonidentical beads introduce additional phenomena which should not affect the outcome in the sense of scaling and power-law relationships, and this expectation was tested with plastic beads. The mass of the pile as a function of time reflects the evolving dynamics of the sandpile. These mass values may be used to infer the size, as well as something of the development, of avalanches induced by the addition of a single bead to the pile.

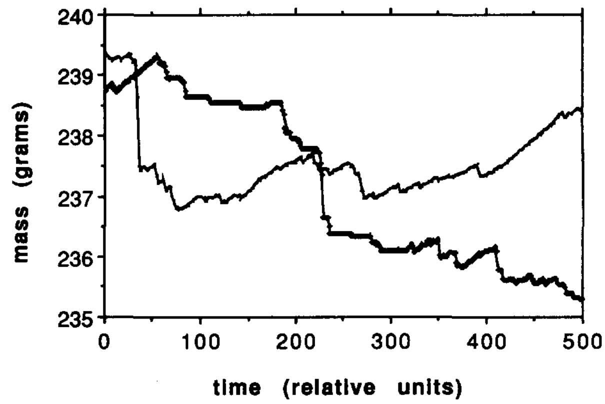

Fig. 2. The mass of the bead pile varies unpredictably with time. These two data runs are typical of the 70 runs reported in this article. While the mass of the initial pile was roughly the same, avalanches of all sizes determined whether the mass would increase or decrease as additional beads were added to the pile. A time unit is the time between bead additions and corresponds to about 1 s.

Our experiment is similar to that done by Held *et al.*:[11] a digital pan balance, interfaced to a computer, continuously monitors the mass of a bead pile as beads are added one at a time. The flow off the pan is recorded as an avalanche whose size is assumed to be related to the amount of lost mass. The number of avalanches producing each mass increment provides a distribution of avalanche sizes that is expected to give a power-law behavior. There are two major assumptions that deserve comment at the outset. The first is that any avalanche will cause beads to fall off the edge; however, it is possible for small avalanches to occur yet stop before reaching the edge. Thus, we expect to undercount small avalanches. The second assumption is that the bead pile is large enough that the surface effects on the bottom layer do not effect the avalanches on the pile. Different pan surfaces were used to investigate surface effects on the relatively small piles occurring in this experiment.

The digital pan balance used in this experiment was a Mettler PJ300 with an internal serial interface board. This balance had a resolution of 1 mg and a capacity of 300 g. The balance was set to "send stable" at 2400 baud, which allowed all stable mass values to be sent to the computer for analysis. An Apple Macintosh SE, using the data acquisition language LabVIEW, collected successive mass values of the pile and saved that data as a disk file in a spreadsheet format.[20] Two data runs are shown in Fig. 2 to illustrate mass of the pile as a function of time; even though both runs start with a similar mass, the piles evolve in very different ways. The data were analyzed using the spreadsheet Wingz and plotted using Cricket Graph, as is described in more detail below.

In order that the beads avalanching off the pile did not obstruct the balance pan, an extension rod was made for the pan which elevated it and allowed a donut-shaped bowl to be underneath the pan to catch the fallen beads. Two different pan sizes were used upon which the bead piles were built: the standard 12.8 cm diam and a smaller 8.0 cm diam pan. Covering the metal pan surface was essential to build a pile. Two types of paper towels were tried as was sandpaper, they all worked well with the sandpaper being the easiest to use. No matter the surface, a critical angle of repose for our piles varied between 20° and 22° (up from the horizontal). The small resulting piles limited the maximum size of an avalanche and could allow the pan's surface covering to influence the distribution of avalanches.

