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I. INTRODUCTION

It has been emphasized recently by several authors[1-3] that parametrization of the hadron resonances by Breit-Wigner (B-W) forms result in notoriously long tails. One of us (S.R.D.) also found[4,5] in preliminary studies of isospin-½ nucleon resonance decay to $K\Lambda$ and ηN that the long tails which are associated with the B-W resonance amplitude can cause some trouble. We present here a possible explanation for at least some of the difficulty and a phenomenological modification of the B-W amplitude in the tail region.

II. RESONANCE AMPLITUDE

It has been known for some time that in the immediate resonance region (one half-width on each side of the resonant energy) of an isolated resonance the amplitude can be correctly described by a one level B-W formula.[6] It is also known, but sometimes ignored, that in order to obtain this formula certain simplifying assumptions must be made. These assumptions and the derivation of the resonance amplitude can be found in standard texts.[7] It is generally accepted that the B-W amplitude in the immediate resonance region is adequate. We have no reason to question this region; however, we do question its validity several half-widths away from the peak region. Consider, for example, the assumption that the "hard-sphere" scattering phase shift ξ is small and can be neglected. It appears in a unitary amplitude A as [7]

$$A = e^{2i\xi} \left(\frac{\gamma}{\epsilon - i\gamma} + e^{-i\xi} \sin \xi \right), \tag{1}$$

where γ is the half width $\Gamma/2$, and ϵ is given in terms of the resonant energy and the total c.m. energy by $\epsilon = W_r - W$. Clearly, if $\xi \to 0$ we obtain the usual B-W amplitude. (For our purposes here we need not concern ourselves with inelasticity or energy dependence of Γ.) That A is unitary, can easily be seen by

$$\text{Im } A = |A|^2 . \tag{2}$$

Suppose the phase shift ξ is small, but not zero everywhere. Even if ξ is as large as $\pm 2^0$ or $\pm 3^0$ a very definite effect can be seen in the resonance amplitude. Let us assume that ξ varies linearly with $-\epsilon$ such that $\xi = -5^0$ when $\epsilon = 5\gamma$ and $\xi = +5^0$ when $\epsilon = -5\gamma$. Then the deviation from a B-W form is significant in the tail region as can be seen from Table I. Thus, one can find a possible mechanism for damping (or raising if ξ varies as $+\epsilon$) the resonance tails, namely the presence of nonzero "hard-sphere" phase shifts.

Equivalently, if one objects to positive "hard-sphere" phase shifts, it is possible to consider a variation of ξ from say -185^0 to -175^0 as ϵ goes from 5γ to -5γ. (Note, A is invariant under the transformation $\xi \to \xi - \pi$.) As a point of interest, if one takes literally the expression for ξ from potential scattering[7]

$$\xi_\ell = \tan^{-1} \left(\frac{j_\ell(ka)}{n_\ell(ka)} \right) , \tag{3}$$

where ℓ denotes the ℓ^{th} partial wave and the arguments are the usual spherical Bessel functions, then $\xi \simeq -180^0$ is not really too unreasonable. For if one assumes that the interaction radius a is 1.4 Fermi, and that k represents the c.m. momentum of the incident pion in πp scattering, then a table of the zeros of j_ℓ [8] yields immediately the energy at which ξ_ℓ given by (3) is equal to -180^0.

$$\xi_0 = -180^0 \quad \text{at} \quad W_r = 1505 \text{ MeV}$$
$$\xi_1 = -180^0 \quad \text{at} \quad W_r = 1780 \text{ MeV}$$
$$\xi_2 = -180^0 \quad \text{at} \quad W_r = 2070 \text{ MeV}$$

One cannot help but compare these results with the S_{11} (1535), P_{11} (1780), and D_{13} (2040) nucleon resonances.[9] Our inclination at present, however, is to regard this result as a coincidence, and to consider ξ as a parameter which can vary in the neighborhood of $0 \pm n\pi$ ($n = 0, \pm 1, \pm 2, \ldots$), and thus induce a slight modification in the B-W amplitude.