The bead delivery system, which drops individual beads onto the top of the pile, forms an integral part of the experiment. The goal with the bead delivery system was to drop an individual bead from a short (1–2 cm) distance above the top of the pile and then to wait before dropping the next for any avalanches to occur. Two designs were used in obtaining the data reported here. The first was similar to that used by Held *et al.*:[11] A flask with a long neck was rotated in an almost horizontal direction allowing beads to spiral down the neck. Both glass and polystyrene beads were delivered with this system but it had two major faults. The orientation of the flask was critical to having beads spiral out individually and after several hundred beads had been delivered, the beads came out sporadically. The second fault was in occasionally having more than one bead roll out together or in having a bead come out before the previous avalanche had stopped. These problems were overcome by using a different bead delivery system.[21] A long (1 m) tube with an internal diameter between one and two bead diameters was filled with beads and placed vertically above two horizontal disks each of thickness slightly more than one bead diameter. The top disk had a hole the diameter of a bead and rotated the hole under the tube of beads capturing one bead. The bottom disk was stationary and immediately below the top disk. As the bead rolled around in the top disk, it would pass over a hole in the bottom disk and fall onto the pile. Individual beads could be delivered to the pile at a consistent rate that was fast enough to allow for efficient data acquisition yet slow enough to assure avalanche completion. The beads were dropped on the pile at a rate of about 1/s.

Many sets of data were taken under different conditions to test the universality of the phenomena. The bead material, balance pan surface upon which the pile formed, diameter of the base of the pile, and the influence of air drafts were all investigated. A bead pile would be formed by pouring the beads through a funnel onto the balance pan surface, placing the bead delivery system about 1 cm above the top of the pile, allowing several beads to fall onto the pile (to allow large avalanches if the pile was supercritical), then running the data acquisition software which would capture mass values as they changed. Each data run consisted of 380 to 2800 avalanche events that were saved on disk. A large number of avalanches must be observed in order to get "good" statistics on the occurrence of the infrequently occurring large avalanches; the definition of "good" varies but 350–500 avalanche events just allow one to see the power-law relation over one decade of avalanche sizes. The number of avalanches of a given size were determined by using a bin feature in the spreadsheet Wingz which provided the avalanche distribution used in subsequent figures. The distributions are normalized to one so that the fractional occurrence of an avalanche size can be observed and compared. There was no apparent effect on the avalanche distributions from the type of bead delivery

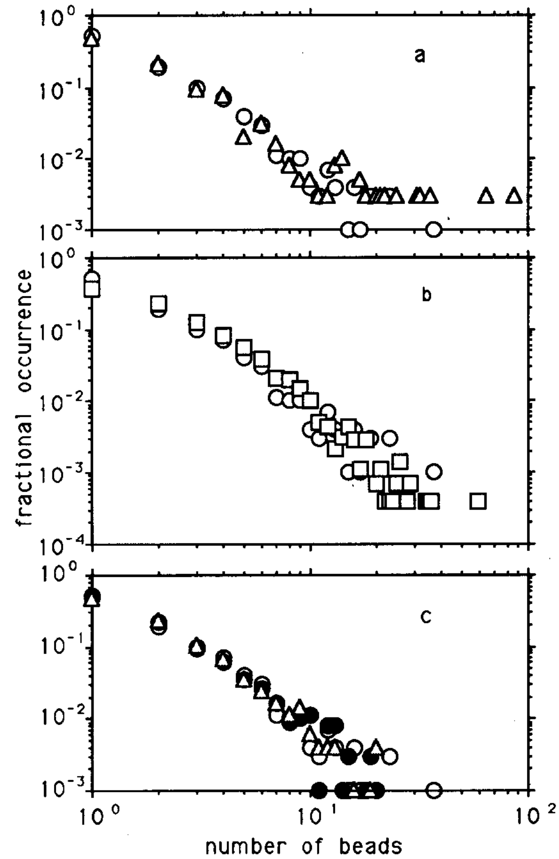

Fig. 3. A log–log plot of the normalized distribution function shows the universality of the avalanche phenomena. Plot (a) compares the iron spheres (triangles) with glass spheres (circles) of the same size and on a balance pan of equal diameter and covering material. Data was taken on a smaller number of bead additions for iron than for other bead materials so that the few large avalanches appear to have a higher fractional occurrence. Plot (b) compares polystyrene (squares) with glass (circles) beads. Using glass beads, a windscreen (dark circles) or no screen (open circles) has no effect on the distribution of avalanches as shown in plot (c), nor does the pan diameter (triangles). The open circles in all three plots are the same data and illustrate the universality of the avalanche distribution: the same shape is observed independent of bead material, pan size, or windscreen.

system used. Other parameters which one might naively expect to cause a difference are described below.

Three different types of beads were used to build sandpiles. Uniform glass spheres[22] with a 3 mm diam were used to take most of the data. Some data was taken with fairly uniform iron spheres (size 4 steel shot for shotgun shells) and with irregularly shaped polystyrene beads.[23] The iron spheres were the same diameter as the glass spheres which allowed the newer single-bead-drop apparatus to be used for both. While the mass of these two types of beads were vastly different, the distribution of avalanche sizes were virtually identical [see Fig. 3(a)]. The light-weight polystyrene beads were ellipsoidal with about a one- to two-millimeter long axis, and the rotating flask was necessary in order to deliver these beads to the pile's top. The polystyrene beads were too light to resolve individual beads and so an avalanche of 10 mg (the average mass of five beads) was used as a "one bead avalanche." The distribution of the resulting avalanche sizes is the same as when glass beads were used [see Fig. 3(b)].

The diameter of the base of the pile and air drafts were

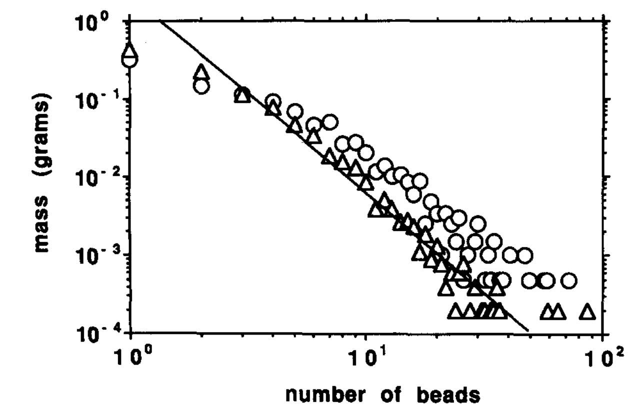

Fig. 4. The covering on the balance pan upon which the bead pile rested seemed to cause fewer small avalanches if a paper towel was used (circles) than if sandpaper was used (triangles). The triangles are the combined distributions of glass, iron, and polystyrene shown in Fig. 3. The straight line is a least-squares fit to the triangles ($N > 3$) and has a slope of -2.5 which is the critical exponent λ. Note that both sets of data have the same slope when larger ($N > 6$) avalanches occurred. The deviation of the line from the data at both extreme ends is due to finite size effects as discussed in the text.

not found to affect the results of our experiments. For all but one run of data, the pile's base diameter was 12.8 cm. For that one run, the diameter was reduced by 40% to 8.0 cm with glass beads forming the pile, but without effect on the distribution of avalanches [see Fig. 3(c)]. A windscreen, that surrounded the entire apparatus for most runs, was removed and the resulting distributions remained unchanged [see Fig. 3(c)]. There was only one parameter which seemed to result in a systematic shift of the distribution: the balance pan's surface upon which the bead pile was formed.

Both sandpaper and paper towels were used as coverings for the balance's pan when glass beads were used to form a pile. The sandpaper base gave consistent results with other bead materials or pile diameters (as illustrated in Fig. 3). However, a paper towel base resulted in fewer small avalanches and a larger proportion of big avalanches (see Fig. 4). It is interesting (and we feel significant) that while the intercept is different for the different surfaces, the slope of the curve in Fig. 4 is the same whichever surface was used. Future experiments might look at this effect in more detail and try a surface with a glued monolayer of beads upon it.