III. A MODIFICATION

The resonance form in the tail region is still important even when additional effects are taken into consideration, such as inelasticity, energy dependent width, and background effects, all of which influence the resonance circle.[10] It appears that at present the data are not sufficient to determine much more than a possible need for a damping of the B-W resonance tails. In most analyses where resonance forms have to be assumed there are already more parameters than one would like. Therefore, it seems almost hopeless at the present time to introduce still more unknowns and determine the various values of ξ for several different partial waves. Hence, we seek a temporary modification[11] of the B-W amplitude in the tail region which (i) damps out the resonance tails away from the immediate resonance region, (ii) does not introduce new parameters, (iii) is unitary, (iv) is continuous, and (v) is smooth. By the last requirement we mean $dA/d\epsilon$ shall be continuous. An amplitude which satisfies these requirements is

$$A' = \begin{cases} \dfrac{\gamma}{\epsilon - i\gamma} & \text{for} \quad |\epsilon| \leqslant \gamma \\ \\ \dfrac{\gamma}{\pm\gamma e^{\mu} - i\gamma} & \text{for} \quad \epsilon \gtrless \pm\gamma \end{cases} \tag{4}$$

where

$$\mu = \tfrac{1}{2}\left(\frac{\epsilon^2}{\gamma^2} - 1\right). \tag{5}$$

In Table I we compare this amplitude with the normal B-W amplitude A_{BW}.

Further work, some of which is in progress, hopefully will lead to a better understanding of resonances in the tail region.

IV. DISCUSSION

In view of the difficulty with the long tails associated with B-W resonance forms we looked for a mechanism which could account for damping away from the immediate resonance region. We found that only small contributions from the "hard-sphere" phase shifts could cause a significant modification in the usual B-W amplitude. For possible use in current calculations we found a modification of the B-W form in the tail region. Only through further detail study of this region will we be able to determine the actual deviation of the experimental results from a pure

Table I. The Amplitudes

ϵ/γ	ξ	ReA	ImA	ReA_{BW}	ImA_{BW}	ReA′	ImA′
± 5	∓ 5°	± 0.11	0.01	± 0.19	0.04	± 0.00	0.00
± 4	∓ 4°	± 0.17	0.03	± 0.24	0.06	± 0.00	0.00
± 3	∓ 3°	± 0.26	0.07	± 0.30	0.10	± 0.02	0.00
± 2	∓ 2°	± 0.38	0.17	± 0.40	0.20	± 0.21	0.05
± 1	∓ 1°	± 0.50	0.48	± 0.50	0.50	± 0.50	0.50
0	0°	0.00	1.00	0.00	1.00	0.00	1.00

B-W shape.

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REFERENCES

1. A. Donnachie, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968 (CERN, Geneva, 1968).
2. Y. Kohsaka, O. Miyamura, F. Takagi, and K. Itabashi, Lett. Nuovo Cimento 1, 408 (1969).
3. R.P. Bajpai and A. Donnachie, Nuclear Physics B12, 274 (1969).
4. S.R. Deans, Bull. Am. Phys. Soc. II 15, 1374 (1970).
5. S.R. Deans, Bull. Am. Phys. Soc. II 16, 64 (1971).
6. M. Gell-Mann and K.M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).
7. P. Roman, Advanced Quantum Theory (Addison-Wesley Publishing Co., Reading, Mass., 1965).
8. P.M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill Book Co., New York, 1953) p. 1576.
9. "Review of Particle Properties," Physics Letters 33B, 1 (1970).
10. R.H. Dalitz, Ann. Rev. Nucl. Sci. 13, 339 (1963).
11. We wish to thank Professor A.E.S. Green for calling our attention to the fact that exponential factors in the line shape are important in other areas. See A.J. Bennett, C.B. Duke, and S.D. Silverstein, Phys. Rev. 176, 969 (1968).