It is remarkable that all of the data runs had a roughly linear dependence when the fractional occurrence was plotted on a log–log plot versus the number of beads in the avalanche (see Fig. 3). Such universal behavior illustrating a power-law relationship is exactly the prediction of SOC. If we take the avalanches to be universal, then the data runs can be combined to give better statistics (less fluctuation) as shown in Fig. 4. The common slope, shown by the straight line in Fig. 4, was determined by a simple least-squares fit to the data and has a value of -2.5 which is the same as observed by Held *et al.*[11] and represents the critical exponent λ. This line also makes apparent the undercounting of small avalanches which can occur in the experiment. However, there is another reason for the one- and two-bead avalanches to be below the line that fits the rest of the data: a finite size effect.

Two finite size effects are important in an experiment of this type. The first is due to the finite size of the beads used to form the pile. Real sandpiles have a large range of particle sizes that leads to virtually continuous avalanches of the finest grains when the pile is near critical; a phenomena quite visible on sand dunes. For such real sandpiles, the avalanche distribution would be of mass, instead of number of beads, and would allow many more small avalanches to be observed. The one- and two-bead avalanche probability we observed is small because of the size of our beads and the limitations that imposes on the avalanches.

The second finite size effect influences the number of large avalanches and is due to the size of the pile. The largest avalanche observable is directly related to the number of beads in the pile so that small piles necessarily limit the observation of large avalanches. While we did not observe any difference in our data as we varied the diameter of the pile's base, such an effect is observable when larger numbers of beads make up the pile, resulting in larger size avalanches being observed. Held *et al.*[11] observed this effect and were able to scale their data onto a universal curve to correct for the size of the pile. Such a scaling is further indication that real "sandpiles" exhibit SOC.

IV. CONCLUSION

Self-organized criticality is a recent theory which holds great potential for unifying a wide range of many-body dynamical phenomena. Even though this theory is new and far reaching, it is still very accessible to undergraduates. The experiment described in this article allows students to observe universal phenomena in a "sandpile" which can be explained by a power-law relationship with a critical exponent consistent with more extensive investigations. While our data are consistent with the predictions of SOC, much larger piles must be observed when many more beads are added sequentially before a definitive test of the theory is possible. Nevertheless, a bead pile is found to be remarkably robust and to illustrate many of the features of self-organized criticality.

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Confusion by representation: On student's comprehension of the electric field concept

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It is argued that university students' diffuse ideas about the two related concepts of force and force field could be due to lack of mastery of the graphical representation of these and related concepts.

I. INTRODUCTION

The idea of interaction at a distance is one of the great contributions to physics made by Isaac Newton. When Michael Faraday introduced the concept of a field that can be graphically represented by field lines we were given a powerful tool for our thinking and communicating about interaction at a distance. But Faraday seemed to have attributed more reality to the field lines than we nowadays find acceptable. Maxwell writes in *Preface to A Treatise on Electricity and Magnetism* 1881:

> Faraday in his mind saw lines of forces traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance; Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

The concepts of field and field lines are sources of confusion among physics students at university level. This fact calls for an educational strategy that not only reflects the inherent theoretical structure but also considers the cognitive difficulties encountered by students. The likely increase in the use of computers and computer graphics in physics education and elsewhere calls for a new form of literacy: the ability to interpret field lines. Our aim is to shed some light on the way students apprehend, comprehend, and use fields and field lines. We will only deal with the interaction between a static electric field and a small point-charge.

II. INTRODUCTORY EXPERIMENT

As an introductory experiment students were given a pen-and-pencil test from the collection of physics problems by Wilson and Hacket[1] consisting of a drawing of field lines in the 2D space between three conductors, A and C being charged, B having no net charge. See Fig. 1.

Question: *The figure above is a perpendicular cut through long, parallel pieces of metal. A and C are charged, B is neutral. There are no currents; the system is stationary. In this figure a number of electric field lines are drawn. Find all the errors in the figure and explain why the field lines cannot be drawn in this way.*

The test was given to 566 second-year students as part of the final examination of a 40-h compulsory nonmajor course in electricity and magnetism at the Royal Institute

